# Remote Stabilization via Time-Varying Communication Network Delays: Application to TCP networks

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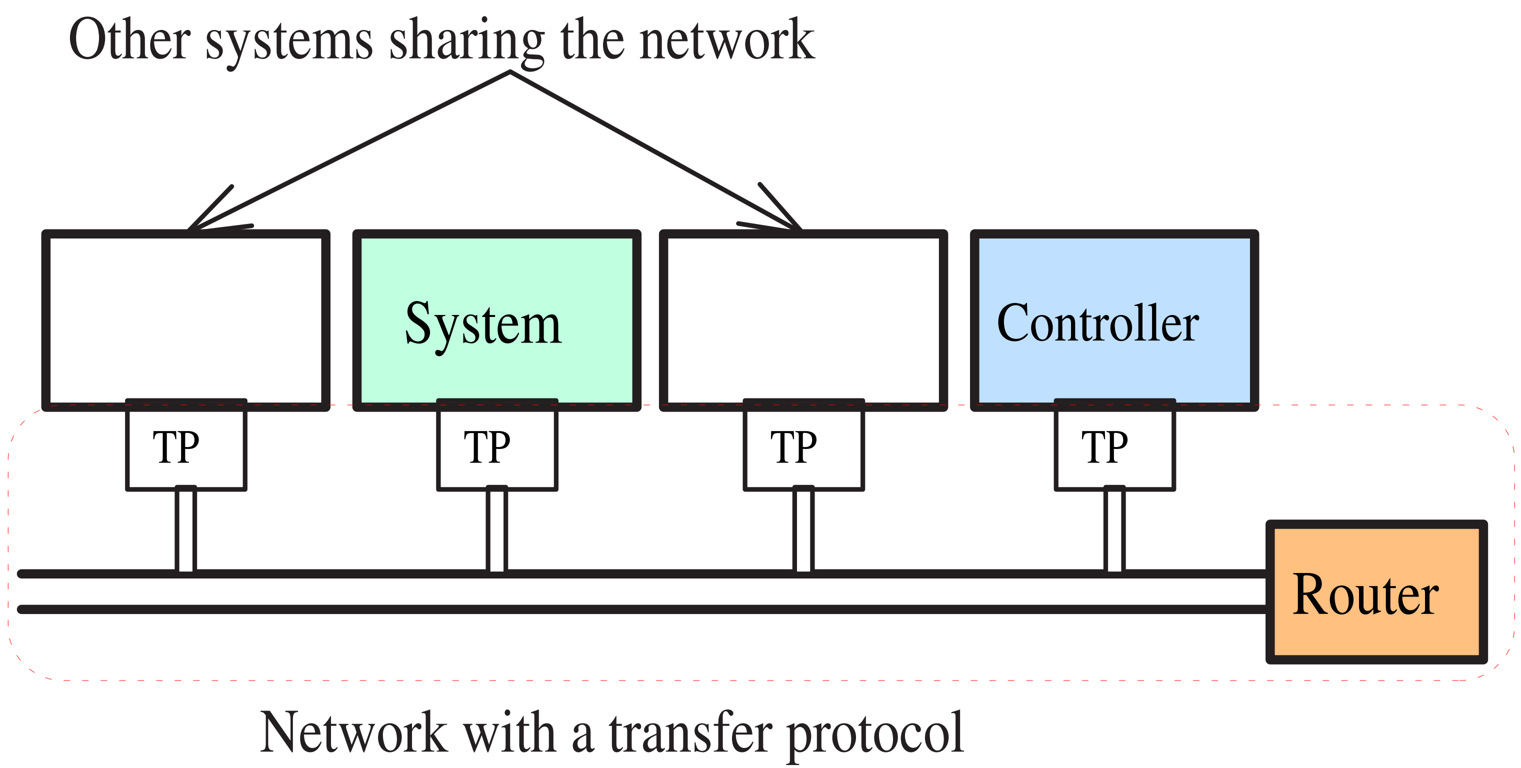



- Open-loop unstable system
- Deterministic model of the network
- Application to secure networks (TCP-SPX-LAN)

# Contents

# I. Background on Time-Delay

- Lyapunov-Krasovskii/Razumikhin approaches *[Niculescu, Kharitonov, Verriest, Yu...]*: constant time-delays/upper bound.

- Passivity *[Anderson/Spong, Niemeyer/Slotine...]*: teleoperation,

$\Rightarrow$ Pole-placement *[Kwon/Pearson, Manitius/Olbrot...]*: state predictor.

- Stability *[Bo Lincoln 03, Meinsma/Zwart, Sename...]*: robustness,

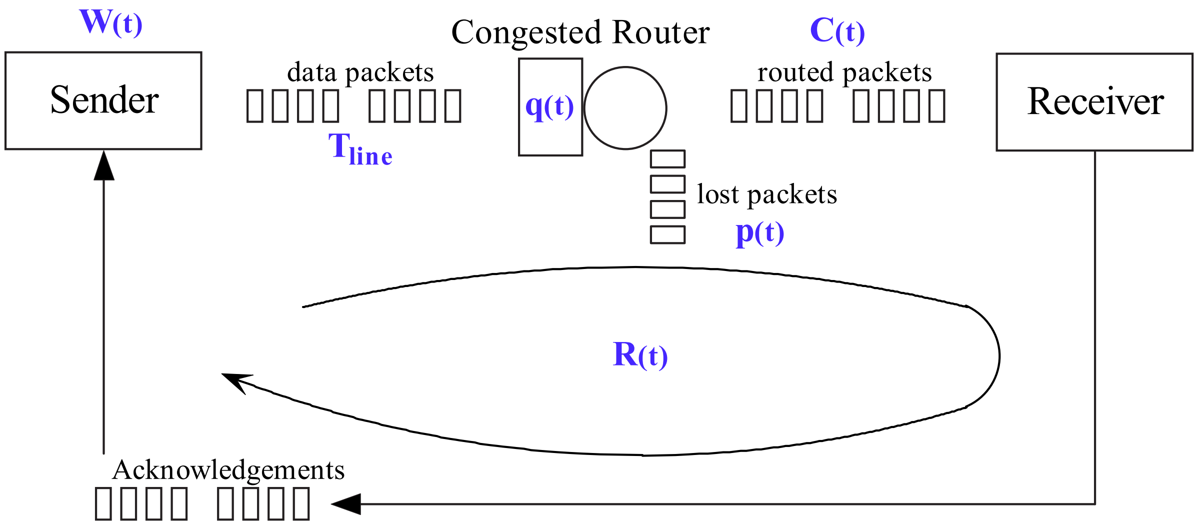
- Stochastic approach *[Nilsson 98...]*: LQG control.

# II. Problem Formulation

- The transmission protocol dynamics write as

$$\begin{aligned} \dot{z}(t) &= f(z(t), u_d(t)), \quad z(t_0) = z_0 \\ \tau(t) &= h(z(t), u_d(t)) \end{aligned}$$

i.e. for secure networks (one flow)




$$z(t) = \begin{bmatrix} W(t) \\ q(t) \end{bmatrix}$$

$$u_d(t) = \begin{bmatrix} p(t) \\ C(t) \end{bmatrix}$$

$$h(.,.) \sim R(t)$$

- The remotely controlled system has the form

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t - \tau(t)) \\ y(t) &= Cx(t) \end{aligned}$$

- Hypotheses
  - $(A, B)$ and $(A, C)$ controllable and observable
  - the network dynamics is such that

$$0 \le \tau(t) \le \tau_{max}, \quad \forall t \ge 0$$

$$\dot{\tau}(t) < 1, \qquad for\ almost\ all\ t \ge 0$$

# III. Control design

State feedback stabilization:

- based on a state predictor with a time-varying horizon $\delta(t)$ *[Artstein 82, Nihtilä 89, Uchida & all. 03]*

$$\begin{aligned} x(t+\delta) &= e^{A\delta}x(t) + e^{A(t+\delta)}\int_t^{t+\delta} e^{-A\theta}Bu(\theta-\tau(\theta))d\theta \\ u(t) &= -Kx(t+\delta) \end{aligned}$$

- explicit use of the network dynamics:

$$\delta(t) = \tau(t+\delta(t))$$

- results in the pole placement of the *time-shifted* closed-loop system

$$\dot{x}(t+\delta(t)) = (A-BK)x(t+\delta(t)) = A_{cl}\ x(t+\delta(t))$$

## Dynamic computation of $\delta(t) = \tau(t + \delta(t))$

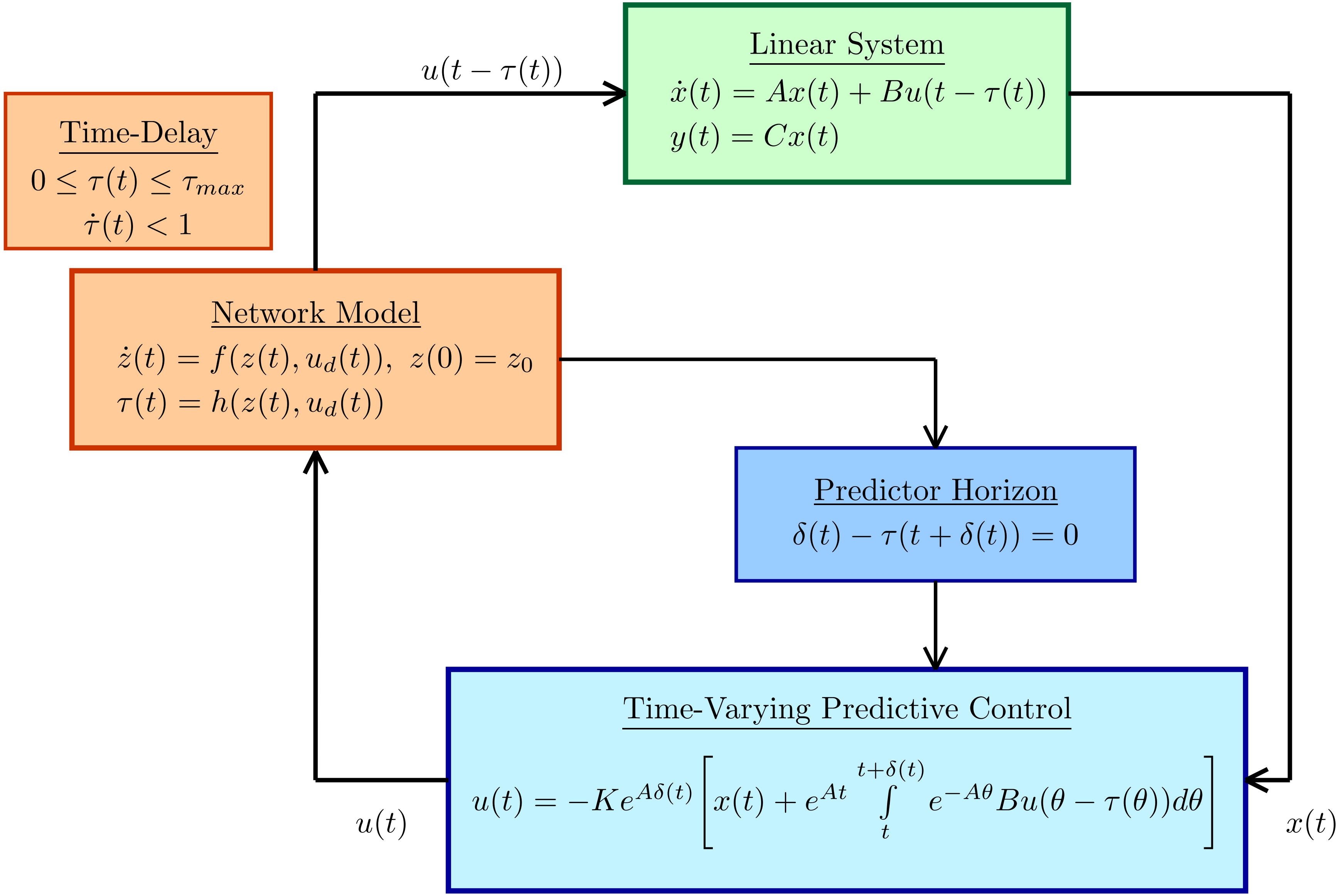
- Let
$$S(t) \doteq \delta(t) - \tau(t + \delta(t))$$
with
$$\dot{S}(t) + \lambda S(t) = 0$$
and $\lambda > 0$, to prevent for the numerical instabilities,

$\Rightarrow$ find $\dot{\delta}(t)$ such that $\delta(t)$ reaches asymptotically the manifold $S(t) = 0$.

Using the assumption $\dot{\tau} \neq 1$, $\delta(t)$ has the following dynamics

$$\dot{\delta}(t) = -\frac{\lambda}{1 - d\tau(\zeta)/d\zeta}\delta + \frac{d\tau(\zeta)/d\zeta + \lambda\tau(\zeta)}{1 - d\tau(\zeta)/d\zeta}$$

where $\zeta = t + \delta$.



Time-Delay
$0 \leq \tau(t) \leq \tau_{max}$
$\dot{\tau}(t) < 1$
$u(t - \tau(t))$
Linear System
$\dot{x}(t) = Ax(t) + Bu(t - \tau(t))$
$y(t) = Cx(t)$
Network Model
$\dot{z}(t) = f(z(t), u_d(t)), \; z(0) = z_0$
$\tau(t) = h(z(t), u_d(t))$
Predictor Horizon
$\delta(t) - \tau(t + \delta(t)) = 0$
Time-Varying Predictive Control
$u(t) = -Ke^{A\delta(t)}\left[x(t) + e^{At}\int_{t}^{t+\delta(t)} e^{-A\theta}Bu(\theta - \tau(\theta))d\theta\right]$
$u(t)$
$x(t)$

Theorem (output feedback): *With the previous hypotheses and*

$$\infty > \upsilon(t) \geq 0,$$
$$|\dot{\upsilon}(t)| < 1 \qquad \forall t$$

*where* $\upsilon(t)$ *is the time-delay of the sensor channel. The observer-based feedback control law*

$$\begin{aligned} u(t) &= -Ke^{A(\delta+\upsilon)}\hat{\hat{x}}(t) - Ke^{A(t+\delta)}\int_{t-\upsilon}^{t+\delta} e^{-A\theta}Bu(\theta-\tau(\theta))d\theta \\ \dot{\hat{\hat{x}}}(t) &= A\hat{\hat{x}}(t) + Bu(t-\upsilon-\tau(t-\upsilon)) + H\{y(t-\upsilon) - C\hat{\hat{x}}(t)\} \end{aligned}$$

*with* $\hat{\hat{x}}(t) \doteq \hat{x}(t-\upsilon(t))$ *ensures that the closed-loop system is bounded, and that the state* $x(t)$ *converges exponentially to zero.*

# IV. Application: control of an inverted pendulum through a TCP network

TCP network:

From the fluid flow model developed by *[Misra & all 00]* and assuming that $N(\zeta)$ is known at $t$, $\delta(t)$ is obtained from

$$\tau(\zeta) = \frac{1}{2}\left[\frac{q(\zeta)}{C_r} + T_{pcs}\right], \quad \frac{d\tau}{d\zeta}(\zeta) = \frac{1}{2C_r}\left[\sum_{i=1}^{N(\zeta)} \frac{W_i(\zeta)}{R_i(\zeta)} - C_r\right] \rightarrow \ \delta(t)$$

T-shape ECP Inverted Pendulum:

- Dynamics: $4^t h$ order, OL unstable, nonminimum phase, coupled nonlinearities...
- Linearized model $\rightarrow A$, $B$
- LQR synthesis $\rightarrow K$

# Experimental setup

Network model (simulated):

$$\frac{dW_1(t)}{dt} = \frac{1}{R_1(t)} - \frac{W_1(t)}{2}\frac{W_1(t-R_1(t))}{R_1(t-R_1(t))}p_1(t),$$

$$\frac{dW_2(t)}{dt} = \frac{1}{R_2(t)} - \frac{W_2(t)}{2}\frac{W_2(t-R_2(t))}{R_2(t-R_2(t))}p_2(t),$$

$$\frac{dq(t)}{dt} = -300 + \sum_{i=1}^{2}\frac{W_i(t)}{R_i(t)}, \quad q(0) = 5$$

$$\tau(t) = R_1(t)/2$$

with

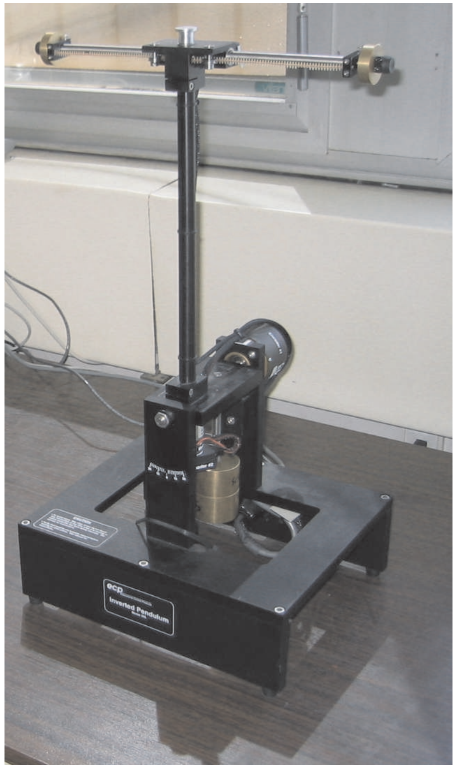
$$R_1(t) \doteq \frac{q(t)}{300} + 0.001$$

$$R_2(t) \doteq \frac{q(t)}{300} + 0.0015$$

$$p_{1,2}(t) = 0.005q(t - R_{1,2}(t))$$

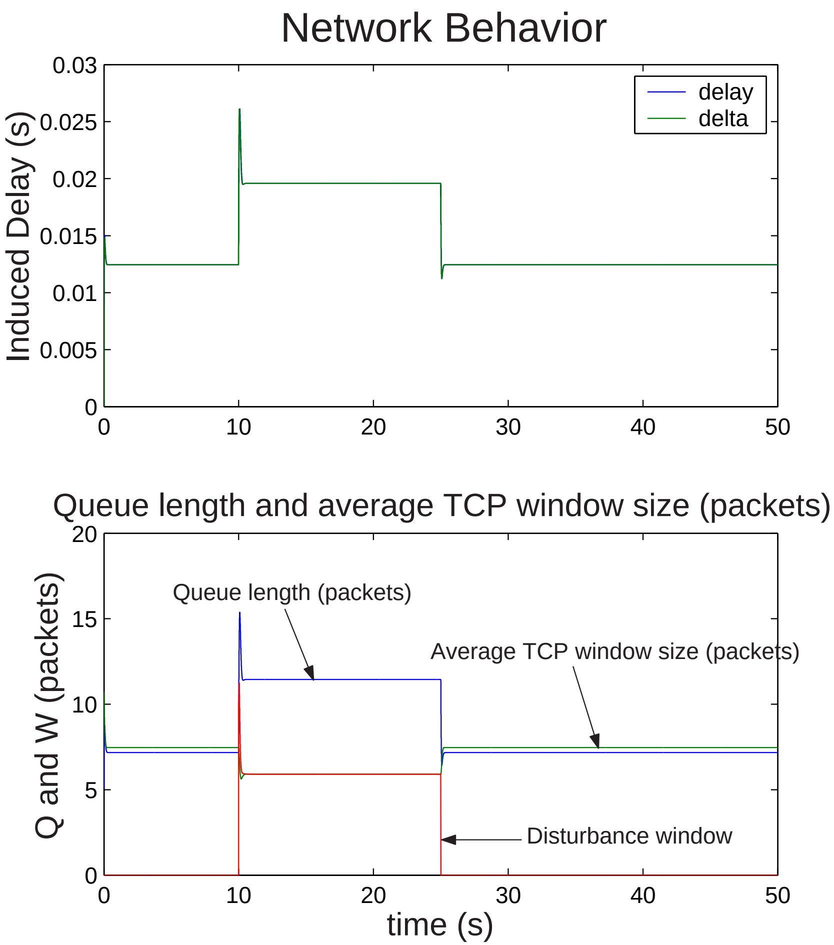
$$W_1(0) = W_2(10) = 10 packets.$$

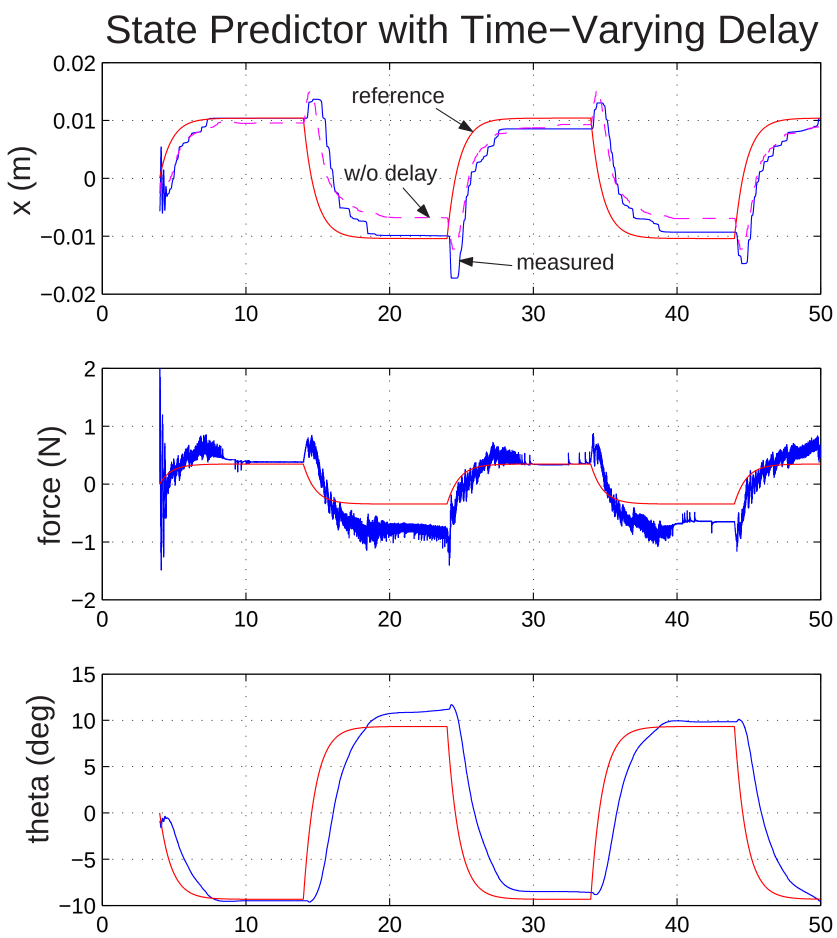
Inverted Pendulum:



$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -21.54 & 0 & 14.96 & 0 \\ 0 & 0 & 0 & 1 \\ 65.28 & 0 & -15.59 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 8.10 \\ 0 \\ -10.31 \end{bmatrix} u(t-\tau)$$

# Experimental results



# Conclusions and Perspectives

- Remote stabilization via communication networks
  $\Rightarrow$ stabilizing an open-loop unstable system with $\tau(t)$.

- The proposed controller:
  - based on a $\delta(t)$-step ahead predictor,
  - results in an exponentially converging closed-loop system and pole placement on the time-shifted system,
  - applied to remote output stabilization and observer-based control.

- Perspectives:
  - robustness with respect to uncertainties on the time-delay (finite spectrum assignment robustness),
  - consider some more specific network features.