Robust Controlled Invariance for UFAD Regulation

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Model description

The 0-D model for the temperature variations in each room is derived from the energy and mass conservation equations.

$\frac{dT_i}{dt} = \sum_j a_{i,j}(T_j - T_i)$	Conduction through walls between room i and neighbors j
$+b_i u_i (T_u - T_i)$	Controlled input: mass flow rate u_i from the underfloor fan
$+\sum_j \delta_{d_{ij}} c_{i,j} * h(T_j - T_i)$	Open doors: temperature gradient create a flow (hot→cold)
$+\delta_{s_i} d_i (T_{s_i}^4 - T_i^4)$	Radiation from heat sources (lamps) of temperature T_{s_i}

- $a,\ b,\ c,\ d > 0$;
- $\delta_s,\ \delta_d$: discrete state of the disturbances (heat sources and doors);
- $h(x \leq 0) = 0,\ h(x > 0) = x^{3/2}$: heat transfer of the door only appears in the colder room;
- detailed description on establishing this model in [2,3].

Process identification

We want to adapt the theoretical model to the measured behavior of our experiment. We run several experiments on the flat to capture the main behaviors modeled in our equation:

- switch the state of a fan, door or lamp;
- wait for an equilibrium;
- switch another element.

Conditions

- Outside temperature: $T_o \approx 30\,°C$;
- underfloor regulation: $T_u = 17\,°C$;
- actuation (fans, doors, lamps) and measurements: LabVIEW™;
- processing data: MATLAB®.

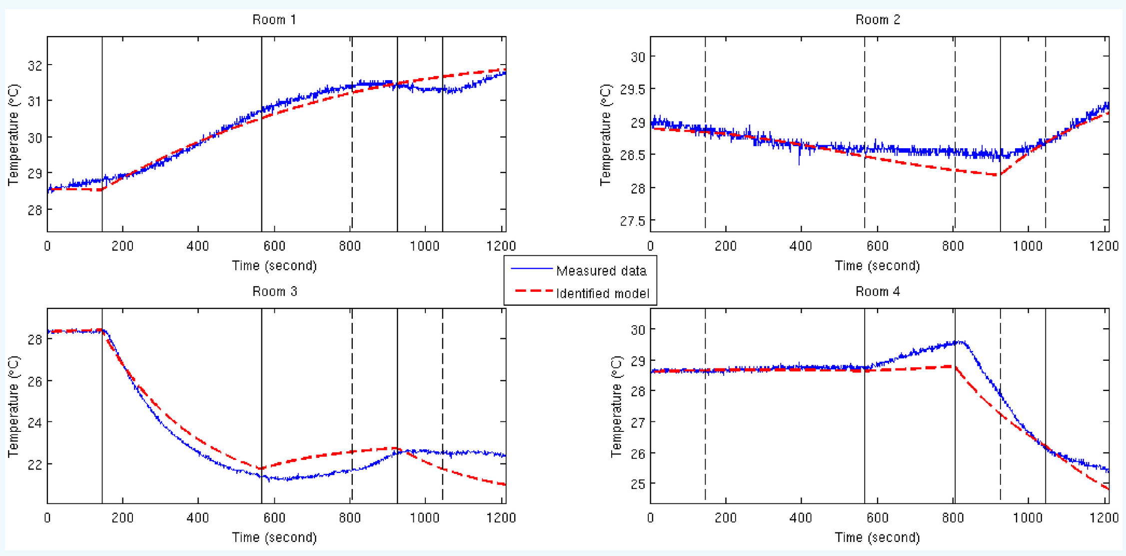
Optimization problem

- 40 parameters (10 per room);
- Recursive Least Squares;
- Initialization based on known physical parameters and observations.

Model evaluation

The identified model is evaluated on an experimental scenario:

- $t = 150s$, lamp 1 and fan 3 on;
- $t = 570s$, lamp 3 on, door $1-4$ open;
- $t = 810s$, fan 4 on;
- $t = 930s$, lamp 3 off, door $1-2$ open;
- $t = 1050s$, door $1-4$ closed.



Data set: 1211 points.
Mean squared error: $0.18\,°C$.
Standard deviation: 0.42.

Monotonicity

Our model verifies the monotonicity property described in [1]. In a monotone system, "each pair of variables may affect each other in either positive or negative forms".

An *increase* in $\frac{dT_i}{dt}$ can be obtained with:

- an *increase* in $T_{j\neq i},\ \delta_{s_i},\ \delta_{d_{ij}}$;
- a *decrease* in the input air flow u_i.

This property allows to focus only on the extremal values of each variable.

Robust Controlled Invariance

RCI = ability to control the system so the state stays in an interval $[\underline{T}, \overline{T}]$, for any value of the external conditions.

Monotonicity ⇒ *RCI* characterized by the sign of the vector field f in the worst conditions of the state (T) and disturbances (w, δ) with the best control (u):

$$\begin{cases} f(\overline{T}, \overline{u}, \overline{w}, \overline{\delta}) \leq 0 \\ f(\underline{T}, 0, \underline{w}, \underline{\delta}) \geq 0. \end{cases}$$

These inequalities define two subspaces in which the boundaries of an interval have to be chosen to ensure the *Robust Controlled Invariance* in this interval.

UnderFloor Air Distribution

The temperature regulation is done by cooling down the air in the underfloor plenum, which is sent into each room of the flat using fans.

This small-scale experiment of a flat equipped with *UnderFloor Air Distribution* is built at the physics department (UFR PHITEM) of University Joseph Fourier, Grenoble, France.

Control implementation

Conditions and interval choice:

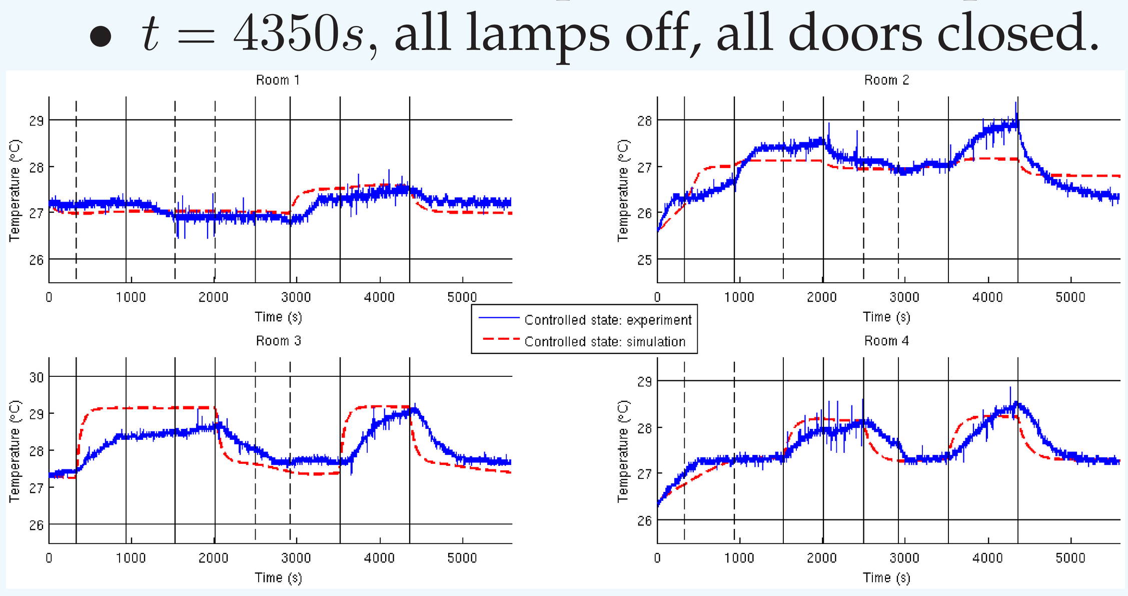
- Outside temperature $\in [27, 30]$;
- Underfloor temperature $\in [17, 21]$;
- $\overline{T} = [29, 28, 30, 29]$;
- $\underline{T} = [26, 25, 26, 26]$.

Decentralized Linear Saturated Controller on the fan voltage:

$$\begin{cases} T_i \leq \underline{T_i} & \Rightarrow V_i = 0 \\ T_i \in [\underline{T_i}, \overline{T_i}] & \Rightarrow V_i(T_i) = \overline{V_i} * \frac{T_i - \underline{T_i}}{\overline{T_i} - \underline{T_i}} \\ T_i \geq \overline{T_i} & \Rightarrow V_i = \overline{V_i} \end{cases}$$

Disturbance schedule:

- $t = 330s$, lamps 2 and 3 on;
- $t = 930s$, doors $1-2$ and $2-3$ open;
- $t = 1530s$, lamp 4 on, door $3-4$ open;
- $t = 2010s$, lamp 3 off, doors $2-3$ and $3-4$ closed;
- $t = 2500s$, lamp 4 off, door $4-1$ open;
- $t = 2910s$, lamp 1 on, door $4-1$ closed;
- $t = 3510s$, all lamps on, all doors open;
- $t = 4350s$, all lamps off, all doors closed.



Even in the extremal conditions, the controlled system (blue data) is indeed invariant in the chosen interval (horizontal lines).

Bibliography

[1] D. Angeli and E.D. Sontag, Monotone Control Systems, *IEEE Transactions on Automatic Control*, Vol. 48, No. 10, pp 1684-1698, 2003.
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[3] E. Witrant, P. Di Marco, P. Park and C. Briat, Limitations and performances of robust control over WSN: UFAD control in intelligent buildings, *IMA Journal of Mathematical Control and Information*, Vol. 27, No. 4, pp 527-543, 2010.