Adaptive Multi-Observer Design for Systems with Unknown Long Input Delay ⋆

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Abstract: This paper deals with the problem of stabilization of systems with a constant delay in the actuator. The time-delay is unknown and can be considerably long, involving an interesting challenge from the control point of view. The proposed control scheme is based on a set of infinite dimensional observers with an adaptive time-delay estimation. The convergence of the observers is studied considering the time-varying delays introduced by the delay estimation. A stability analysis of the closed-loop control system is provided. The proposed observer-based controller is tested by means of numerical simulation, considering an unstable plant.

Keywords: Stability and stabilization, predictor based control, observation, control design.

1. INTRODUCTION

The stabilization of time-delay systems has been considered as an interesting problem and is a topic of numerous studies since the last decades. Delays appear in control systems due to the transport of material or information, yielding to oscillations, complex behavior or even instability, especially when the lag is large (Niculescu, 2001; Richard, 2003; Sipahi et al., 2011).

Classic control strategies have been analyzed in order to deal with time-delay systems. For instance, PID controllers for delayed processes have been extensively studied and are still of interest (Visioli and Zhong, 2011; Vilanova and Visioli, 2012). Considering unstable open-loop plants, the stability conditions are restricted by the relationship between the delay size and the system parameters, (Lee et al., 2010; Hernández-Pérez et al., 2015).

A different control approach is provided by the Smith predictor (Smith, 1957; Palmor, 1996), which consists of removing the delay from the loop by predicting the state evolution. The main drawback of the classical Smith predictor is that it is restricted to stable systems with known constant delays. Different modifications have been proposed on the classical Smith predictor in order to control unstable systems (Zhong, 2006; Normey-Rico and Camacho, 2009; Matausek and Ribic, 2012), obtaining satisfactory results. However, in some cases the use of modified Smith predictors involve control laws with distributed delays and matrix exponentials, making their practical implementation a numerical challenge.

The state prediction idea has been extended by the finite spectrum assignment (FSA) methods, (Manitius and Olbrot, 1979; Kwon and Pearson, 1980; Artstein, 1982; Witrant et al., 2003). The control law is based on a prediction of the state variable over one delay interval. The prediction is generated and a feedback of the predicted state is used, compensating the effect of the time-delay. Numerical implementation of such control laws implies approximations, which should be handled carefully in order to prevent undesirable effects on the closed loop system (Mondie and Michiels, 2003).

The use of observer based controllers for time-delay systems represents another option. The main idea is to obtain free delay signals from the process model and use them as control feedback to the plant (Del-Muro-Cuéllar et al., 2011; Márquez-Rubio et al., 2012; Novella Rodríguez et al., 2014). Recent results are devoted to the stabilization of systems with input delay by means of multiple observers that are sequentially connected (Besançon et al., 2007), showing improvements in the corresponding stability conditions (Najafi et al., 2013; Zhou et al., 2017). The main drawback of this methodology is the need for an accurate process model, including the delay size.

This work proposes an adaptive observation scheme in order to address the problem of stabilization of a linear system, potentially unstable, with a unknown input delay. Similar results are presented in (Bresch-Pietri and Krstic, 2010) using a FSA predictor feedback. In the proposed scheme, the time-delay is split in small intervals $d = D/N$, where D is the actual delay and N is the number of observers. Therefore, the main advantage of the use of sequential observers is that only an interval d is needed to estimate the value of the actual time-delay.

The paper is organized as follows. In Section 2, the problem formulation and the class of systems considered are stated. Section 3 introduces the multi-observer control scheme, including time-varying delays on its structure as well as the update law used in the delay adaptation. Section 4 is devoted to provide a stability analysis, including the error convergence and the stability of the controlled plant. Section 5 provides a numerical example, involving an unstable plant. The control performance is evaluated by means of simulations.

⋆ This work was supported by the Secretary of Education, Sciences, Technology and Innovation of Mexico City, under the grant SECITI/079/2017.

2. PROBLEM FORMULATION

Considering the linear time invariant system with input delay given by:

$$\dot{x}(t) = Ax(t) + Bu(t - D), \quad \forall t \geq 0, \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the internal state, $u(t) \in \mathbb{R}^r$ is the control input, $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times r}$ are constant and known matrices; and $D > 0$ is a constant but unknown time-delay.

Assumption 1. The pair $(A,\ B)$ is controllable, but the matrix A could be unstable. Namely, the delay free system

$$\dot{x}(t) = Ax(t) + Bu(t), \tag{2}$$

can be stabilized by means of the state feedback controller $u(t) = Kx(t)$ with the matrix of gains $K \in \mathbb{R}^{r\times n}$. The matrix K can be computed using conventional design methods.

Assumption 2. The value of the delay size D is unknown. However, a lower bound D_{min} and a upper bound D_{max} are both known.

Under the previous assumptions, this work proposes a multi-observer scheme with an adaptation to the delay estimation to stabilize system (1).

3. MULTI-OBSERVER SCHEME

An interesting scheme for the stabilization of time-delay systems of the form (1) was presented in (Najafi et al., 2013; Zhou et al., 2017), considering the case of constant known delays. This control strategy can compensate arbitrary long delays and can be applied without the computation of neither distributed terms nor matrix exponentials. Moreover, the exponential stability of linear time-varying systems with time-varying delays controlled with this method is proved by (Mazenc and Malisoff, 2017) by considering that the time-delay function is known.

3.1 Multi-observer for time-varying delays

Considering the input delay system (1), our work proposes an observer-based controller of the form

$$\begin{aligned}
\dot{z}_1(t) &= Az_1(t) + Bu(\beta^{N-1}(t)) + L_1\left(z_1(\beta(t)) - x(t)\right),\\
\dot{z}_i(t) &= Az_i(t) + Bu(\beta^{N-i}(t)) + L_i\left(z_i(\beta(t)) - z_{i-1}(t)\right),\\
\dot{z}_N(t) &= Az_N(t) + Bu(t) + L_N\left(z_N(\beta(t)) - z_{N-1}(t)\right),
\end{aligned} \tag{3}$$

for all $i = \in \{2, \cdots N-1\}$, where N is the number of observers used in the scheme and $L_j \in \mathbb{R}^{n\times n}$, for all $j \in \{1, \cdots N\}$, are gain matrices of appropriate dimensions.

The function $\beta(t) = t - d(t)$ represents the delayed function, where the time-varying delay segment $d(t)$ is bounded by:

$$\frac{D_{min}}{N} \leq d(t) \leq \frac{D_{max}}{N}. \tag{4}$$

The justification of the provided bounds is detailed later in Section 3.2. The superscript j in $\beta^j(t)$ represents the functional power of $\beta(t)$. Namely, the iterated composition of the delayed function. From this observation scheme, the predictive control law is defined by:

$$u(t) = Kz_N(t). \tag{5}$$

According to the previous definitions, the observation error can be defined as:

$$\begin{aligned}
e_1(t) &= z_1(\beta(t)) - x(t),\\
e_i(t) &= z_i(\beta(t)) - z_{i-1}(t).
\end{aligned} \tag{6}$$

Then, the error dynamics is given by

$$\begin{aligned}
\dot{e_1}(t) &= \dot{\beta}(t)\left\{Az_1(\beta(t)) + L_1e_1(\beta(t)) + Bu(\beta^{N-1}(\beta(t))\right\}\\
&\quad - Ax(t) - Bu(t-D),\\
\dot{e_i}(t) &= \dot{\beta}(t)\left\{Az_i(\beta(t)) + L_ie_i(\beta(t)) + Bu(\beta^{N-i}(\beta(t))\right\}\\
&\quad - Az_{i-1}(t) - L_{i-1}e_{i-1}(t) - Bu(\beta^{N-i+1}(t),
\end{aligned}$$

for all $i \in \{2, \cdots, N\}$. Taking into account the relation between composed functions $\beta^j(\beta(t)) = \beta^{j+1}(t)$, the error is:

$$\begin{aligned}
\dot{e_1}(t) &= Ae_1(t) + L_1e_1(\beta(t)) + Bu(\beta^N(t)) - Bu(t-D)\\
&\quad -\dot{d}(t)\left\{Az_1(\beta(t)) + L_1e_1(\beta(t)) + Bu(\beta^N(t))\right\}\\
\dot{e_i}(t) &= Ae_i(t) + L_ie_i(\beta(t)) - L_{i-1}e_{i-1}(t)\\
&\quad -\dot{d}(t)\left\{Az_i(\beta(t)) + L_ie_i(\beta(t)) + Bu(\beta^{N+1-i}(t))\right\}.
\end{aligned} \tag{7}$$

Assumption 3. In order to avoid numerical problems in the controller design, the derivative of the delay segment is assumed to be bounded, $|\dot{d}(t)| < 1$ and we consequently have that $\dot{\beta}(t) \neq 0$ for all $t \geq 0$. The following section details this assumption.

Remark 1. It is worth stressing out that in the nominal case, ***i.e.*** when the delay segment $d(t) = D/N$ is known, the sequential observers (3) have the same structure as the ones proposed in (Najafi et al., 2013; Zhou et al., 2017). However, for the unknown delay case, an update law is proposed in the next section.

3.2 Delay Update Law

Let us consider the multi-observer scheme (3), where each observation stage can be seen as a predictor of the original internal state $x(t)$. In the constant nominal case dealt in (Najafi et al., 2013; Zhou et al., 2017), the corresponding prediction allows to obtain an estimation d units of time ahead of the state $x(t)$, where $d = D/N$ with N observers in the scheme.

However, when the value of the delay is uncertain, the multi-observer designed with a constant time-delay can exhibit bad performance. Even when the plant parameters are exactly known, a small mismatch on the time-delay could yield instability on the closed-loop system. To overcome this problem, an adaptive law for the time-delay is proposed, following the methodology given in (Bresch-Pietri, 2012; Bresch-Pietri et al., 2012).

The main idea is to estimate a segment of the delay, such that $\hat{d}(t) \rightarrow D/N$ as $t \rightarrow \infty$. To determine the value of the unknown input delay, the measured state signal $x(t)$ is compared with its predicted version $z_1(t)$, taken from the multi-observer system (3). The steepest descent algorithm is used to estimate the delay. Then, we can define:

$$\tau(t) = -\left(z_1\left(t-\hat{d}(t)\right) - x(t)\right) \times \frac{\partial z_1\left(t-\hat{d}(t)\right)}{\hat{d}(t)}, \quad (8)$$

where the gradient is defined as:

$$\frac{\partial z_1\left(t-\hat{d}(t)\right)}{\partial d_e(t)} \approx \frac{z_1\left(t-\hat{d}(t)\right) - z_1\left(t-\hat{d}(t)-\delta_d\right)}{\delta_d}, \quad (9)$$

where δ_d is a constant time period. Under the assumption 2, the following condition can be stated.

Condition 1. There exist positive constants $\gamma > 0$ and $\mu > 0$ such that

$$\dot{\hat{d}}(t) = \gamma Proj_{[\underline{d},\, \overline{d}]}\{\tau(t)\}, \quad (10)$$

$$\forall t \geq 0, \quad |\tau(t)| \leq \mu, \quad (11)$$

where the $Proj_{[\underline{d},\, \overline{d}]}$ is the standard projection operator on the interval $[\underline{d},\ \overline{d}]$, with $\underline{d} = D_{min}/N$ and $\overline{d} = D_{max}/N$.

The standard projection operator is defined as follows:

$$Proj_{[\underline{d},\, \overline{d}]}\{\tau(t)\} = \begin{cases} 0, & if \ \ \hat{d}(t) = \overline{d} \ \ and \ \ \tau(t) > 0 \\ 0, & if \ \ \hat{d}(t) = \underline{d} \ \ and \ \ \tau(t) < 0 \\ \tau(t), & else \end{cases} \quad (12)$$

The descent algorithm provides an accurate estimation of the unknown input delay and guarantees that no extraneous local minimum interferes with the minimization process, considering that the initial guess of the delay is sufficiently close to the real value. In contrast to (Bresch-Pietri et al., 2012), the adaptive update law for the time-delay proposed in this work is implemented without the use of distributed terms and exponential matrices.

4. STABILITY ANALYSIS

4.1 Convergence Error

Taking into account the error dynamics (7), it can be noticed that the multi-observer scheme has a triangular structure. According to (5), the delayed control term in (7) can be written as follows:

$$u(\beta^{N+1-i}(t)) = Kz_N(\beta^{N+1-i}(t)). \quad (13)$$

According with the error definition given by (6), we find:

$$Bu(\beta^{N+1-i}(t)) = BK\left(z_{i-1}(t) + e_i(t) + \Delta_{e_i}(t)\right), \quad (14)$$

with:

$$\Delta_{e_i}(t) = \sum_{k=i+1}^{N} e_k(\beta^{k-i}(t)) \quad (15)$$

After algebraic manipulations, we can rewrite the error dynamics (7) as follows:

$$\begin{aligned} \dot{e}_i(t) = {} & Ae_i(t) + L_i e_i(\beta(t)) - L_{i-1}e_{i-1}(t) \\ & -\dot{\hat{d}}(t)\left\{(A+BK)e_i(t) + L_i e_i(\beta(t))\right. \\ & \left. +(A+BK)z_{i-1}(t) + BK\Delta_{e_i}(t)\right\}. \end{aligned} \quad (16)$$

The triangular structure of the error dynamics allows us to independently study the stability of each observer. To analyze the convergence of the error dynamics we focus on the terms in the diagonal of system (16), *i.e.* only the elements identified with subscript i and defining the following auxiliary system:

$$\begin{aligned} \dot{\epsilon}(t) = {} & A\epsilon(t) + L\epsilon(t-\hat{d}(t)) \\ & -\dot{\hat{d}}(t)\left\{(A+BK)\epsilon(t) + L\epsilon(t-\hat{d}(t))\right\}. \end{aligned} \quad (17)$$

Then, the convergence analysis can be derived from the following system with time-varying structured uncertainties:

$$\dot{\epsilon}(t) = (A+\Delta A(t))\epsilon(t) + (A_d+\Delta A_d(t))\epsilon(t-\hat{d}(t)) \quad (18)$$

with:

$$\hat{d}(t) < h \ \ and \ \ \dot{\hat{d}}(t) < \mu.$$

The uncertainties are assumed to be on the form:

$$[\Delta A(t) \quad \Delta A_d(t)] = DF(t)\,[E_a \quad E_{ad}]\,.$$

where D, E_a, and E_{ad} are constant matrices with appropriate dimensions; and $F(t)$ is an unknown, real, and possibly time-varying matrix with Lebesgue measurable elements satisfying

$$F^T(t)F(t) \leq I \ \ \forall t.$$

Associating (17) with (18) we have $A_d = L$, according to condition 1 there exists a positive constant μ and considering assumption 2 we have $h = d_{max}$. Then, the function $F(t) = \dot{\hat{d}}(t)/\mu$ and the matrices $D = I$, $E_a = -\mu(A+BK)$ and $E_{ad} = -\mu L$. Therefore, the following Theorem can be used to analyze the convergence of system (17).

Theorem 1. (Wu et al. (2010)). Consider system (18) with a delay, $\hat{d}(t)$, that satisfies both assumption 2 and condition 1. Given scalars $h = d_{max} \geq 0$ and $\mu > 0$, the system is robustly stable if there exist matrices $P > 0$, $Q \geq 0$, $Z > 0$,

$$X = \begin{bmatrix} X_{11} & X_{12} \\ \star & X_{22} \end{bmatrix} \geq 0,$$

$Y \geq 0$, any appropriately dimensioned matrices N_1 and N_2, and a scalar λ such that the following LMIs hold:

$$\Psi = \begin{bmatrix} X_{11} & X_{12} & N_1 \\ \star & X_{22} & N_2 \\ \star & \star & Z \end{bmatrix} \geq 0, \quad (19)$$

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & hA^T Z & PD \\ \star & \Phi_{22} & hA_d^T Z & 0 \\ \star & \star & -hZ & hZD \\ \star & \star & \star & -\lambda I \end{bmatrix} < 0, \quad (20)$$

with

$$\Phi_{11} = PA + A^TP + N_1 + N_1^T + Q + hX_{11} + \lambda E_a^T E_a,$$
$$\Phi_{12} = PA_d + A_d^T - N1 + N_2^T + hX_{12} + \lambda E_a^T E_{ad},$$
$$\Phi_{22} = -N_2 - N_2^T - (1-\mu)Q + hX_{22} + \lambda E_{ad}^T E_{ad},$$

A complete proof of previous Theorem can be found in (Wu et al., 2010).

Remark 2. The values of d_{max} and μ are considered as known constants from assumption 2 and condition 1 stated previously. Then, the result from Theorem 1 can be used to design the observers gains L_i such that the error in the multi-observer scheme converges asymptotically to 0. Notice that increasing the number of observers reduces in general the values of d_{max} and μ, contributing to relax the solutions of the LMIs in Theorem 1.

4.2 *Control System*

Considering a state observer given by (3), the feedback control signal is set by (5), where $z_N(t)$ is the vector corresponding to the last observed states provided by (3). The closed-loop dynamics is then obtained as:

$$\dot{x}(t) = Ax(t) + BKz_N(t-D), \tag{21}$$

where we can define:

$$z_N(t-D) = z_N(\beta^N(t)) + \Delta_z(t), \tag{22}$$

with $\Delta_z(t) = z_N(t-D) - z(\beta^N(t))$. Considering the relation among the observers error, we have:

$$\begin{aligned} z_N(\beta(t)) &= z_{N-1}(t) + e_N(t), \\ z_N(\beta^2(t)) &= z_{N-2}(t) + e_{N-1}(t) + e_N(\beta(t)), \\ &\vdots \\ z_N(\beta^N(t)) &= x(t) + \Delta_e(t), \end{aligned} \tag{23}$$

where:

$$\Delta_e(t) = \sum_{k=1}^{N} e_k(\beta^{k-1}(t)).$$

The resulting closed-loop process is given by:

$$\dot{x}(t) = A_{cl}x(t) + BK\left(\Delta_z(t) + \Delta_e(t)\right), \tag{24}$$

where K can be computed using any conventional methods such that $A_{cl} = A + BK$ is a Hurwitz stable matrix. Then, the following result can be stated.

Lemma 1. Considering the system (24): if condition 1 and the following conditions hold:

- I all the real part of the eigenvalues of A_{cl} are in the open left hand side of the complex plane,
- II the observers are designed such that the error signals converges asymptotically to 0,

then,

$$\|x(t)\| \le 2\|BK\|\xi_{max}\frac{\lambda_{max}(P)}{\lambda_{min}(Q)}\sqrt{\frac{\lambda_{max}(P)}{\lambda_{min}(P)}} \quad \forall t \ge t_0 + T,$$

where $P > 0$, $Q = Q^T > 0$ and ξ_{max} is a positive constant. $\lambda_{min}(Q)$, $\lambda_{max}(P)$ are the minimum and the maximum eigenvalues of Q and P, respectively.

Proof 1. Condition 1 implies that the derivative of the time-delay estimation is bounded and guarantees that there is not extraneous local minimum in the optimization process. Considering the delayed iterative function $\beta^N(t) = t - \overline{D}(t)$ as the total delay estimation, where according to the proposed update law this estimation has well defined bounds $D_{min} \le \overline{D}(t) \le D_{max}$. Then, the following relation holds:

$$\|\Delta_z(t)\| = \|z(t-D) - z(t - \overline{D}(t))\| \le \xi_z, \tag{25}$$

where ξ_z is a positive constant. Moreover, condition I is related to the controller design. Finally, once convergence error is guaranteed, it is possible to introduce the following bound $\|\Delta_e(t)\| \le \xi_e$, with the constant $\xi_e > 0$. Then, defining $\xi_a(t) = \Delta_z(t) + \Delta_e(t)$ and rewriting the system (1) as follows:

$$\dot{x}(t) = A_{cl}x(t) + BK\xi_a(t), \tag{26}$$

the disturbances yield by the delay estimation can be represented by $\|\xi_a(t)\| \le \xi_{max}$, where ξ_{max} is a positive constant. Hence, we can choose a matrix $Q = Q^T > 0$ and consider a quadratic positive-definite Lyapunov function:

$$V(x) = x^T(t)Px(t), \tag{27}$$

where P is the unique positive-definite symmetric solution of the algebraic equation:

$$PA_{cl} + A_{cl}^TP = -Q. \tag{28}$$

It is worth stressing on that the solution of (28) exists for any symmetric positive definite Q since A_{cl} is Hurwitz according to condition I. The time derivative of $V(x)$ along the trajectories satisfies:

$$\begin{aligned} \dot{V}(x) &= -x^T(t)Qx(t) + 2x^TPBK\xi_a(t) \\ &\le -\|x\|\left(\lambda_{min}(Q)\|x\| - \lambda_{max}(P)\|BK\|\xi_{max}\right). \end{aligned} \tag{29}$$

From (29), it follows that for all $x(t)$ located outside of the compact set:

$$B_r = \left\{x \in D \;:\; \|x(t)\| \le 2\frac{\lambda_{max}(P)}{\lambda_{min}(Q)}\|BK\|\xi_{max} = r\right\} \tag{30}$$

where it is assumed $\xi_{max} > 0$ is sufficiently small for the inclusion $B_r \subset D$ to hold. Therefore, according to (29), $\dot{V}(x) < 0$ for all $x(t)$ from the annulus

$$\Lambda = \{x \in \mathbb{R}^n \;:\; c_{min} \le V(x) \le c_{max}\}, \tag{31}$$

where c_{max} represents the maximal level set of $V(x)$ in D and c_{min} is the minimal level set of $V(x)$ that contains B_r. The corresponding ultimate bound R can be estimated as follows. Let us introduce the smallest sphere that contains Ω_{min} such that:

$$B_R = min_c\{x \in \Omega_{min} \;:\; \|x(t)\| \le c\}$$

Then, for all $x(t) \in B_R$, we have:

$$\lambda_{min}(P)\|x^2(t)\| \le x^T(t)Px(t) = \lambda_{max}(P)r^2, \tag{32}$$

and therefore

$$\|x(t)\|^2 \le \frac{\lambda_{max}(P)}{\lambda_{min}(P)}r^2 = R^2. \tag{33}$$

Then, the radius of the smallest sphere B_R that surrounds Ω_{min} is:

$$R = 2\|BK\|\xi_{max}\frac{\lambda_{max}(P)}{\lambda_{min}(Q)}\sqrt{\frac{\lambda_{max}(P)}{\lambda_{min}(P)}}$$

■

The resulting closed-loop plant (26) can be seen as a system with disturbances $\Delta_z(t)$ and $\Delta_e(t)$ induced by the delay estimation and the state prediction, respectively. The disturbances are unknown but are assumed to be bounded since the delay estimation is based on a local minimization and the convergence error is guaranteed.
The uniformly ultimate boundedness states that all the trajectories of the closed-loop system (26) will enter and remain inside a ball with radius B_r despite of the initial state $x(t_0)$ and the unknown uncertainties introduced by the state prediction $z_N(t)$, (Lavretsky and Wise, 2013).

5. NUMERICAL EXAMPLE

Let us consider an example used in (Huang and Chen, 1997; Bresch-Pietri, 2012). A second order system given by:

$$G(s) = \frac{e^{-Ds}}{(as-1)(Ts+1)},$$

with the following state-space representation:

$$\dot{x}(t) = \begin{bmatrix} 0 & \frac{1}{aT} \\ 1 & \frac{a-T}{aT} \end{bmatrix} x(t) + \begin{bmatrix} \frac{1}{aT} \\ 0 \end{bmatrix} u(t-D)$$

$$y(t) = [0\ 1]x(t)$$

The parameters of the system are $a = 5$, $T = 2.07$, and the time-delay $D = 0.939$. The multi-observer control scheme is designed considering an unknown input delay bounded by $\underline{D} = 0.8$, $\overline{D} = 1.1$. An initial condition $D(0) = 1$ is set for the delay update law, with a constant $\gamma = 3.4$.

The prediction is obtained by means of a multi-observer scheme with $N = 20$. Following the analysis given in Section 4, the computed stabilizing gains for the controller are $K = [-65.03,\ -71.16]$. For the observer, the same gain is selected for all the observers, obtaining :

$$L = \begin{bmatrix} -4 & -0.1 \\ -1 & -4.3 \end{bmatrix}.$$

The system is excited by means of a step reference, introduced at instant $t = 0\ s$ and removed at instant $t = 15\ s$. The closed-loop behavior of the state-space variables is shown in Fig. 1. The solid line indicates the state variables of the controlled plant, $x(t)$. The dashed line shows the predicted variables taken from the multi-observer scheme z_{20}. Due to the unknown delay, a significant difference between the state variables and their predictions on the first transient response can be noticed. On the contrary, the prediction is more accurate during the transient response when the step input is removed at the time instant $t = 15\ s$.

The update of the time-delay is displayed in Fig. 2. The top panel shows the time-varying delay estimation of the unknown input delay whereas the lower panel shows time derivative of the delay segment $\hat{d}(t)$ bounded with $\mu \leq 0.02$.

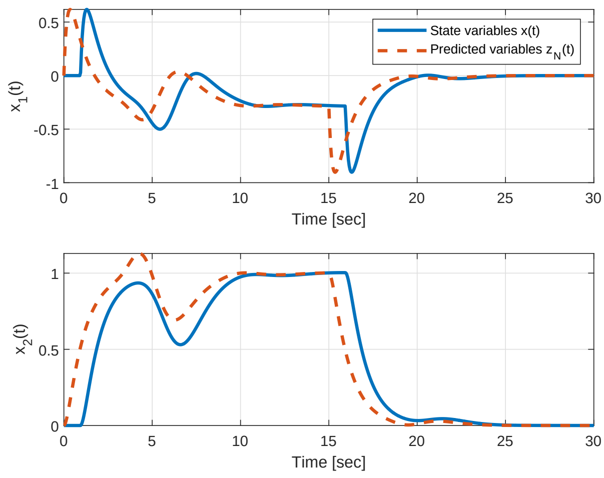

Fig. 1. Closed-loop response of the state variables $x_1(t)$ and $x_2(t)$.

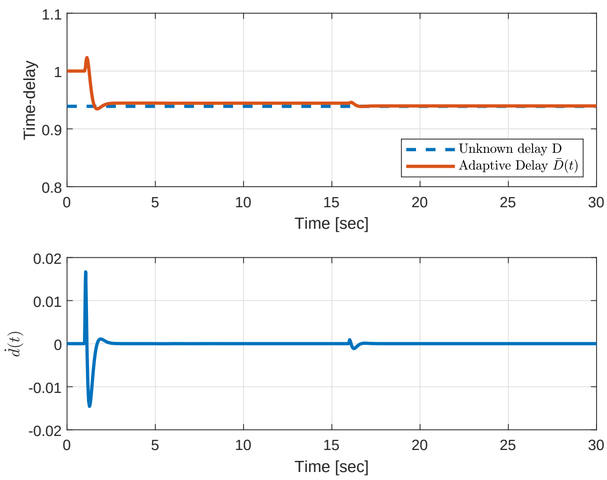

Fig. 2. Time- delay estimation.

To illustrate the performance of the multi-observer scheme, top panel in Fig. 3 shows the a comparison between the convergence error of the *first* observer, $e_1(t)$, and the last observer, $e_{20(t)}$. The corresponding control signal $u(t)$ is plotted on the bottom panel.

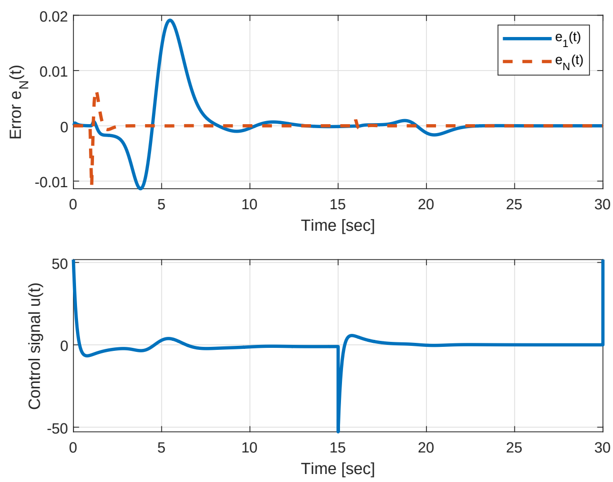

Fig. 3. Error convergence and control signal.

6. CONCLUSIONS

This work has presented a control strategy to stabilize plants with an unknown and potentially long input delay. The proposed controller is based on a multi-observer scheme including adaptation to a time-delay estimation. The estimation of the delay has been performed by means a gradient descendent method. The main advantage of the proposed scheme is that it allows us to estimate the time-delay D using just a small segment $\hat{d}(t)$. The magnitude of $\hat{d}(t)$ is inversely proportional to the number of observers N. However, it should be considered that large amount of observers increases the computational effort. A stability analysis of the controlled plant is provided considering the uniformly ultimate boundedness (UUB) concept. Moreover, the error dynamics convergence is studied by considering the time-varying delays introduced with the delay estimate.

Further works on this topic include a multi-observer adaptive controller considering unknown process parameters and unknown delay. Another interesting challenge is the use of multi-observer schemes to deal with systems with uncertain time-varying time-delays, a common case in networked control systems.

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