Symbolic Control of Monotone Systems
Application to Ventilation Regulation in Buildings

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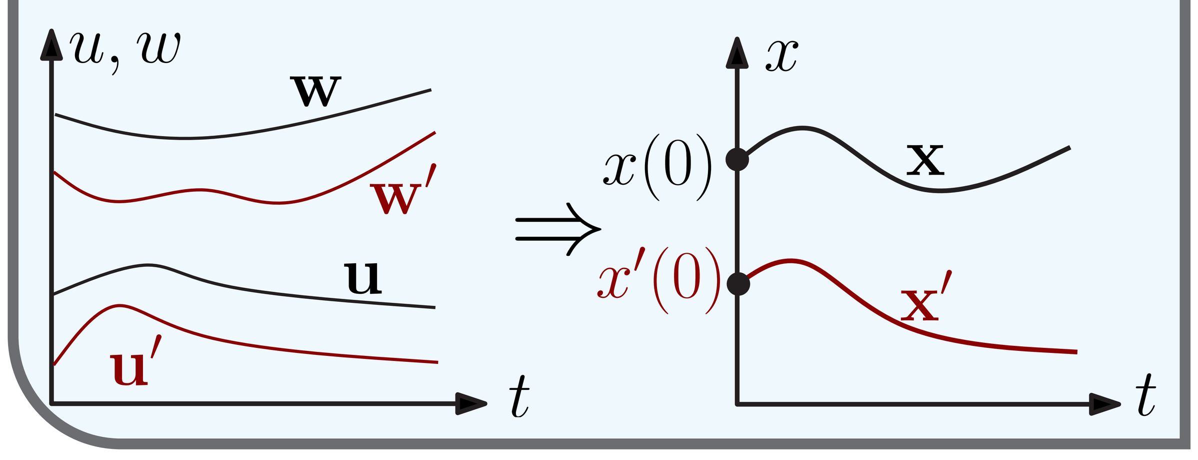
Monotone system

Dynamical system of the form:

$$\dot{x} = f(x, u, w), \quad (1)$$

where $x \in \mathbb{R}^n$, $u \in [\underline{u}, \overline{u}] \subseteq \mathbb{R}^p$, $w \in [\underline{w}, \overline{w}] \subseteq \mathbb{R}^q$ are the state, control and disturbance inputs.

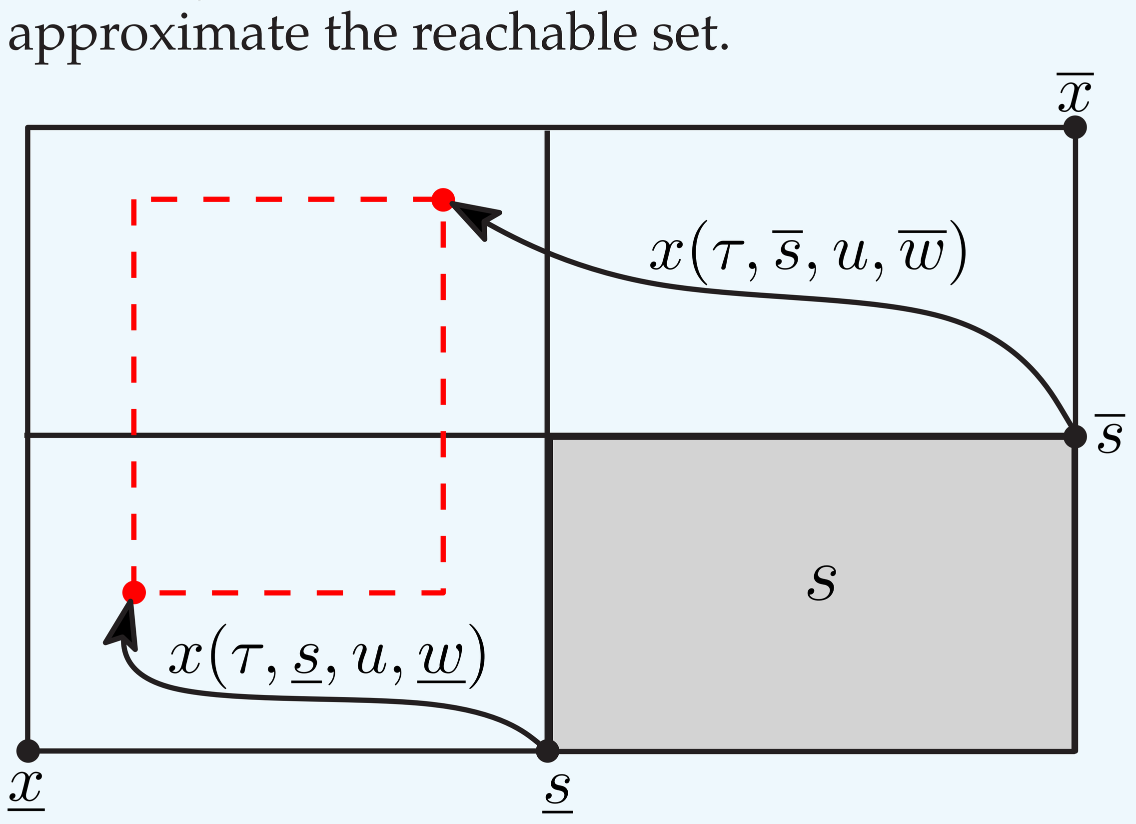
Definition 1 [1] *The* **system is monotone** *if for all $x(0) \geq x'(0)$, $\mathbf{u} \geq \mathbf{u}'$, $\mathbf{w} \geq \mathbf{w}'$, it holds for all $t \geq 0$, $x(t) \geq x'(t)$.*



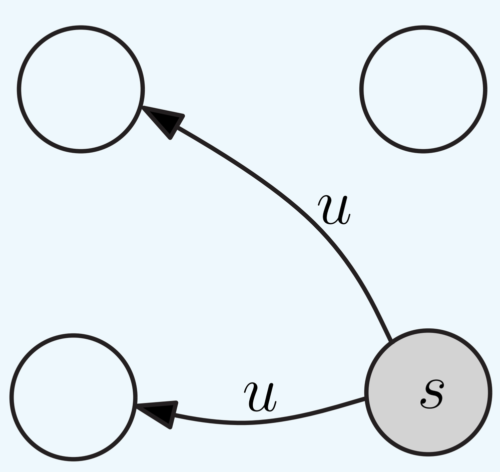
Symbolic abstraction

$\mathcal{P}$: partition of $[\underline{x}, \overline{x}]$ into intervals $s = [\underline{s}, \overline{s}]$.

Given symbol $s \in \mathcal{P}$ and control u, we over-approximate the reachable set.



Symbolic abstraction of (1): finite transition system S_a with transitions to the symbols in $\mathcal{P}$ overlapped by the over-approximation.



The symbolic abstraction S_a is **alternatingly simulated** by system (1) [3].

Bibliography

[1] D. Angeli and E.D. Sontag, Monotone Control Systems, *IEEE Transactions on Automatic Control*, Vol. 48, No. 10, pp 1684-1698, 2003.

[2] P.-J. Meyer, A. Girard and E. Witrant, Controllability and invariance of monotone systems for robust ventilation automation in buildings, *IEEE Conference on Decision and Control*, 2013.

[3] P. Tabuada, *Verification and control of hybrid systems: a symbolic approach*, Springer, 2009.

Controller synthesis

Safety specification: $x(t) \in [\underline{x}, \overline{x}]$ for all $t \geq 0$.
Fixed-point algorithm to synthesize a controller $C : \mathcal{P} \to 2^{U_a}$ keeping the state of S_a in $\mathcal{P}$.

Performance criteria: J_0 defined iteratively by

$$J_N(s) = \hat{g}(s)$$

$$J_k(s) = \min_{u \in C(s)} \left(g(s, u) + \lambda \max_{s \xrightarrow{u} s'} J_{k+1}(s') \right)$$

where N is the time horizon and $\lambda \in (0, 1)$.

Receding horizon control scheme: controller C_a^* of S_a and associated controller C^* of (1).

Performance guarantee

Proposition 1 *Let $(x_0, u_0, x_1, u_1, \dots)$ be a trajectory of (1) controlled with C^*, then*

$$\forall k \in \mathbb{N}, \; x_k \in [\underline{x}, \overline{x}].$$

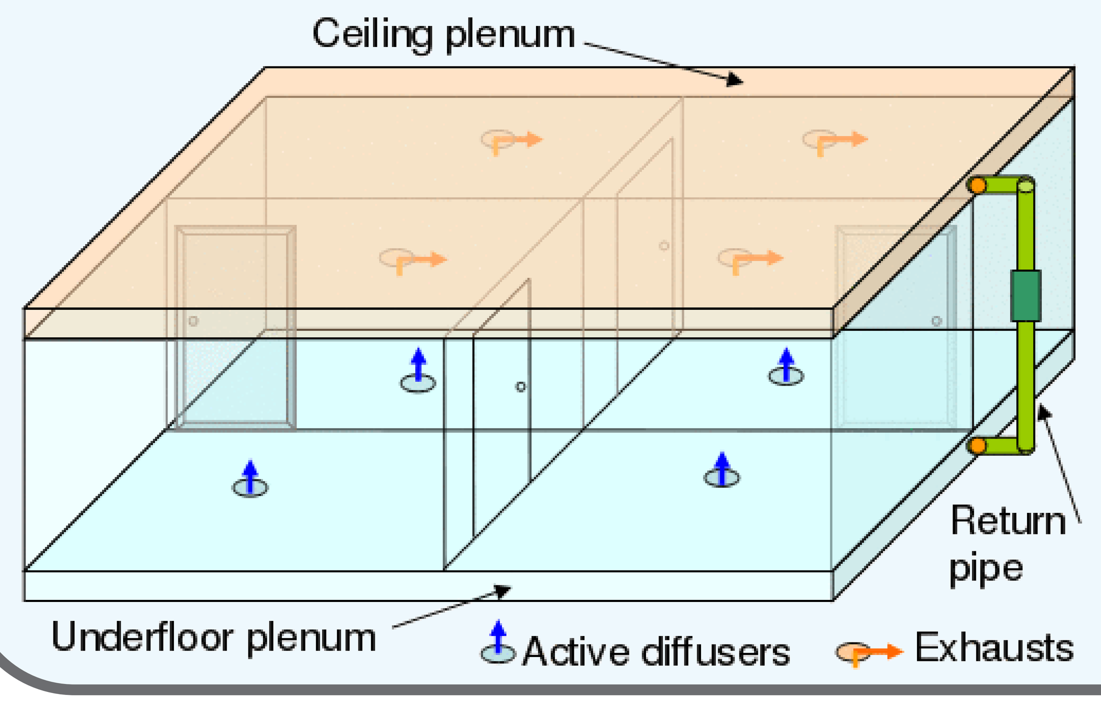
Moreover, let $s_0, s_1, \dots \in \mathcal{P}$ such that for all $k \in \mathbb{N}$, $x_k \in s_k$. Then, for all $k \in \mathbb{N}$,

$$\sum_{i=0}^{+\infty} \lambda^i g(s_{k+i}, u_{k+i}) \leq J_0(s_k) + \frac{\lambda^{N+1}}{1 - \lambda} M$$

where M is an upper bound of functions g and $\hat{g}$.

UnderFloor Air Distribution

The **temperature regulation** is done by cooling down the air in an underfloor plenum and sending it into each room of the flat using fans.



Experimental building

Based on the 4-room UFAD **experimental building** hosted by UFR PhITEM, the model for the temperature variations in each room is derived from the energy and mass conservation equations. Parameters are identified using experimental data. The model is monotone [2].

Experimental conditions

Symbolic abstraction S_a of the 4-room experimental building with:

- 4 fan control values per room
- state interval partitioned into 10^4 symbols
- control synthesis over a horizon of $N = 5$ time periods with a discount factor of $\lambda = 0.5$
- optimization of a tradeoff between the magnitude of the control inputs, their variations and the distance of the state to the center of the interval.

Control implementation

The symbolic method is applied to the 4-room building model.
The obtained controller C^* of (1) is implemented on the experimental building.
We can see that the measured temperatures (dashed blue) are correctly kept in their prescribed bounds (horizontal red lines) by the controlled ventilation (plain green).