A Time-delay Approach for Modeling and Control of Mist in a Poiseuille Flow

Miguel A. Hernandez Perez, Emmanuel Witrant and Olivier Sename

Abstract— **Moisture control in advective flow systems with time delay is studied in this work and assessed in a process-control system (test bench). A control-oriented model is proposed by considering advection as a transport time-delay and the sensitivity to operating conditions as structured uncertainties. The model parameters and variations are identified from experimental results. A guaranteed cost control strategy is then used to handle both time-varying delays and uncertainties in an optimal design, and improved with a linear change of variables and an integral action. The results are compared with a traditional PI strategy and evaluated on both simulation and experimental results.**

I. INTRODUCTION

The ability to manipulate flow properties (density, concentration, pressure, etc.) to improve efficiency and performance in the transport of materials is of major technological importance and is currently an active research topic in control engineering. Flow control is often encountered in industrial and commercial applications, such as hydraulic networks [1], gas flow in pipelines [2] and flow regulation in mining [3]. The air flow must be regulated to ensure the proper operating conditions, often described in terms of temperature or moisture. Flow control can also be applied to the automotive industry, for instance to regulate the quantity of fresh air in the intake manifold of an engine with exhaust gas recirculation, which is a critical factor for reducing emissions [4]. The flow dynamics are generaly modeled by Partial Differential Equations (PDE), inferred from Euler or Navier-Stokes equations. Reduction and discretization strategies are often used to formulate the corresponding lumped model and solve the control problem in a framework of ordinary differential equations, e.g. in [5] and [6]. In [7] the control problem is directly addressed in the PDE framework by considering an hyperbolic system with n rightward convecting transport PDEs. Using the information from the boundary control and the boundary conditions, an observer is designed by Lyapunov based techniques. Another approach consists in approximating the transport phenomena by a time delay model, as in [8] where the method of characteristics is used to express the PDE model as a Functional Differential Equations (FDE) with a distributed delay kernel.

Flow transport is a phenomenon that induces time-delay. This time-delay is caused by the path followed by an elementary mass in the pipeline. In closed-loop this problem is more difficult to handle. The existence of time delays may result in instability, oscillation and poor performance of the control system. Therefore, significant efforts have been done during the last decades to study time-delay systems, and a large number of results on analysis and control synthesis have been reported in the literature, as presented in [9], [10], among others. In addition, various control methods have been proposed in the literature to stabilize uncertain control systems with time delay [11], [12]. One approach to deal with this problem is to minimize a cost function that sets the performance and robustness objectives [13], [14], which seems to be attractive from the application point of view.

This paper is focused on the modeling and control of moisture in a test-bench available at University Joseph Fourier, Grenoble. The test-bench, presented in Figure 1, consists of a heating column encasing a resistor, a tube, two fans, a wind speed meter and three distributed moisture sensors. The actuators are the resistance power, the rotational velocities of the fans and the injection of mist. The mixture of dry inflow and mist injection generates a change of moisture along the tube. The contribution of this paper is to propose a simplified model and a control methodology to handle the inherent time-delay in the dynamical behavior of the moisture inside of the tube. Variations of the operating conditions (in terms of temperature and fan rotational velocity) are also considered. First, a simplified model for moisture regulation in the test bench is obtained, as a linear system with parameter uncertainties and a time-delay.In order to minimize a performance cost we design a state-feedback controller and minimizes an integral-quadratic cost function such that the resulting closed-loop system is robustly stable [15]. The sufficient conditions for stability are presented in a linear matrix inequality (LMI) framework. Finally, to improve the tracking performance, a state-feedback integral control is developed and compared with a classical PI control, using simulation and experimental results on the test bench.

The paper is organized as follows. Section II describes the test bench identification and the resulting model for moisture control. The control problem is formulated in Section III, explicitely taking into account the parameters uncertainties and a time-delay. The regulation problem is solved in Section IV and the control strategy is applied for the regulation of the moisture in the test bench in the Section V.

II. MODELING AND IDENTIFICATION

A. Test bench Identification

To investigate the phenomenon of mass transport in Poiseuille flow and to obtain a model which represents the dynamics of moisture in the system, several experiments

Grenoble Images Parole Signal Automatique (GIPSA-Lab), Universite Joseph Fourier, 11 rue des Mathematiques, Saint Martin d'Heres, Grenoble, France. miguel-angel.hernandez-perez, olivier.sename, emmanuel.witrant@gipsa-lab.fr

were carried out on the test bench. The description of the test bench is presented in Figure 1.

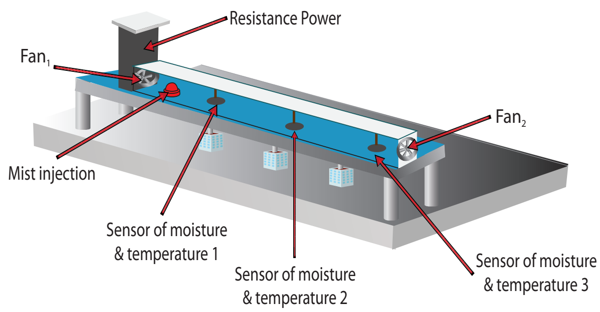

Fig. 1. *Flow control test bench.*

This device is equipped with 7 sensors (3 moisture sensors distributed along the tube, 3 temperature sensors distributed along the tube and 1 wind speed sensor) and 4 actuators (1 mist injection into the tube (Mist Control), 1 heater (Resistance power) and 2 fans for circulating the air (controlled by the rotational velocity)).

The objective of this plant is to illustrate the transport of mist (concentration of water particles in dry air) along of the tube, which is characterized by a time-delay between the mist injection and the measurement of the moisture. In this section, an identification procedure has been conceived to obtain the control-oriented model between the mist injection and the humidity measurement given by sensor 3. Moreover, the dynamics of such a plant exhibit large variations according to changes of heating and wind conditions. We thus also analyze the system performance on the whole range of operation and deduce an uncertain control-oriented model.

As first step, it is important to know how the moisture dynamics interact with the temperature and wind speed, which is evaluated by setting the mist injection at 100%, the resistance power at 0% and the rotational velocity of the fan at 40%. Figure 2 shows that the temperature does not change significantly (from $31°C$ to $30.8°C$) when the mist injection is set at 100% at constant velocity $(0.4m/s)$ and the resulting maximum value of moisture is 53%. We can thus conclude that the evaporation of the droplets of water composing the mist (which would reduce the temperature due to the energy that is necessary for the phase change) is negligible at ambient temperature.

A set of experiments has been done to identify a nominal model and the parameter uncertainties. Variations of mist injection, resistance power and rotational velocities of the fans are thus necessary. This was achieved according to the following stages.

1) Varying mist injection from 20% to 100%, without changing the resistance power nor the fan rotational velocity.
2) Increasing the resistance value from 0% to 100% by intervals of 20%, in order to observe the maximum change in the moisture (no change in the fan rotational velocity).

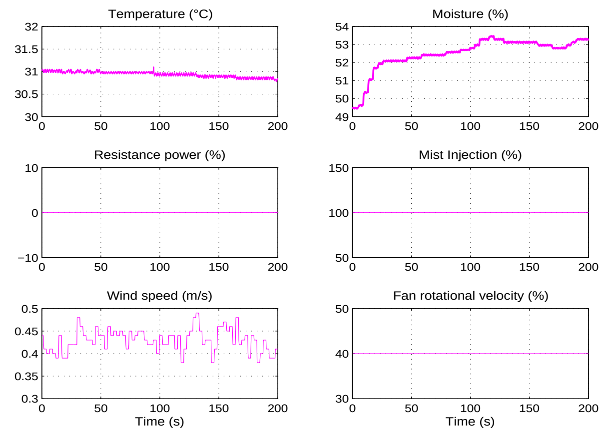

Fig. 2. *Test Bench response when mist injection, resistance power and fan rotational velocity are set to* 100%, 0% *and* 40%, *respectively*

3) Increasing the fan rotational velocity from 20% up to 100% by intervals of 20%.

Table I shows the parameter variations considered (mist injection, resistance power and fan rotational velocity, all variations were done starting from 0% to 100% by intervals of 20%).

TABLE I
TABLE OF EXPERIMENTS

Resistance	**Mist injection**	**Fan rotational velocity**				
		20%	**40%**	**60%**	**80%**	**100%**
0%	$20-100\%$	✓	✓	✓	✓	✓
20%	$20-100\%$	✓	✓	✓	✓	✓
40%	$20-100\%$	✓	✓	✓	✓	✓
60%	$20-100\%$	✓	✓	✓	✓	✓
80%	$20-100\%$	✓	✓	✓	✓	✓
100%	$20-100\%$	✓	✓	✓	✓	✓

To give an overview of the performance of the plant, Figure 3 shows the first stage of the experiments when the operating conditions are: mist injection ranging from 0% to 100%, no resistance power and fan rotational velocity at 40%. Changes in the moisture (with initial condition at 43.4%) are large when the fan rotational velocity is slow (approximately between 10% and 40%) and weakly affect the temperature (which varies between $25.9°C$ and $26.4°C$).

The second and third stages of the experiments are illustrated in Figure 4, which shows the possible range of moisture (the initial condition with which the experiments were performed is for the moisture at 40% and the temperature around of $28°C$). The temperature does not vary significantly (from $28°C$ to $31°C$), while the moisture ranges from 39% to 56% at a wind speed of $0.2\,m/s$ approximately. Note that the range of moisture at steady state is significantly larger than in Figure 3, due to the decreased flow velocity.

B. Time-delay Model

As shown in the previous section, the test bench has strong dynamical variations according to the operating point (mist injection, resistance power and fan rotational velocity). A complete model should account for the energy exchange and transport phenomenon inside the tube, but could be efficient

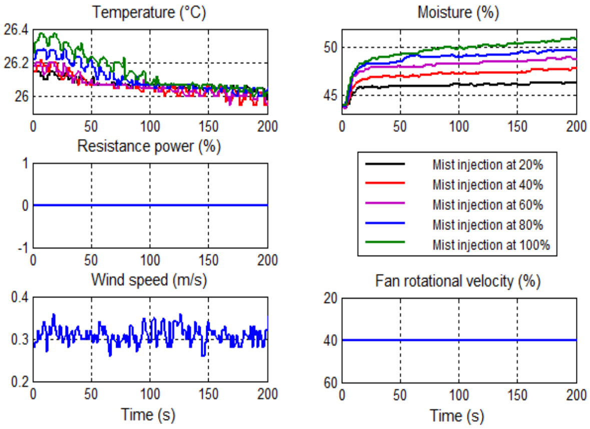

Fig. 3. *Test Bench response with different values of mist injection (from* 0% *to* 100% *by intervals of* 20%*).*

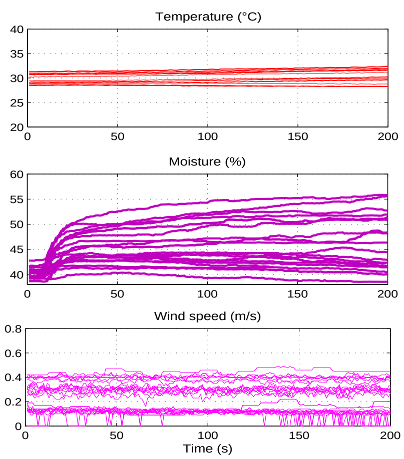

Fig. 4. *Operating range of the moisture dynamics in the test bench.*

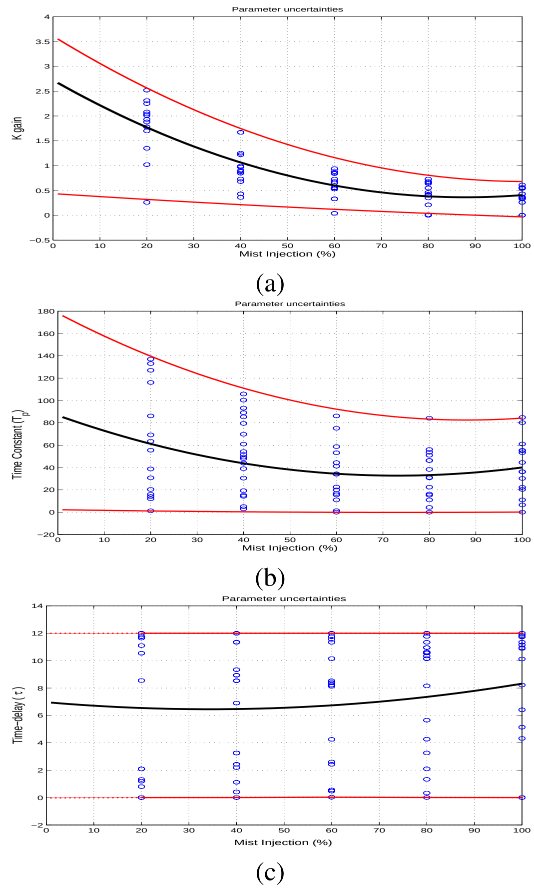

Fig. 5. Parameters uncertainties $\{k, T_p, \tau\}$ respectively.

to handle for control design. To get a simplified control-oriented model, we consider a first-order time-delay system that captures the main dynamics of the moisture in the plant.

To obtain the model parameters and the admissible range of uncertainties, all the system responses gathered from the experiments described above have been considered. The variations of the mist injection, resistance power and fan rotational velocity are presented by mapping 3 parameters (gain, time constant and time-delay) to generate a first-order transfer function with time delay given by

$$\frac{Moisture(\%)}{MistControl(\%)} = \frac{K}{1 + T_p s} e^{-\tau s}, \tag{1}$$

where K is the gain, T_p is the time constant and τ is the time-delay. This parameters are illustrated in Figure 5 and are presented as follows

- $K = k_0 + \delta K$ with $K_0 = 1.2$ and $\delta K \in [-0.6; 1.34]$.
- $T_p = T_p^0 + \delta T_p$ with $T_p^0 = 40$ and $\delta T_p \in [-33.75; 76.27]$.
- $\tau = \tau_0 + \delta\tau$ with $\tau_0 = 8.3$ and $\delta\tau \in [-3.17; 3.7]$.

Taking into account all the dynamics presented in Figure 5, it is possible to choose one operation point (when the experiment is working at 40% in the mist injection, the resistance power at 20% in the resistance power and 20% in the fan rotational velocity), in order to illustrate the behavior of the nominal mathematical model. Figure 6illustrate that there exists a good approximation between the nominal mathematical model and the dynamics of the test bench.

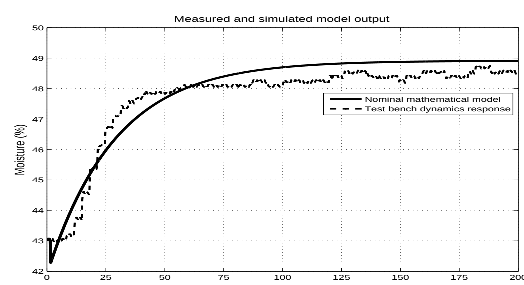

Fig. 6. *Comparison between nominal model and the experimental results obtained from the test bench*

Remark 1: It is important to note that the gain decreases when the mist input increases, independently of the variation of the fan rotational velocity and of the resistance power, which implies some nonlinearity in the mist control. Also, the time-delay (τ) is inversely proportional to the fan rotational velocity as expected.

III. FORMULATION AND SOLUTION OF THE CONTROL PROBLEM

As identified in Section II, the considered model is an input delay system with a time-varying delay. This class of systems has been considered in several studies for stabilization and robust feedback design [17], [18]. As show later in Section IV, the plant model (1), can be represented in the state space form with uncertain parameters and a time-varying input delay as follows:

$$\sum \begin{cases} \dot{x}(t) = (\boldsymbol{A} + \boldsymbol{\Delta A}(t))\, x(t) \\ \qquad + (\boldsymbol{B} + \boldsymbol{\Delta B}(t))\, u\,(t - \tau(t)) \\ x(t) = \phi(t), \quad \forall t \in [-h, 0] \end{cases} \tag{2}$$

where $x(t) \in \mathbb{R}^n$ is the state and $u(t) \in \mathbb{R}^m$ is the control input. $\boldsymbol{A}$ and $\boldsymbol{B}$ are known real constant matrices with appropriate dimensions. The scalar h is defined as $h = \sup\{\tau(t)\}$ and $\phi(t)$ is a given differential initial function on $[-h, 0]$. $\boldsymbol{\Delta A}(t)$ and $\boldsymbol{\Delta B}(t)$ are unknown real matrices representing time-varying parametric uncertainties, which are assumed to be of the form:

$$[\boldsymbol{\Delta A}(t) \quad \boldsymbol{\Delta B}(t)] = \boldsymbol{M}\boldsymbol{F}(t)\,[\boldsymbol{N}_1 \quad \boldsymbol{N}_2] \tag{3}$$

where $\boldsymbol{M}$, $\boldsymbol{N}_1$, and $\boldsymbol{N}_2$ are known constant matrices, and $\boldsymbol{F}(t) \in \mathbb{R}^{l \times q}$ is an unknown real time-varying matrix satisfying:

$$\boldsymbol{F}(t)^T \boldsymbol{F}(t) \leq \boldsymbol{I} \tag{4}$$

$\boldsymbol{\Delta A}(t)$ and $\boldsymbol{\Delta B}(t)$ are said to be admissible if both (3) and (4) hold. The time-varying input delay is assumed to be a continuous and bounded function, satisfying for all $t \geq 0$

$$0 < \tau(t) \leq h. \tag{5}$$

The objective is to design a robust and optimal controller that to minimizes the quadratic cost function:

$$\boldsymbol{J} = \int_0^\infty \left[x(t)^T \boldsymbol{R}_1 x(t) + u(t)^T \boldsymbol{R}_2 u(t) \right] dt \tag{6}$$

where $\boldsymbol{R}_1 > 0$ and $\boldsymbol{R}_2 > 0$ are given constant matrices, considering the linear state-feedback controller:

$$u(t) = \boldsymbol{K} x(t), \qquad \boldsymbol{K} \in \mathbb{R}^{m \times n} \tag{7}$$

The control problem is formulated as follows: given a scalar $h > 0$, design a state-feedback controller (7), such that for any time-varying delays $\tau(t)$ satisfying (5), the closed-loop system (2)-(7) is asymptotically stable and the cost function in (6) is upper bound for all admissible uncertainties. In this case, (7) is said to be a guaranteed cost state feedback controller [16]. Sufficient conditions for the solvability of the guaranteed cost control problem can be obtained following [15], considering some simplification for the studied plant.

Theorem 1: Consider the uncertain time-delay system (2) and the cost function (6). Then, for given a scalar $h > 0$, the guaranteed cost control problem is solvable if there exist matrices $\boldsymbol{X} > 0$, $\boldsymbol{Z} > 0$, $\boldsymbol{Y}$, $\boldsymbol{W}_1$, $\boldsymbol{W}_2$, $\boldsymbol{S}_1$, $\boldsymbol{S}_2$ and a scalar $\epsilon > 0$ such that the following matrix inequality holds:

$$\begin{bmatrix} \boldsymbol{H}_1 & \boldsymbol{\Omega}_1 & \boldsymbol{\Omega}_2 & \boldsymbol{\Omega}_3 & \boldsymbol{\Omega}_4 \\ \boldsymbol{\Omega}_1^T & \boldsymbol{H}_2 & \boldsymbol{\Omega}_5 & \boldsymbol{\Omega}_6 & 0 \\ \boldsymbol{\Omega}_2^T & \boldsymbol{\Omega}_5^T & \boldsymbol{H}_3 & 0 & 0 \\ \boldsymbol{\Omega}_3^T & \boldsymbol{\Omega}_6^T & 0 & \boldsymbol{H}_4 & 0 \\ \boldsymbol{\Omega}_4^T & 0 & 0 & 0 & \boldsymbol{H}_5 \end{bmatrix} < 0 \tag{8}$$

where:

$$\begin{aligned} \boldsymbol{H}_1 &= \boldsymbol{X}\boldsymbol{A}^T + \boldsymbol{A}\boldsymbol{X} + \boldsymbol{B}\boldsymbol{Y} + \boldsymbol{Y}^T\boldsymbol{B}^T \\ &\quad - \boldsymbol{W}_1 - \boldsymbol{W}_1^T - \boldsymbol{S}_1 - \boldsymbol{S}_1^T + \epsilon \boldsymbol{M}\boldsymbol{M}^T \\ \boldsymbol{H}_2 &= \text{diag}\left(\boldsymbol{W}_2 + \boldsymbol{W}_2^T, \boldsymbol{S}_2 + \boldsymbol{S}_2^T\right) \\ \boldsymbol{H}_3 &= \text{diag}\left(0, h\boldsymbol{X}\boldsymbol{Z}^{-1}\boldsymbol{X}\right), \ \boldsymbol{H}_4 = \begin{bmatrix} \epsilon h^2 \boldsymbol{M}\boldsymbol{M}^T - h\boldsymbol{Z} & 0 \\ 0 & \epsilon \boldsymbol{I} \end{bmatrix} \\ \boldsymbol{H}_5 &= \text{diag}\left(-\boldsymbol{R}_1^{-1}, -\boldsymbol{R}_2^{-1}\right) \\ \boldsymbol{\Omega}_1 &= \left[\boldsymbol{A}_1\boldsymbol{X} + \boldsymbol{W}_1 - \boldsymbol{W}_2^T \qquad \boldsymbol{B}_1\boldsymbol{Y} + \boldsymbol{S}_1 - \boldsymbol{S}_2^T\right] \\ \boldsymbol{\Omega}_2 &= \left[0 \qquad h\boldsymbol{S}_1\right] \\ \boldsymbol{\Omega}_3 &= \left[h\boldsymbol{X}\boldsymbol{A}^T + h\boldsymbol{Y}^T\boldsymbol{B}^T + \epsilon h \boldsymbol{M}\boldsymbol{M}^T \quad \boldsymbol{X}\boldsymbol{N}_1^T\right] \\ \boldsymbol{\Omega}_4 &= \left[\boldsymbol{X} \qquad \boldsymbol{Y}^T\right], \quad \boldsymbol{\Omega}_5 = \text{diag}\left(0, h\boldsymbol{S}_2\right) \\ \boldsymbol{\Omega}_6 &= \begin{bmatrix} h\boldsymbol{X}\boldsymbol{A}_1^T & \boldsymbol{X}\boldsymbol{N}_1^T \\ h\boldsymbol{Y}^T\boldsymbol{B}_1^T & \boldsymbol{Y}^T\boldsymbol{N}_2^T \end{bmatrix} \end{aligned}$$

In this case, the desired guaranteed cost state-feedback controller can be chosen as

$$u(t) = \boldsymbol{Y}\boldsymbol{X}^{-1} x(t) \tag{9}$$

and the corresponding cost function in (6) satisfies:

$$\boldsymbol{J} \leq \phi(0)^T \boldsymbol{X}^{-1} \phi(0) + \int_{-h}^0 \int_\beta^0 \dot{\phi}(\alpha)^T \boldsymbol{Z}^{-1} \dot{\phi}(\alpha)\; d\alpha\; d\beta \tag{10}$$

Remark 2: Theorem 1 provides a sufficient condition for the solvability of the guaranteed cost control problem for the time-delay system (Σ). Note that the matrix inequality in (8) is not an LMI because of the term $\boldsymbol{X}\boldsymbol{Z}^{-1}\boldsymbol{X}$. In order to solve this non-convex problem, we introduce a new variable $\boldsymbol{P}$ such that $\boldsymbol{X}\boldsymbol{Z}^{-1}\boldsymbol{X} \geq \boldsymbol{P}$. Then, we have:

$$\boldsymbol{P}^{-1} \geq \boldsymbol{X}^{-1}\boldsymbol{Z}\boldsymbol{X}^{-1}, \tag{11}$$

by Schur complement, it follows from (11) that

$$\begin{bmatrix} \boldsymbol{P}^{-1} & \boldsymbol{X}^{-1} \\ \boldsymbol{X}^{-1} & \boldsymbol{Z}^{-1} \end{bmatrix} \geq 0.$$

The following non-linear optimization problem can be stated, involving LMI conditions (for more details see [19]) subject to

$$\begin{bmatrix} \boldsymbol{H}_1 & \boldsymbol{\Omega}_1 & \boldsymbol{\Omega}_2 & \boldsymbol{\Omega}_3 & \boldsymbol{\Omega}_4 \\ \boldsymbol{\Omega}_1^T & \boldsymbol{H}_2 & \boldsymbol{\Omega}_5 & \boldsymbol{\Omega}_6 & 0 \\ \boldsymbol{\Omega}_2^T & \boldsymbol{\Omega}_5^T & \mathcal{H}_3 & 0 & 0 \\ \boldsymbol{\Omega}_3^T & \boldsymbol{\Omega}_6^T & 0 & \boldsymbol{H}_4 & 0 \\ \boldsymbol{\Omega}_4^T & 0 & 0 & 0 & \boldsymbol{H}_5 \end{bmatrix} < 0 \tag{12}$$

where $\boldsymbol{H}_1,\ \boldsymbol{H}_2,\ \boldsymbol{H}_4,\ \boldsymbol{H}_5,\ \boldsymbol{\Omega}_i,\ i = 1, \ldots, 6$, are the same as those used for the Theorem 1, and $\mathcal{H}_3 = \text{diag}\,(0, h\boldsymbol{P})$. Using Theorem 1, it can be seen that the guaranteed cost control problem is solvable and a desired guaranteed cost state-feedback controller can be obtained as in (9).

IV. APPLICATION TO THE TEST BENCH

A. Control design

The plant model represented by (1) is first represented as the state-space delay system (2) with

$$\boldsymbol{A} = \begin{bmatrix}-0.025\end{bmatrix},\ \boldsymbol{B} = \begin{bmatrix}1.2\end{bmatrix},$$
$$\boldsymbol{M} = \begin{bmatrix}1\end{bmatrix},\ \boldsymbol{N_1} = \begin{bmatrix}0.017\end{bmatrix},\ \boldsymbol{N_2} = \begin{bmatrix}1.34\end{bmatrix}$$

The values of N_1 and N_2 are taken from the relationship between the maximum value and the nominal value of T_p and K, respectively. The time-delay $\tau(t)$ is assumed to satisfy (5), with $h = 12$. Using Theorem 1 and by (10), a guaranteed cost state-feedback control can be obtained with a maximum cost of $\boldsymbol{J} < 37.28$. Nevertheless, the associated tracking performance is not accurate enough: a state-feedback control with integral action is then required. The aim is to regulate the output of the moisture $x(t)$ around a reference operating point value (x_{ref}). In order to have a zero steady-state error ($x_{ref}(t) - x(t)$), an integrator is added and the extended system is:

$$\begin{aligned}\begin{bmatrix}\dot{x}(t)\\ \dot{E}(t)\end{bmatrix} &= \begin{bmatrix}A(t) & 0\\ -I & -\lambda(t)\end{bmatrix}\begin{bmatrix}x(t)\\ E(t)\end{bmatrix}\\ &+ \begin{bmatrix}B(t)\\ 0\end{bmatrix}u(t-\tau) + \begin{bmatrix}0\\ 1\end{bmatrix}x_{ref}\end{aligned} \tag{13}$$

where $E = \int_0^t [x_{ref}(t) - x(t)]dt$ is the integral of the error. A new parameter $0 < \lambda(t) < \frac{1}{T_p}$ has been introduced as a "forgetting factor" for the integrator. The purpose of this term is to avoid high overshoots when changing the operating point by weighting down past accumulated errors. This parameter is designed to vanish in finite time (e.g. see [20] for its use in optimal control of tokamak plasmas).

Using the Theorem 1, a guarantee cost state-feedback integral control (GC-IC) is obtained as

$$u(t) = [-0.031 \quad 0.0005][x(t) \quad E(t)]^T$$

The corresponding closed-loop GC-IC satisfies $\boldsymbol{J} < 46.25$.

B. Simulation results

To illustrate the performance of our controller, the operating conditions are given by the nominal model and the objective is to track a reference of $x_{ref} = 50\%$, is show in Figure 7. To compare the GC-IC efficiency with a classical feedback design strategy, we consider a PI feedback tuned using the internal model control (IMC) method proposed by [21]. The IMC-PI tuning rules have the advantage of using only a single tuning parameter to achieve a clear trade-off between closed-loop performance and robustness against model inaccuracies.

Consider the model of the test bench described by (1) and the PI controller given by:

$$PI(s) = K_p\left(1 + \frac{1}{T_i s}\right) \tag{14}$$

The IMC-PI tuning rules for time-delayed system are given by [21] as $K_p = \frac{1}{K}\frac{T_p}{T_c+\tau}$ and $T_i = \min\left(T_p, 4\left(T_c+\tau\right)\right)$, where T_c is the trade-off on output performance (small), usually set as $T_c = \tau$.

Considering the nominal model of the test bench described in the Section II. B. The values of the PI control are: $K_p = 2.03$ and $T_i = 0.025$.

The GC-IC and IMC-PI controllers are applied on the nominal and extreme models, in order to compare their performance on moisture regulation. The nominal responses are illustrated in the Figure 7. In order to evaluated the performance in the extreme models case, we use the integral absolute error (IAE) as an indicator of efficiency (illustrated in Figure 8). The results show that IMC-PI has better performances on the nominal model the GC-IC has a better robust performance on the extreme models.

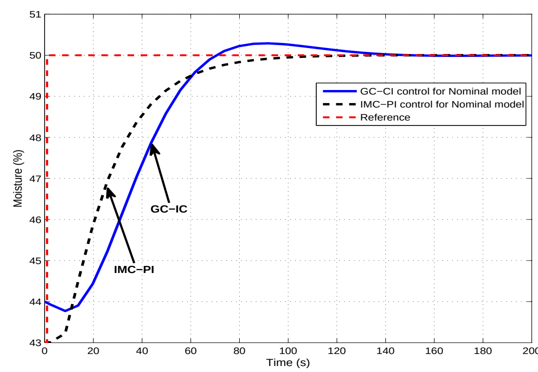

Fig. 7. *Comparison between IMC-PI and GC-IC tracking efficiencies on the nominal model for ref*=50%

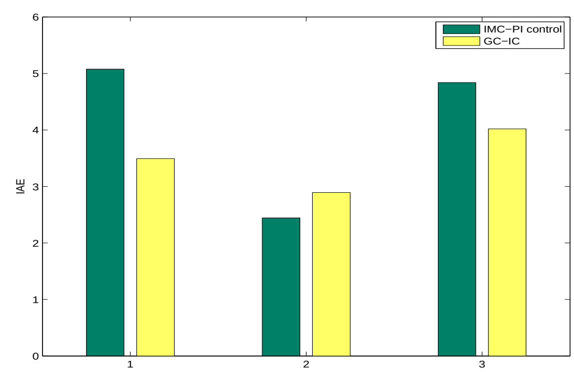

Fig. 8. *Performance indicator (integral absolute error) of IMC-PI and GC-IC for the nominal and extremal models*

C. Experimental results

To investigate the effectiveness of the proposed control strategies, the IMC-PI and GC-IC feedbacks have been implemented on the test bench. The first experiment is done on the nominal case and has the following operating condition: no resistance power, an initial temperature at $26°C$ and a fan rotational velocity at 20%. Figure 9 shows the response of the controlled moisture while tracking a step rference of 50%.

In the second experiment (extreme case), the resistance control is increased at 40% and the fan rotational velocity

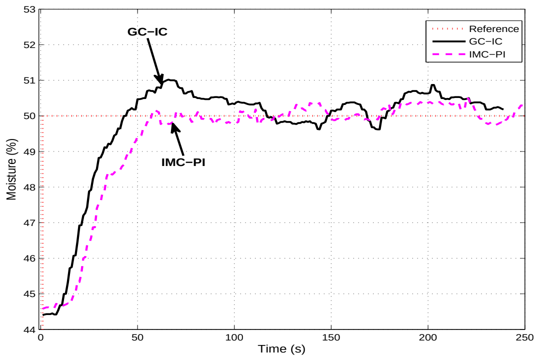

Fig. 9. *Experiment 1 (nominal): Response of IMC-PI and GC-IC on the test bench*

at 40% with an initial temperature of $25°C$. The moisture reference is the same as in the previous experiment (50%) and the experimental results are shown in Figure 10.

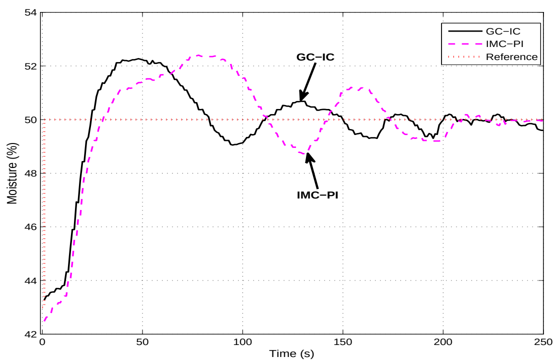

Fig. 10. *Experiment 2: Response of IMC-PI and GC-IC on the test bench*

These experimental results show that the both controller ensure the reference tracking with small steady-state error. However the performances of the IMC-PI are more sensitive to uncertainties that the GC-IC.

V. CONCLUSION

A complete modeling and control study of moisture regulation in an experimental test bench has been proposed in this paper. First, a model was obtained from different experiments in order to identify the dynamics of moisture in the system. The dynamics of mist transport were modeled as a first order system with uncertain parameters and input delay (which is mostly given by the ratio between the length of the tube and the airflow velocity) between the measured moisture and the mist injected. A guaranteed cost control was then proposed for the uncertain time-delay system. The state-feedback controller guarantees the robust stability of the closed-loop system and an upper bound of the specific cost function for the maximum uncertainty in the test bench. The reference design strategy was improved by introducing a linearizing change of variables (which provides a convex constructive method using LMIs) and an integral action with a forgetting effect. Finally the efficiency of that control methodology is illustrated and compared with a classical IMC-PI design using simulation and experimental results.

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