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Further investigations on energy saving by jet impingement in bread baking process ⋆

X.-D. Li * **M. Alamir** * **E. Witrant** * **G. Della-Valle** **
O. Rouaud *** **L. Boillereaux** *** **Ch. Josset** ****

* *CNRS, UJF, INPG, Gipsa-lab, University of Grenoble. 11 rue des Mathmatiques, 38402 Saint-Martin d'Hères, France*
** *INRA, UR 1268 Biopolymère Interactions et Assemblage (BIA). 44316 Nantes, France*
*** *LUNAM Université, ONIRIS, CNRS, GEPEA, UMR 6144. Rue de la Géraudière, 44322 Nantes, France*
**** *Laboratoire de Thermocinétique de Nantes, UMR 6607, Polytech Nantes. Rue C. Pauc, 44306 Nantes, France*

Abstract: This paper presents a study of the estimation of potential energy saving in bread baking process using the jet impingement technology. This new technology is developed to increase the heat transfer efficiency during the baking process. Based on a mechanistic heat exchange model identified in the past work, a non convex optimization problem is formulated taking account of a non zero energy cost related to the new technology. The simulation result shows that one can expect to obtain up to 12% of energy saving under some reasonable assumptions.

Keywords: Bread Baking, Energy Saving, Non Linear Programming.

1. INTRODUCTION

This work is a part of a national French research grant that aims at investigating new energy-saving technologies in French bakery. The technology considered in the present work is the jet impingement technique which increases the heat transfer efficiency between the bread crust and the oven atmosphere.

In this paper the study is focused on the estimation of energy efficiency in bread baking process using impinging jet. One aims at optimizing the process with different jet impingement related energy costs. In the past work of [Alamir et al., 2012] a simple and faithful mechanistic model is experimentally identified and validated over the operational range of interest. The main thermal characteristic profiles, such as temperature and water content on the surface and inside the bread during the baking process, can then be used in the optimization process.

The basic assumption used all through this paper is : reproducing the temperature profile of the crust ensures obtaining the main properties of the resulting bread, regardless our understanding of complex phenomena that take place during the baking process.

In the simplest case where the jet impingement related energy cost is free, a Linear Programming (LP) problem is formulated to express the tracking of the crust temperature. This allows us to determine the upper bound of potential energy saving. At the second step, a nonlinear optimization problem is considered in the case where the jet impingement technology has a small energy cost but not negligible compared to the basic heating process.

The paper is organized as follows: the identified thermal model of a standard oven is presented in section 2, followed by the explanation of the jet impingement technology and the basic assumption mentioned above. In section 3 an optimization problem with nonlinear constraints is formulated to compute the upper bound of energy saving using the new technology. The problem is then transformed into an equivalent nonlinear optimization problem followed by its discretized problem in standard form. Section 4 is devoted to some explanations and comments on the numerical results for both cases where jet impingement related energy cost is free or significant.

2. DYNAMIC MODEL FOR CONTROL

In this section, the dynamic model is introduced taking account of the jet impingement effect. Some brief recalls are given in order to understand the role of the jet impingement technique in the thermal model.

2.1 Thermal model of a standard oven

Consider a standard french baking oven. The thermal model of oven temperature T_3 is described by:

$$\dot{T}_3 = -\gamma_l \cdot T_3 + \gamma_h \cdot u_1 \tag{1}$$

with the heating actuator set-point $u_1 \in [0, 1]$ as a control input and γ_l, γ_h the experimentally identified parameters. The linear differential equation (1) is easy to solve and

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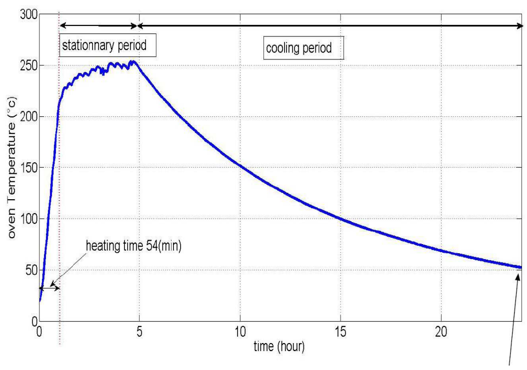

Fig. 1. Temperature evolution profile in BONGARD oven with a scenario of heating-stabilization-cooling phases. The heat control $u_1 = 1$ is applied during the heating phase and $u_1 = 0$ during the free-cooling phase.

the explicit solution will be useful in the resolution of the optimization problem which will be stated later.

The thermal parameters can be identified using the oven temperature evolution profile presented in Fig. 2.1. The data is provided by the experiment which took place in a widely used BONGARD oven. Take first the free-cooling phase in which there is no heat control $u_1 = 0$. The temperature decreases exponentially from 250^oC to 100^oC during $t_1 = 5h$ and $t_2 = 15h$. This allows us to estimate

$$\begin{aligned}\gamma_l &\approx -(t_2 - t_1)^{-1}\ln\Big(\frac{T(t_2)}{T(t_1)}\Big)\\ &= -(10 \times 3600)^{-1}\ln\Big(\frac{100+273}{250+273}\Big)\\ &= 9.4 \times 10^{-6}[s^{-1}]\end{aligned}$$

Similarly, using the heating phase in which the control $u_1 = 1$, one can estimate γ_h. Notice that the oven temperature rises from 25^oC to 210^oC during the first $54min$. Hence one has

$$\begin{aligned}\gamma_h &\approx \frac{\gamma_l(210+273-e^{-\gamma_l(54\times60)})(25+273)}{1-e^{-\gamma_l(54\times60)}}\\ &= 6.1 \times 10^{-2}[kg \cdot s^{-1}]\end{aligned}$$

2.2 Jet impingement technique

Jet impingement technique has been studied over the years due to its wide applications in industrial food or drying process. Considerable reduction in process times and improvement in product quality can be obtained using this technique. For example, [Li and Walker, 1996] compared large impingement ovens and the combination of impingement and microwaves and found large differences to industrial hot air ovens.

In the considered bread baking process, the boundary layer occurring at sole and bread surfaces constitutes a great resistance to convective heat transfer between hot air and the bread surface. Impinging jet can then be used to reduce the thickness of the boundary layer and thus increases the convection [Banooni et al., 2008]. In this study, a multiple impinging jet oven has been developed to investigate the energy consumption during bread baking process. Its geometrical characteristics were chosen as a compromise between technical considerations and literature reviews [Zuckerman and Lior, 2006].

According to the correlations available in the literature, the average Nusselt number due to impingement on a flat plate could represent 5 - 10 times the Nusselt number due to natural convection [Attalla and Specht, 2009, Chang et al., 2006, Geers et al., 2008]. It is more difficult to evaluate the gain due to impingement on bread because its volume changes during the process. However the expected gain should be of the same order of magnitude as the one obtained for the sole.

The use of jet impingement can be seen as an additional control $u_2 \in [0, u_2^{max}]$ that increases the coefficient which reflects the efficiency of heat transfer between the bread crust and the oven atmosphere.

2.3 An assumption on the baking process

The baking model using jet impingement can be simplified by accepting the following

ASSUMPTION *All the phenomena that take place during the baking process inside the dough is driven by the crust temperature profile.*

In other words, if one aims at producing an equivalent product using an energetically less demanding process, it suffices to reproduce a crust temperature profile, denoted by $T_2(\cdot)$, which is identical to the profile obtained using the standard protocol.

Denote the targeted crust temperature profile by $T_2^r(\cdot)$ and recall that the protocol used in this experiment corresponded to a constant oven temperature $T_3^r(\cdot) \equiv 250^oC$ (to obtain T_2^r in a standard protocol). The above target would be reached if the following equality is satisfied:

$$\forall t \in [0, t_f];\;\; (1+u_2)[T_3(t) - T_2(t)] = T_3^r(t) - T_2^r(t) \quad (2)$$

where t_f denotes the baking process duration. Indeed, it would guarantee that the evolution of T_2 tracks exactly T_2^r, and hence by virtue of the Assumption, all the key variables will follow their corresponding reference profiles.

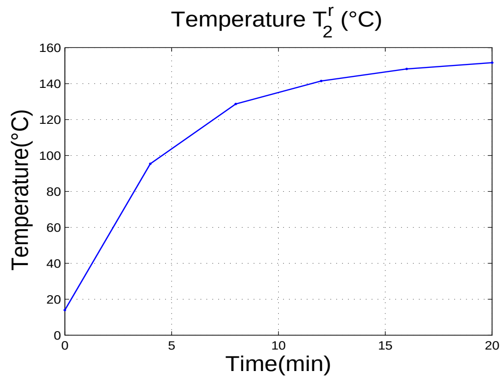

Fig. 2. Targeted crust temperature profile T_2^r obtained in a standard protocol (typical duration in 20 min)

3. OPTIMAL CONTROL PROBLEM

3.1 Optimal control problem formulation

In this section, a particular optimal control problem is considered in order to minimize the energy cost during the baking process.

Let be given

- · reference profiles $T_2^r(\cdot)$ and $T_3^r(\cdot)$ defined on $[0, t_f]$, (3)
- · maximum efficiency coefficient u_2^{max}, (4)
- · coefficient $C_2 = \mu C_1$ that defines the unitary energy cost of the use of u_2, (5)

where the coefficient $C_1 \approx 6.8 \times 10^4$ is the gain related to the necessary heating power consumed to maintain the oven reference temperature $T_3^r = 250^oC$.

Based on the Assumption, the optimal control problem can be formulated as follows :

Problem 1.
Consider the dynamic model (1) and let be given (3)-(5). Compute the profiles $u_1^*(\cdot)$ and $u_2^*(\cdot)$ as well as the initial values $T_3(0)$ of the oven temperature which minimize the energy related cost function

$$J^* := \min_{u_1(\cdot),u_2(\cdot),T_3(0)} \int_0^{t_f} \Big[u_1(t) + \mu \cdot u_2(t)\Big] dt \qquad (6)$$

subject to

$$\forall t \in [0, t_f];\ (1 + u_2)(T_3(t) - T_2^r(t)) = T_3^r(t) - T_2^r(t) \quad (7)$$

$$\forall t \in [0, t_f];\ u_2(t) \in [0, u_2^{max}] \quad (8)$$

$$\forall t \in [0, t_f];\ u_1(t) \in [0, 1] \quad (9)$$

$$T_3(t_f) = T_3(0) \quad (10)$$

♢

Notice that in this formulation, the cost function to be minimized is proportional to the energy being consumed with the ratio C_1. Indeed, by (5) the unitary energy cost of the jet impingement technology u_2 is given as a fraction μ of the unitary energy cost of the classical heating technology u_1.

The problem (6)-(9) is adapted to an isolated process. However it is possible that the final oven temperature $T_3(t_f)$ would be not high enough for the next process. For this purpose, an additional equation should be considered which interprets the cyclic character. The constraint (10) enables a cyclic process in which a baking process can start at the end of the preceding one with the same initial oven temperature $T_3(0)$.

It is clear that Problem 1 is not a LP problem because of the bilinear term $u_2(t) \cdot T_3(t)$ in the constraint (7) which involves a product of a control input and a state component. One can define an equivalent optimization problem by a change of input variable which transforms (7) in linear constraint and solve the equivalent problem using the MATLAB subroutine FMINCON.

3.2 Equivalent problem with linear constraints

By the change of input variable

$$\eta_2(t) := \frac{1}{1 + u_2(t)} \in [\eta_2^{min}, 1]; \quad \eta_2^{min} := \frac{1}{1 + u_2^{max}}, \quad (11)$$

the equivalent optimization problem associated to Problem 1 can be defined as follows:

Problem 2.
Consider the dynamic model (1) and let be given (3)-(5). Compute the profiles $u_1^*(\cdot)$ and $\eta_2^*(\cdot)$ as well as the initial values $T_3(0)$ of the oven temperature which minimize the energy related cost function

$$J^* := \min_{u_1(\cdot),\eta_2(\cdot),T_3(0)} \int_0^{t_f} \Big[u_1(t) + \mu \cdot \frac{1 - \eta_2(t)}{\eta_2(t)}\Big] dt \quad (12)$$

subject to

$$\forall t \in [0, t_f];\ T_3(t) - (T_3^r(t) - T_2^r(t)) \cdot \eta_2(t) = T_2^r(t) \quad (13)$$

$$\forall t \in [0, t_f];\ \eta_2(t) \in [\eta_2^{min}, 1] \quad (14)$$

$$\forall t \in [0, t_f];\ u_1(t) \in [0, 1] \quad (15)$$

$$T_3(t_f) = T_3(0) \quad (16)$$

♢

Notice that by now, the constraints become all linear and the nonlinearity is moved to the cost function containing the fractional term in η_2. The advantage of this formulation is that it can be immediately used to compute an upper bound on the potential energy saving. Indeed, taking $\mu = 0$ in the formulation of Problem 2 yields a LP problem that corresponds to the case where the jet impingement cost is free. Thus, the corresponding energy saving obtained by solving the resulting problem would be an upper bound on the energy saving that one can expect when using this technology. LP problems can be solved by well developed numerical tools and the details will be omitted here, interested readers are referred to [Dantzig, 1998].

3.3 Discretized problem in standard form

The discretized problem can be formulated in standard form :

$$\begin{aligned} &\min_p \Big[\tau c^T p - \mu(N-1)\tau\Big] \\ &\text{subject to} \quad \begin{aligned} &A_{eq} p = b_{eq} \\ &p^{min} \le p \le p^{max} \end{aligned} \end{aligned} \quad (17)$$

where

- $p \in \mathbb{R}^{2N+1}$ denotes the unknown vector

  $$p := (u_1(t_1), \ldots, u_1(t_N), \eta_2(t_1), \ldots, \eta_2(t_N), T_3(t_1))^T$$

  by adopting an uniform discretization over the baking process duration $[0, t_f]$ with a sampling period τ

  $$t_k = (k-1)\tau; \quad \tau = \frac{t_f}{N}$$

- the upper and lower bounds p^{min} and $p^{max} \in \mathbb{R}^{2N+1}$ satisfy the constraints (14) and (15)

  $$p^{min} := (0, \ldots, 0, \eta_2^{min}, \ldots, \eta_2^{min}, T_3^{min})^T$$
  $$p^{max} := (1, \ldots, 1, 1, \ldots, 1, T_3^{max})^T$$

- the vector $c \in \mathbb{R}^{2N+1}$ is defined such that $(\tau c^T p - \mu(N-1)\tau)$ equals to the criterion minimizing (12)

$$c := \left(1, \dots, 1, \frac{-\mu}{\eta_2^2(t_1)}, \dots, \frac{-\mu}{\eta_2^2(t_N)}, 0\right)^T$$

- the matrices $A_{eq} \in \mathbb{R}^{(N+1)\times(2N+1)}$ and $b_{eq} \in \mathbb{R}^{(N+1)}$ are defined such that $A_{eq}p = b_{eq}$ implements the constraint (13) and (16)

$$A_{eq} := \begin{pmatrix} 0_{1\times N} & -\Delta^r(t_1) & 0 & \cdots & \cdots & 0 & 1 \\ \Psi_1 & 0 & \ddots & \ddots & \ddots & \vdots & a \\ \Psi_2 & \vdots & \ddots & \ddots & \ddots & \vdots & a^2 \\ \vdots & \vdots & \ddots & \ddots & \ddots & 0 & \vdots \\ \Psi_{N-1} & 0 & \cdots & \cdots & 0 & -\Delta^r(t_N) & a^{N-1} \\ \Psi_N & 0 & \cdots & \cdots & 0 & 0 & a^N - 1 \end{pmatrix}$$

and

$$b_{eq} := (T_2^r(t_1), \dots, T_2^r(t_N), 0)^T$$

with the matrices $\Psi_i \in \mathbb{R}^{1\times N}$, Δ^r and the scalar a defined by

$$\Psi_i := (a^{i-1}b, \dots, ab, b, 0, \dots, 0)$$
$$a := \exp(-\gamma_l \cdot \tau)$$
$$b := \frac{\gamma_h}{\gamma_l}(1 - \exp(-\gamma_l \cdot \tau))$$
$$\Delta^r := T_3^r - T_2^r$$

In the above formulation, the first n lines implement the constraint (13) for an isolated process. The explicit solution of (1) is used. The last line

$$(\Psi_N \ 0 \ \dots \ 0 \ a^N - 1)p = 0$$

implements the cyclic constraint (16), that is, the the final temperature $T_3(t_f)$ should be equal to initial temperature $T_3(t_1)$.

4. NUMERICAL RESULTS

4.1 The $\mu = 0$ case

This case corresponds to the case where jet impingement has no cost. The optimal solutions of Problem 2 for $\mu = 0$ and different values of the maximal achievable transfer improvement coefficient u_2^{max} are investigated and commented, for more related details see [Alamir et al., 2012].

When using $\mu = 0$ in (12), the resulting problem is a LP problem. Using a sampling period of $\tau = 24$ *sec* and a baking scenario of length $t_f = 20 \times 60$ *sec*, a LP problem of dimension 150 is formulated and solved using the MATLAB LINPROG subroutine. The results are shown in Figure 3.

This figure shows the optimal time profile of the control variables for four different maximum achievable value of the heat transfer increase u_2^{max} due to jet impingement technology. Figure 3 also shows the evolution of the corresponding oven temperature T_3. The zoomed profiles are more shown in Figure 4 where one can see more clearly how the heat transfer improvement enables lower oven temperatures and leads to a possible significant energy saving.

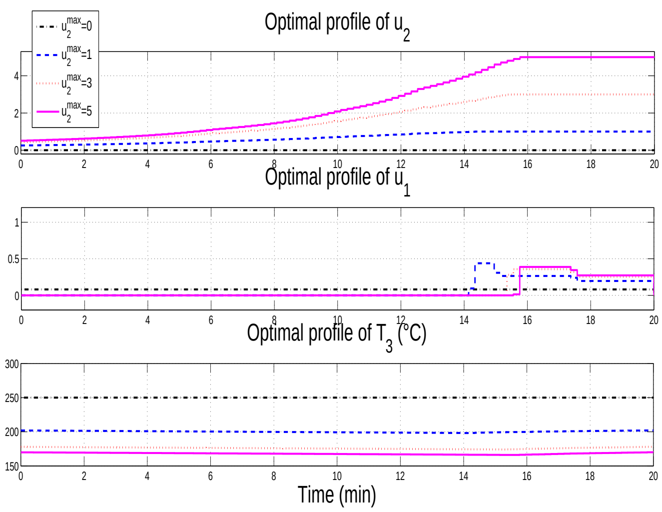

Fig. 3. Computation of the optimal control profile of the heating set-point $u_1(\cdot)$ and the increased heat efficiency transfer coefficient $u_2(\cdot)$ for different values of the maximum efficiency increase coefficient u_2^{max}, in the case where $\mu = 0$.

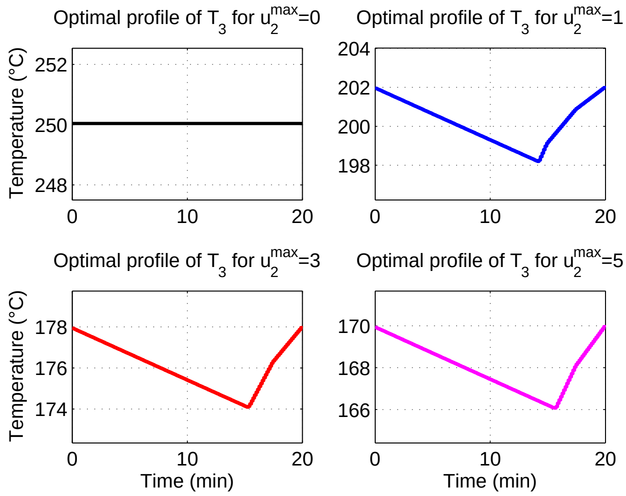

Fig. 4. Details of the optimal oven temperature profiles for different values of the maximum achievable heat transfer improvement u_2^{max}, in the case where $\mu = 0$.

Denote by p^{opt} the optimal solution of (17). In order to compute the corresponding energy saving, the optimal cost given by $J(p^{opt})$ has to be compared to the reference cost corresponding to the oven being maintained at the reference temperature $T_3^r = 250^oC$ that has been used to define the tracking problem. The reference energy level is then given by

$$J^r := t_f \times \frac{\gamma_l}{\gamma_h} \times T_3^r \tag{18}$$

which leads to the following percentage energy saving computation formula:

$$G := \frac{J^r - J(p^{opt})}{J^r} \times 100 \tag{19}$$

Jet impingement coefficient u_2^{max}	Energy saving (%) ($\mu = 0$)	Energy saving (%) ($\mu = 1.13 \times 10^{-3}$)
0	0	0
1	9.5	8.6
2	12.6	10.8
3	14.2	11.7
4	15	11.9
5	15.7	12.2
10	17.1	12.2

Table 1. Percentage of energy saving as a function of the jet impingement related heat transfer improvement in both cases where the energy cost of jet impingement is supposed free or not. Computation is based on a reference scenario in which a constant oven temperature $T_3^r = 250^oC$ is used in the BONGARD oven corresponding to the thermal characteristics investigated in section 2.

fan set point (%)	factor F	absorbed power P_{abs} (W)
25	2.5	44.8
50	3.125	59.7
75	3.875	92.2
100	4.875	154

Table 2. The ratio F between the jet impingement heat transfer and the natural convection transfer coefficient and the power absorbed P_{abs} by the fan under different velocities.

The results are shown on Table 1 where it can be seen that under the assumption used above, the gain can be close to 16% for a realistic and experimentally validated increase in heat transfer of $u_2^{max} = 5$.

4.2 The $\mu \neq 0$ case

Now consider the case where a non zero energy cost is used for the jet impingement technology u_2. In order to estimate the energy saving using u_2, one needs to determine the coefficient μ in the cost function (11). Equivalently, one needs to know the necessary power consumed by u_2 which can double the thermal transfer on the bread surface.

Define the multiplicative factor F as the ratio between the jet impingement heat transfer coefficient and the natural convection transfer coefficient. According to the first experimental results using a natural convection transfer coefficient of $8W/(m^2.K)$ around the cylinder of a circular jet, the expected transfer coefficient could be about 4.9 times higher if one uses the jet to full power. The experience is realized at room temperature with the velocity of jet is on the order of $6m/s$ at 100%. The raltion between the velocity (m/s) and the percentage (%) is considered linear. The uncertainty in the identified F is around ± 0.5. The experimental values of the power absorbed (denoted by P_{abs} in $Watts$) by the fan under different velocities are given in Table 2.

Consider the case where the fan is at maximum flow rate corresponding to the power absorbed equals to $154W$, one can determine the value of

$$\mu = \frac{154}{2 \times 6.8 \times 10^4} \approx 1.13 \times 10^{-3}$$

when linearity between the power and the efficiency gain u_2 is assumed.

The resulting optimization problem can be solved using the MATLAB subroutine FMINCON. Concerning the initial value setting, a reasonable choice would be $\eta_2^0(t) = 1$ which means only the heating input u_1 is considered. To guarantee the optimality, different initial conditions are also tested, including $\eta_2^0(t) = \eta_2^{min}$ or a random sequence between η_2^{min} and 1.

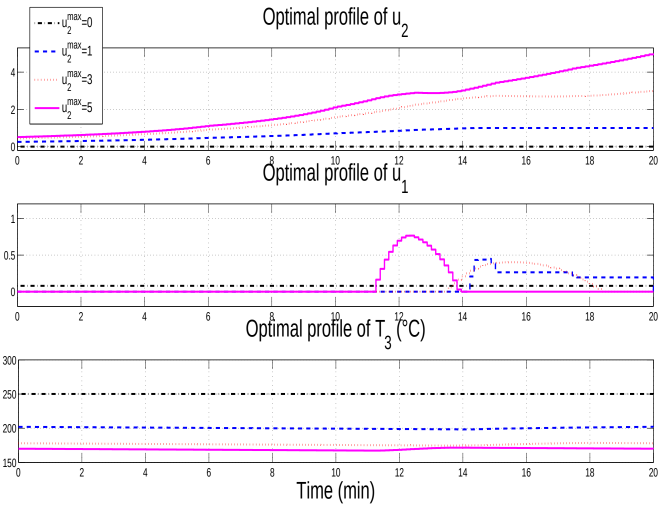

Fig. 5. Computation of the optimal control profile of the heating set-point $u_1(\cdot)$ and the increased heat efficiency transfer coefficient $u_2(\cdot)$ for different values of the maximum efficiency increase coefficient u_2^{max}, in the case where $\mu = 1.13 \times 10^{-3}$.

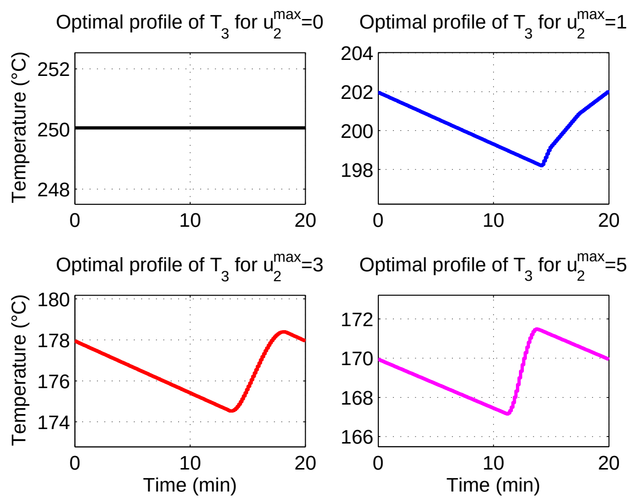

Fig. 6. Details of the optimal oven temperature profiles for different values of the maximum achievable heat transfer improvement u_2^{max}, in the case where $\mu = 1.13 \times 10^{-3}$.

The percentage of corresponding energy saving with different values of maximal achievable transfer coefficient u_2^{max} is computed by using (18)-(19) in the previous case and the results are given in Table 1.

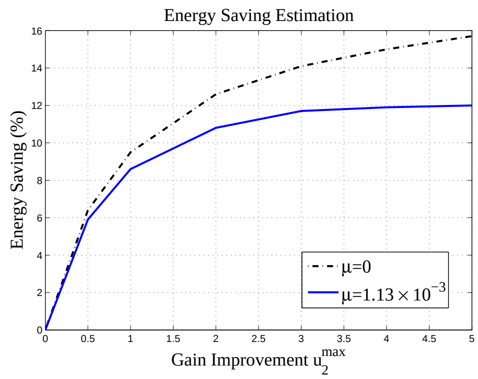

Fig. 7. Comparison of energy saving as a function of the maximum allowable jet impingement related coefficient in both cases where $\mu = 0$ and $\mu = 1.13 \times 10^{-3}$.

Notice that for a maximum allowable transfer coefficient $u_2^{max} = 5$ the potential energy saving in the $\mu = 1.13 \times 10^{-3}$ case is close to 12 % which is an encouraging result for the use of the jet impingement technology.

5. CONCLUSION

In this paper we have investigated the potential energy saving in the French baking process by using the new jet impingement technology. A simple mechanistic model on the oven temperature is used. A Non Linear Programming optimization problem is formulated. The problem is reduced to a LP problem in the case where the jet impingement technology has a free energy cost. Under some assumption, an upper bound of expected energy saving has been computed showing an approximated saving level of 16% when the gain improvement coefficient corresponding to the new techology equals to 5. Moreover, the experimental data shows that the energy cost of the jet impingement technique is negligible when compared to the classical baking process with a coefficient of level 10^{-3}. In this case, the expected energy saving can reach up to 12%. Future investigation should address the non convex optimization problem with the radiation and the infrared based technologies included in the model.

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