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Common rail injection system controller design using input-to-state linearization and optimal control strategy with integral action

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***Abstract*— The goal is to develop a controller for a common rail injection system where the calibration process for the implementation is systematic and simpler. A 0D nonlinear model of the common rail system is designed. The common rail model contains strong nonlinearities which might be hard to handle from the control point of view. In order to overcome this constraint, an input-to-state linearization is applied to the common rail nonlinear model. This procedure yields with a virtual linear system, where two optimal Linear Quadratic Regulator LQR strategies are applied. Very good results are obtained, the calibration process is significantly simplified and the controller stability is ensured.**

I. Introduction

In the common rail systems, a high-pressure pump allows storing fuel in a distribution rail at high pressure and the goal is to control the injection advance, duration and pressure, in order to manage accurately the combustion depending on the engine operating conditions. Very advanced controllers for the common rail system are presented by Gauthier in his PhD dissertation and in several publications: [5], [6], [8], [7] and [9]. As well, the works of [2], [1], [3] and [4] are very representative approaches for the common rail injection system.

In spite of the existence of such complex and complete common rail control strategies, a proportional-integrative (PI) controller is traditionally used to control the common rail system in the industry. A main problem of this strategy is that it is difficult to find the PI parameters for the whole operating range, such process requires on an expensive and non systematic calibration process. Besides, the PI does not guarantee the robustness and stability of the common rail system which implies that it might not be reliable in some operating conditions.

With the aim to develop a common rail controller to be implemented in the current industry, where the calibration process is simplified and the stability can be ensured, a model based control strategies are proposed. This strategy is based in a nonlinear common rail system model, which is transformed with an input-to-state linearization into a virtual linear system, where two LQR controllers are synthesized. This strategy leads to an advanced control law which has the advantage to be simple to implement. To validate the results, an industrial-like methodology is used, thus the pressure reference obtained through data maps is used.

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II. Common rail injection system

A common rail system is composed by several devices arranged in two main circuits: a low pressure (LP) circuit ($4-10$ bar) and a high pressure (HP) circuit ($300-1600$ bar for diesel engines and $20-200$ bar for gasoline engines). Figure 1 presents the common rail system scheme used in this work.

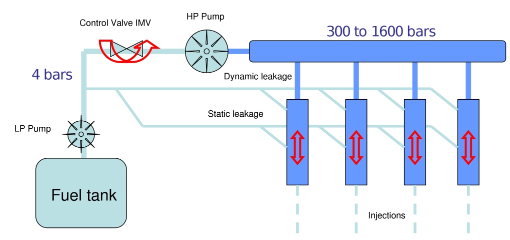


Fig. 1: Scheme of the common rail system.

The HP pump boosts the fuel from the LP circuit to the HP circuit. The fuel flow passing through the inlet metering valve IMV is the variable that controls the pressure in the rail. The rail is a distribution pipe where the fuel is stocked before the injection. The injectors provide the amount of fuel required per cycle to the engine depending on the injection timing and the actual rail pressure p_{rail}. Two leakages are associated to the injectors; *static leakage* and *dynamic leakage*. The static leakage is used to overcome functioning failures and/or to regulate the rail during the slow driving, it guarantees a minimum p_{rail} when the flow through the IMV is too low to drive the HP pump. The dynamic leakage guarantees the functioning of the injector needle. Those leakages are modeled through static data maps, which do not depend on the system control inputs.

The problem to handle is to ensure the appropriate rail pressure p_{rail} to get the accurate amount of fuel injected into the engine in all the operating conditions. The pressure that ensures such performance is given by a pressure reference p_{ref}, which is mapped with respect to several parameters, where the most relevant are the engine speed N and the torque demand.

A. 0D common rail injection model

Based on the architecture shown in Figure 1, a model for the common rail injection system is proposed. The model has one state that is p_{rail} and one control input $f_{q_{pump}}(N, p_{rail}, I_{imv})$, which is the mass flow passing through the IMV and the HP pump, which is driven by the IMV control electric current I_{imv} and also depends on the engine speed N in $r.p.m.$ The dynamics of the rail pressure is obtained through a mass balance in the rail and is given by:

$$\begin{aligned} \frac{dp_{rail}}{dt} &= \Gamma_{rail}\frac{\beta_f}{V_{rail}}\Big(f_{q_{pump}}(N, p_{rail}, I_{imv}) \\ &- f_{l_{dyna}}(N, p_{rail}, T_{inj}) - f_{l_{stat}}(N, p_{rail}, T_{inj}) \\ &- f_{q_{inj}}(N, p_{rail}, T_{inj})\Big) \end{aligned} \quad (1)$$

where V_{rail} is the rail volume in m^3 and Γ_{rail} is a unit conversion constant. β_f is the fuel compressibility coefficient in Pa. This coefficient depends on the pressure and temperature of the rail. However, this parameter is considered as constant to simplify the model, and the deviations due to this assumption are mitigated with the LQR controller, which is intrinsically robust against model uncertainties. $f_{l_{dyna}}(N, p_{rail}(t), T_{inj}(t))$ is the volumetric flow of the dynamic leakage, $f_{l_{stat}}(N, p_{rail}(t), T_{inj}(t))$ is the volumetric flow of the static leakage and $f_{q_{inj}}(N, p_{rail}(t), T_{inj}(t))$ is the volumetric injected fuel flow. All the volumetric flows are in m^3/s. $T_{inj}(t) = f_{T_{inj}}(p_{rail}(t), m_{inj})$ is the injection timing, where m_{inj} is the fuel mass to be injected per cycle. All these functions are data maps that represent the physical behavior of the different devices in the common rail system. All the data maps are bijective statical function. The data maps are obtained from the information of the devices supplier and experimental calibration.

III. INPUT-TO-STATE LINEARIZATION TRANSFORMATION

Because of the inherent complexity and nonlinear character of system (1), the control synthesis is not easy and may be involved. A transformation of the nonlinear system into a virtual linear system is used, allowing the design of a linear control strategy. Indeed an input-to-state linearization is used to cancel out the nonlinearities of the system dynamics which results in a virtual linear controllable system.

A. Brief background on Input-to-state linearization [10]

Definition 1: A nonlinear system:

$$\dot{x} = f(x) + G(x)u \quad (2)$$

where $f : D_x \to \mathbf{R}^n$ and $G : D \to \mathbf{R}^{n\times p}$ are sufficiently smooth on a domain $D_x \subset \mathbf{R}^n$, is said to be input-state linearizable if there exists a diffeomorphism $T : D_x \to \mathbf{R}^n$ such that $D_z = T(D_x)$ contains the origin and the change of variables $z = T(x)$ transforms System (2) into the form

$$\dot{z} = Az + B\beta^{-1}(x)(u - \alpha(x)) \quad (3)$$

where $A \in \mathbf{R}^{n\times n}$, $B \in \mathbf{R}^{n\times p}$, and such that the pair (A, B) is controllable, and the functions $\alpha(x) : \mathbf{R}^n \to \mathbf{R}^p$ and $\beta(x) : \mathbf{R}^n \to \mathbf{R}^{p\times p}$ are defined in a domain $D_x \subset \mathbf{R}^n$ that contains the origin. The matrix $\beta(x)$ is assumed to be non-singular for every $x \in D_x$. Notice that β^{-1} denotes the inverse of the matrix $\beta(x)$ for every x, and not the inverse map of the function $\beta(x)$.

The basic idea behind using the input-state linearization is to find a control input u such that the system nonlinearities are compensated for, resulting in a controllable linear system. Indeed, for system (3) a state feedback is chosen as:

$$u = \alpha(x) + \beta(x)v, \quad (4)$$

then system (2) is rewritten as the linear system:

$$\dot{x} = Ax + Bv \quad (5)$$

B. Common rail model transformation

As seen in section II, the dynamics of p_{rail} is controlled varying the mass flow through the IMV, thus the system input can be defined as:

$$u = f_{q_{pump}}(N, p_{rail}, I_{imv}) \quad (6)$$

One can define the nonlinear function $\alpha(p_{rail})$ such that:

$$\begin{aligned} \alpha(p_{rail}) &= f_{l_{dyna}}(N, p_{rail}, T_{inj}) \\ &+ f_{l_{stat}}(N, p_{rail}, T_{inj}) + f_{q_{inj}}(N, p_{rail}, T_{inj}) \end{aligned} \quad (7)$$

and using the following equivalences, which match the components of Equation (3):

$$z = p_{rail}, \quad B = \Gamma_{rail}\frac{\beta_f}{V_{rail}} \quad \beta(x) = 1, \quad A = 0 \quad (8)$$

System (1) becomes:

$$\frac{dp_{rail}}{dt} = \Gamma_{rail}\frac{\beta_f}{V_{rail}}\Big(u - \alpha(p_{rail})\Big) \quad (9)$$

which presents the same form specified in Definition 1 in Equation (3). Thus, according to Equation (4), the system state feedback is:

$$u = \alpha(p_{rail}) + v \quad (10)$$

and the transformed linear system is:

$$\frac{dp_{rail}}{dt} = \Gamma_{rail}\frac{\beta_f}{V_{rail}}v \quad (11)$$

where v is the system virtual input. In this work it is assumed that the fuel compressibility coefficient β_f is constant. Under this assumption, System (11) is linear time invariant.

The common rail system is controlled with the mass flow passing through the IMV and therefore through the HP pump $f_{q_{pump}}$ (Equation (6)). Indeed, the mass flow itself cannot be set directly and the HP pump is actually controlled through the IMV control electric current I_{imv}. To obtain an analytical expression for I_{imv} it can be assumed the existence of the function $f^{-1}_{q_{pump}}(N, p_{rail}, f_{q_{pump}})$ such that:

$$I_{imv} = f^{-1}_{q_{pump}}(N, p_{rail}, f_{q_{pump}}) \tag{12}$$

This assumption is realistic while a bijetive relation between the I_{imv} current and q_{pump} exists and the data map is statical. From equations (7) and (10), the virtual input v is:

$$v = f_{q_{pump}} - f_{l_{dyna}}(N, p_{rail}, T_{inj}) \tag{13}$$
$$- f_{l_{stat}}(N, p_{rail}, T_{inj}) - f_{q_{inj}}(N, p_{rail}, T_{inj})$$

thus, using Equation (13), a relation between the virtual control input v and the system actual input I_{imv} is obtained as:

$$I_{imv} = f^{-1}_{q_{pump}}\Big(N, p_{rail}, v + f_{l_{dyna}}(N, p_{rail}, T_{inj})$$
$$+ f_{l_{stat}}(N, p_{rail}, T_{inj}) + f_{q_{inj}}(N, p_{rail}, T_{inj})\Big) \tag{14}$$

Thanks to the input-to-state linearization, the linear system (11) is designed and different linear control strategies can be used to find v to control p_{rail}. The actual system input I_{imv} can be obtained from the virtual input v through the transformation (14). In this work, two optimal LQR control approaches are developed.

IV. Common rail controller: Linear Quadratic Regulator (LQR) application

The common rail control problem is that p_{rail} should track p_{ref}. Two LQR controllers have been considered: an LQR tracking controller (feedforward) with integral action referred to as the LQR tracking and an LQR with integral action referred to as the LQR. The main difference on both controllers lies on the feedforward:

- In the LQR tracking, the pressure reference p_{ref} is considered as an exogenous input for the system, referred to as the feedforward. Thus, the controller follows the changes in the reference tracking the trajectory of the error and of the reference itself.
- In the LQR, the feedback is the error between the pressure reference and the rail pressure $p_{rail} - p_{ref}$, thus the controller follows the changes in the reference only tracking the trajectory of the error.

Remark 1: The development and proof of these controllers have been extensively reported in the literature, thus only the application results of these controllers are presented.

A. LQR controllers application

The interest is that p_{rail} reaches the pressure reference p_{ref} according to the specifications in Table I, through a controller which guarantees the stability in all operating conditions and which has a simpler calibration process.

zero steady-state error	$p_{rail}(\infty) = p_{ref}$
overshoot	$< 5\%$
$p_{rail_{max}}$	$1600\ bar$
settling time	$< 1\ s$
$p_{rail} - p_{ref}$ tolerance	$\pm 25\ bar$ transitory (after the rise time), $\pm 8\ bar$ stable

TABLE I: Dynamics specifications for the common rail system controller.

1) LQR tracking with integral action: Given the common rail system (11), the new state $e = \int(p_{ref} - p_{rail})$ is defined. The goal of this term is to add an integral action to the controller to obtain zero steady state error. Including the new state, the extended state space system is:

$$\dot{x_e} = A_e x_e + B_e v + \Omega_e p_{ref}, \quad x_e = \begin{bmatrix} p_{rail} \\ e \end{bmatrix} \tag{15}$$

where $A_e = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}$, $B_e = \begin{bmatrix} \Gamma_{rail}\frac{\beta_f}{V_{rail}} \\ 0 \end{bmatrix}$ and $\Omega_e = \begin{bmatrix} 0 & 1 \end{bmatrix}^T$.
The cost function to minimize is:

$$J = \frac{1}{2}\int (x_e^T Q x_e + R v^2) dt \tag{16}$$

Q and R are parameters of the controller. In this case, these parameters are chosen such that the weight is on the extended state e in order to decrease the error between p_{rail} and p_{ref}, and giving the necessary weight to v such that the feedforward term has the desired effect on the controller.
The problem of finding a feedback controller v for system (15) subject to the cost function (16) has the solution:

$$v = -K x_e - K_r p_{ref} \tag{17}$$
$$K := \begin{bmatrix} K_p & K_i \end{bmatrix}^T = R^{-1} B_e^T P,$$
$$K_r = R^{-1} B_e^T \left(A_e^T - P B_e R^{-1} B_e^T\right)^{-1} P \Omega_e$$

where $P = P^T > 0$ is the solution of the Control Algebraic Riccati Equation for the system. The closed loop controller is:

$$\dot{x}_e = (A_e - B_e K) x_e + K_r p_{ref}, \quad x_e(0) = x_{e_0} \tag{18}$$

The virtual controller v is the control input for system (11), thus the transformation in Equation (14) is performed to obtain I_{imv} that is used in the common rail system control input $u = f_{q_{pump}}(N, p_{rail}, I_{imv})$.
The controller architecture is displayed in Figure 2. In the figure, two main elements are distinguished. Enclosed in dotted lines the LQR tracking which gives the virtual input v. This input is transformed through the input-to-state linearization block into the input $u = f_{q_{pump}}(N, p_{rail}, I_{imv})$,

which is used to obtain the actual system input I_{imv}. The feedforward term is observed as an exogenous input in the system, controlled by the gain K_r.

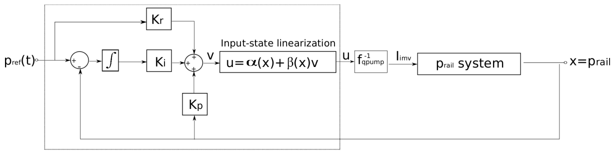


Fig. 2: Optimal tracking control with integral action.

2) LQR with integral action: Similarly to the previous case, the new state $e = \int(p_{ref} - p_{rail})$ is defined. Given the system (11) and the new state e, the extended state space system is:

$$\dot{x}_e = A_e x_e + B_e v_e, \quad x_e = \begin{bmatrix} p_{rail} \\ e \end{bmatrix}, \qquad (19)$$
$$v_e = \begin{bmatrix} v \\ p_{ref} \end{bmatrix}$$

where $A_e = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}$ and $B_e = \begin{bmatrix} \Gamma_{rail}\frac{\beta_f}{V_{rail}} & 0 \\ 0 & 1 \end{bmatrix}$ and v is defined in equation (13). In theory, a feedforward term exists as long as p_{ref} is included in v_e. However, the cost function is designed as:

$$J = \int \frac{1}{2}(x_e^T Q x_e + v_e^T R v_e) \qquad (20)$$

R is chosen such that the weight of p_{ref} in v_e is large enough so that the feedback gain of the input p_{ref} in the controller is almost 0 and can be neglected. Thus, only a gain for v must be defined for the system virtual input instead of a gain for v_e.

Taking into account these precisions, the problem of finding a feedback controller for (19) subject to the cost function (20) has the solution:

$$v = -R^{-1} B_e^T P x_e = -K x_e, \qquad (21)$$
$$K =: \begin{bmatrix} K_p & K_i \end{bmatrix}^T = R^{-1} B_e^T P$$

where $P = P^T > 0$ is the solution of the Control Algebraic Riccati Equation for the system. The control input v is defined similarly to (21) and the closed loop controller is:

$$\dot{x}_e = (A_e - B_e K)\, x_e, \quad x_e(0) = x_{e_0} \qquad (22)$$

The virtual controller v is the control input for system (11), thus the transformation in Equation (14) is performed to obtain I_{imv} that is used in the common rail system control input $u = f_{q_{pump}}(N, p_{rail}, I_{imv})$.

The controller architecture is similar to the displayed in figure 2, but the main difference is that in the case of the LQR tracking controller, there is an additional term K_r in the feedback, which is the feedforward term that multiplies the controller v.

V. LQR CONTROLLER SIMULATION RESULTS

Using the model presented in II-A, the common rail injection system (1) is simulated, with the two developed LQR controllers and a traditional PI. The parameters Q and R are calibrated as a trade-off between the system requirements and the controller response, and are chosen according to the specifications mentioned in Section IV-A. For the LQR tracking:

$$Q = \begin{bmatrix} 10000 & 0 \\ 0 & 25 \times 10^3 \end{bmatrix}, \quad R = 10 \qquad (23)$$

and for the LQR controller:

$$R = \begin{bmatrix} 1 \times 10^{-3} & 0 \\ 0 & 1000 \end{bmatrix} \quad Q = \begin{bmatrix} 0.1 & 0 \\ 0 & 10 \end{bmatrix} \qquad (24)$$

First, the step response of the controllers is analyzed and presented in Figure 3 together with the error. The results with respect to the controller requirements presented in Table I are presented in Table II.

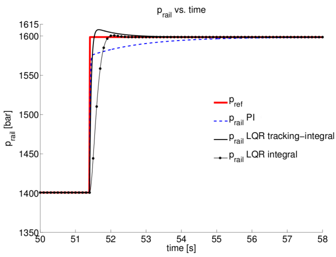
The three approaches achieve an adequate response with similar characteristics. Some differences are presented between the three approaches. The LQR controller has a slower rise time for abrupt changes in p_{ref} than the other two controllers during the transitory, thus the maximum error with respect to the reference is bigger as depicted in Figure 3b. This is due to the fact that the LQR controller has a smother response when compared to the other two methods. However, the mean error in this approach remains below 0.2% which corresponds to the same mean error range than the other two approaches. Besides, the settling time of the PI strategy is longer than the LQR strategies.

	Overshoot	$p_{rail_{max}}$	Settling time
Specification	$< 5\%$	1600 bar	$\leqslant 1\ s$
LQR tracking	4%	1608 bar	1 s
LQR	0	1600 bar	1 s
PI	0	1600 bar	7 s

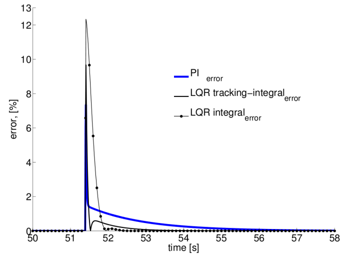
	$p_{ref} - p_{rail}$ tolerance		mean error
Specification	Transitory ($\pm 25\ bar$)	Stable ($\pm 8\ bar$)	
LQR tracking	40 bar	$< 1\ bar$	0.11%
LQR	15 bar	$< 1\ bar$	0.17%
PI	15 bar	8 bar	0.12%

TABLE II: Common rail controller results.

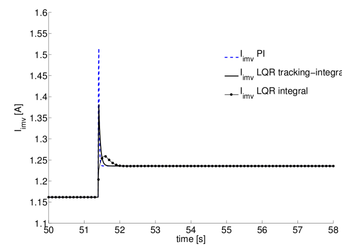
The LQR tracking controller has a smaller transient with respect to abrupt changes in the reference when compared to the LQR controller and the PI. However, it also has a steeper response, which slightly overpasses the maximum pressure 1600 bar and might produce a saturation in the actuator, in the other hand, the LQR and the PI do not have overshoots.



(a) p_{rail} as a function of time for all the control law.

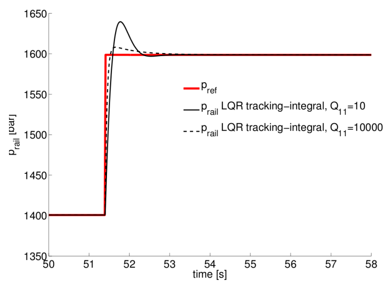


(b) Error $\frac{|p_{rail}-p_{ref}|}{p_{ref_{max}}}$ as a function of time for all the control law.

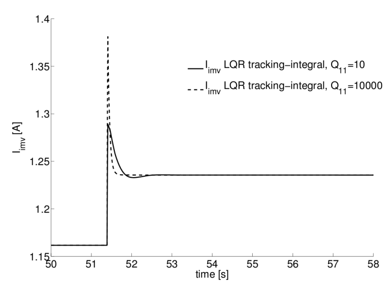


(c) I_{imv} as a function of time for all the control law.

Fig. 3: Step response for the common rail injection pressure controller.

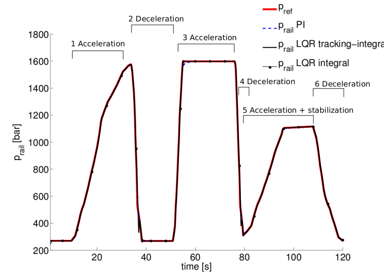


(a) Step response. p_{rail} as a function of time for the LQR tracking controller

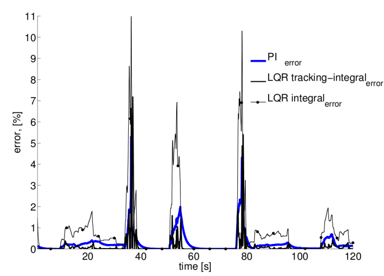


(b) Step response. I_{imv} as a function of time for the LQR tracking controller

Fig. 4: Step response of the common rail injection pressure LQR tracking controller for two different Q parameters.



(a) p_{rail} as a function of time for all the control law.



(b) $\frac{|p_{rail}-p_{ref}|}{p_{ref_{max}}}$ as a function of time for all the control law.

Fig. 5: Simulation results for the common rail injection pressure controller.

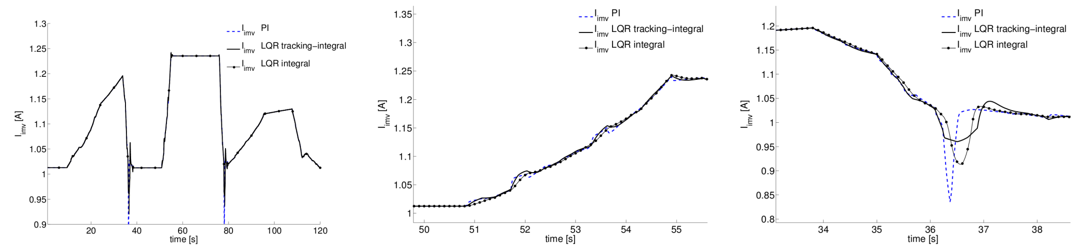
Figure 3c shows the I_{imv} signal of the step response. As it is observed, the PI has the bigger current peak, followed by the LQR tracking controller. Differently, the LQR controller has a considerable smother input signal behavior with respect to the other two approaches.

The parameters Q and R can be adapted to a specific response requirement. For instance, if a smaller transient time is required for the LQR tracking controller, decreasing the first element in the parameter matrix Q (Equation (23)) to 10, decreases the weight of the signal pressure with respect to the cost function (16), which results on a slightly smaller transient time as depicted in Figure 4a. However, it also increases the overshoot up to 20%, which is out of the bounds of the controller requirement ($1600\ bar$). As well, this change decreases the peak magnitude of I_{imv} from $1.38\ A$ to $1.28\ A$ as depicted in Figure 4b.

A second simulation experience is performed using the common rail pressure reference p_{ref}, which is obtained with experimental data maps, where the inputs are the torque demand represented by the mean effective cylinder pressure $IMEP$ and the engine speed N. The engine speed is varied from $700\ rpm$ up to $4000\ rpm$ and p_{ref} varies from $250\ bar$ up to $1600\ bar$. Phases of acceleration followed by phases of deceleration are tested, they are depicted in Figure 5. Main events are presented in phases 2 and 3 :

- Phase 2. Deceleration from $33\ s$ up to $50\ s$. Strong deceleration, between $38\ s$ up to $50\ s$ the rail pressure is forced to its minimum value.
- Phase 3. Acceleration from $50\ s$ up to $76\ s$. Strong acceleration, between $55\ s$ up to $76\ s$ the rail pressure is forced to its maximum value.

From the simulation results shown in Figure 5 it is observed that all the controllers have a similar good tracking performance. The mean error in all the approaches remain lower



(a) I_{imv} as a function of time for all the control laws.

(b) I_{imv} as a function of time for all the control laws, zoom between 50 s to 55 s.

(c) I_{imv} as a function of time for all the control laws, zoom between 33 s to 38 s.

Fig. 6: Simulation results for the IMV control current I_{imv}.

than 0.5%.
The control signal I_{imv} shown in Figure 6a is also analyzed. As illustrated, I_{imv} seems to be alike on the three cases, but some differences can be appreciated. When using the PI, there are phases where the control signal overpasses the other two control signals, suggesting important differences in the transitory response. To give more details, figures 6b and 6c show a close up of Figure 6a in the peaks region. The PI control signal is less smooth compared with the LQR approaches. It confirms the observations from Figure 3c, where the PI current signal overpasses significantly the other controllers. These abrupt peaks current might imply more energy consumption with respect to the power delivered in the solenoid coils of the IMV and a reduction on the actuator's lifetime, because unnecessary power consumption diminishes the valve performance and lowers the valve operating life.
Besides the performance differences remarked until now, the most important difference with respect to the PI lies on the controller synthesis. The LQR controllers are computed solving the Control Algebraic Ricatti Equation (CARE) which guarantees the system stability. Moreover, in the LQR algorithm only the weighting factors Q and R must be calibrated, it can be performed through a simulation process where the designer judges the produced *optimal* controller adjusting the weighting factors to get a controller more in line with the specified design goals. Differently from the PI, those weighting factors are computed once and remain constants for all the operating range and do not compromise the controller stability. Adjusting these parameters empirically is a common procedure in control design.

VI. Conclusions

A common rail injection system model has been designed to develop a control strategy to track the rail pressure. As the common rail model contains strong nonlinearities, an input-to-state linearization strategy has been applied to obtain a virtual linear system, which is controlled by a virtual input. A linear control strategy is performed in the linear system and the virtual input is computed. Using input-to-state linearization makes possible to cancel out the nonlinearities of the injection system and to have a linear controllable system which can be used to create linear control laws.
When applying feedback linearization and the LQR approach, a unique control law has to be created for the whole operating range contrary to the PI strategy which usually needs of a non systematic process to obtain a set of data maps of PI gains to be adjusted.

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