Stabilization of Networked Controlled Systems

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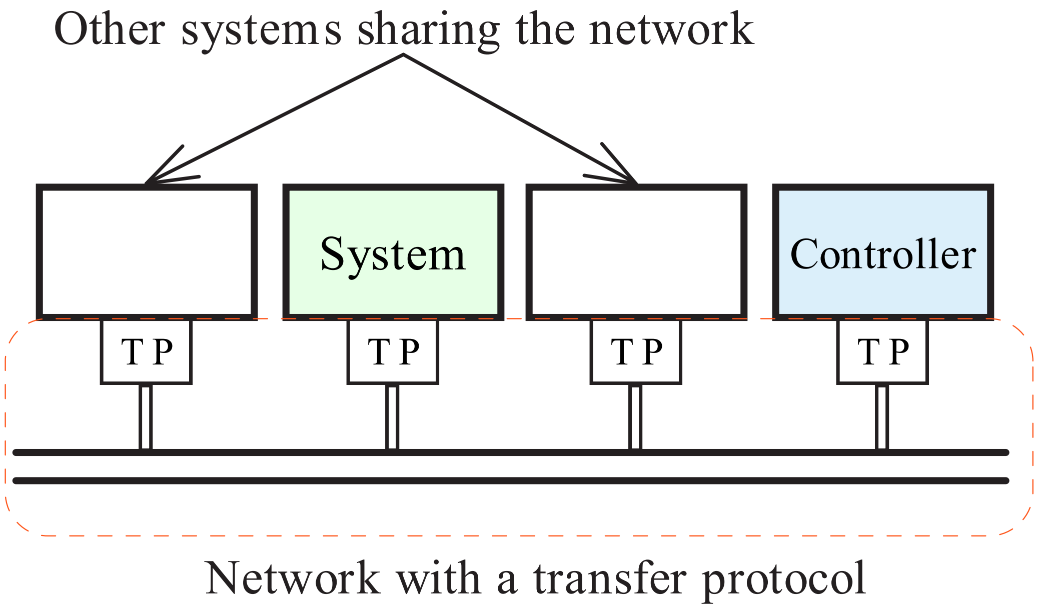


- Deterministic model of the network
  - allow for non-deterministic behavior: robustness

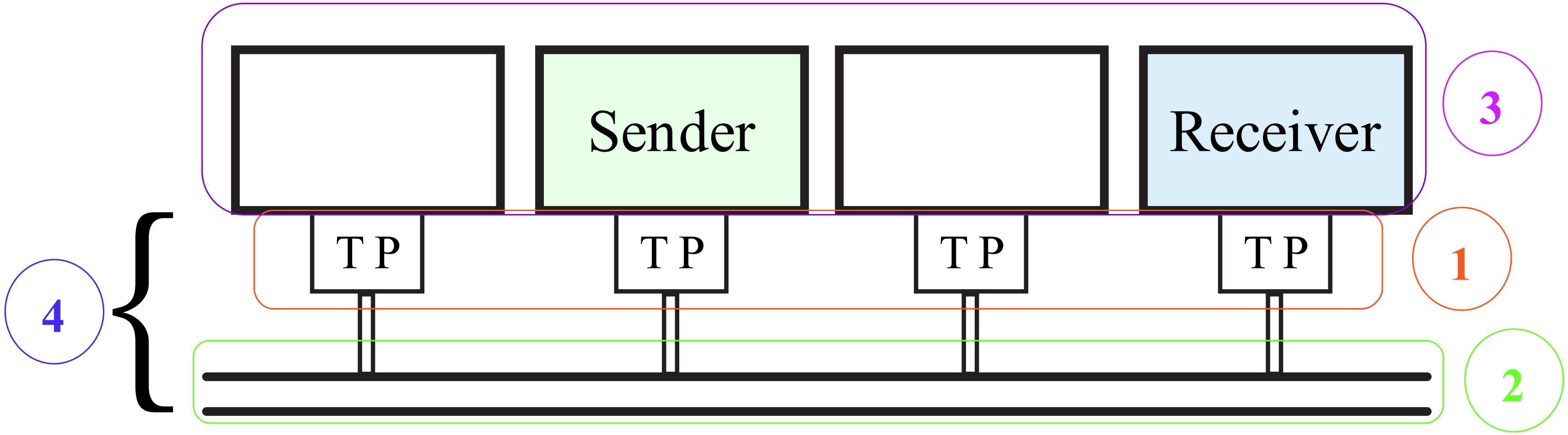
$\Rightarrow$ use system information to increase performance

- Application to secure networks (TCP-SPX-LAN)

- Open-loop unstable system

Contents

I. Overview of Network Problems



1. Quantization, encoding/decoding:
   - related to information theory,
   - control with limited information,
   - time-varying sampling,
   - differential coding - Δ-Modulation.
2. Congestion and packet loss:
   - congestion control,
   - discrete analysis and game theory.
3. Link and bandwidth allocation
   - distributed systems,
   - quality of service,
   - control under communication constraints.
4. Time-delays
   - Lyapunov-Krasovskii/Razumikhin approaches: constant time-delays/upper bound.
   - Passivity: teleoperation,
   - Stability: robustness,
   - Stochastic approach: LQG control.

$\Rightarrow$ Pole-placement: state predictor.

Modern cars:

- multiple safety/comfort devices,
- VAN/CAN,
- high jitter.

SX-29:

- open-loop unstable,
- LAN,
- high performance control.

Global Hawk (UAV):

- local + remote control,
- wideband satellite and Line-Of-Sight data link communications.

ITER:

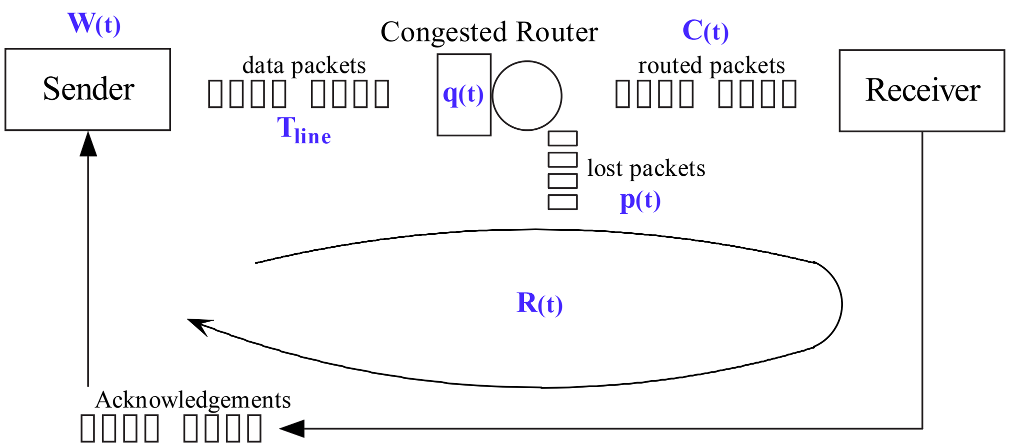
- large multi-systems device,
- LAN: control and data signals,
- scheduled tasks.

II. Problem Formulation

- The network dynamics is described by a dynamical model,

$$\begin{aligned} \dot{z}(t) &= f(z(t), u_d(t)), \quad z(t_0) = z_0 \\ \tau(t) &= h(z(t), u_d(t)) \end{aligned}$$

i.e. for secure networks (one flow)*[Misra & all 00]*: TCP with AQM



$$\begin{aligned} \frac{dW_i(t)}{dt} &= \frac{1}{R_i(q)} - \frac{W_i(t)W_i(t-R_i(q))}{2R_i(q(t-R_i(q)))}p(t) \\ \frac{dq(t)}{dt} &\approx -C + \sum_{i=1}^{N} \frac{W_i(t)}{R_i(q)} \\ R_i(q) &= \frac{q}{C} + T_{pi} \end{aligned}$$

- The remotely controlled system has the form

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t - \tau(t)) \\ y(t) &= Cx(t) \end{aligned}$$

- Hypotheses
  - (A, B) and (A, C) controllable and observable
  - the network dynamics is such that (secure network)

$$\begin{aligned} 0 \leq \tau(t) \leq \tau_{max}, &\quad \forall t \geq 0 \\ \dot{\tau}(t) < 1, &\quad for\ almost\ all\ t \geq 0 \end{aligned}$$

NB: $\tau(t)$ is the delay *experienced* by the signal, i.e. $\dot{\tau}(t) = 1 \Leftrightarrow$ the data never gets to its destination.

III. Control design

- based on a state predictor with a time-varying horizon $\delta(t)$ *[Artstein 82, Nihtilä 89, Uchida & all. 03]* *[Springer 2005]*

$$\begin{aligned} x(t+\delta(t)) &= e^{A\delta(t)}x(t) + e^{A(t+\delta(t))}\int_t^{t+\delta(t)} e^{-A\theta}Bu(\theta-\tau(\theta))d\theta \\ u(t) &= -Kx(t+\delta(t)) \end{aligned}$$

- results in the pole placement of the *time-shifted* closed-loop system

$$\frac{dx(t+\delta(t))}{d(t+\delta(t))} = (A-BK)x(t+\delta(t)) = A_{cl}\ x(t+\delta(t))$$

$\Rightarrow$ Non-linear time transformation $t \mapsto t+\delta(t)$ but exponential convergence if A_{cl} Hurwitz $\&$ hyp. on $\tau(t)$ are satisfied.

- explicit use of the network dynamics: $\delta(t) = \tau(t+\delta(t))$

Dynamic computation of $\delta(t) = \tau(t + \delta(t))$ *[IEEE CCA 2004]*

- Let

$$S(t) \doteq \hat{\delta}(t) - \tau(t + \hat{\delta}(t))$$

with

$$\dot{S}(t) + \lambda S(t) = 0$$

and $\lambda > 0$, to prevent for the numerical instabilities,

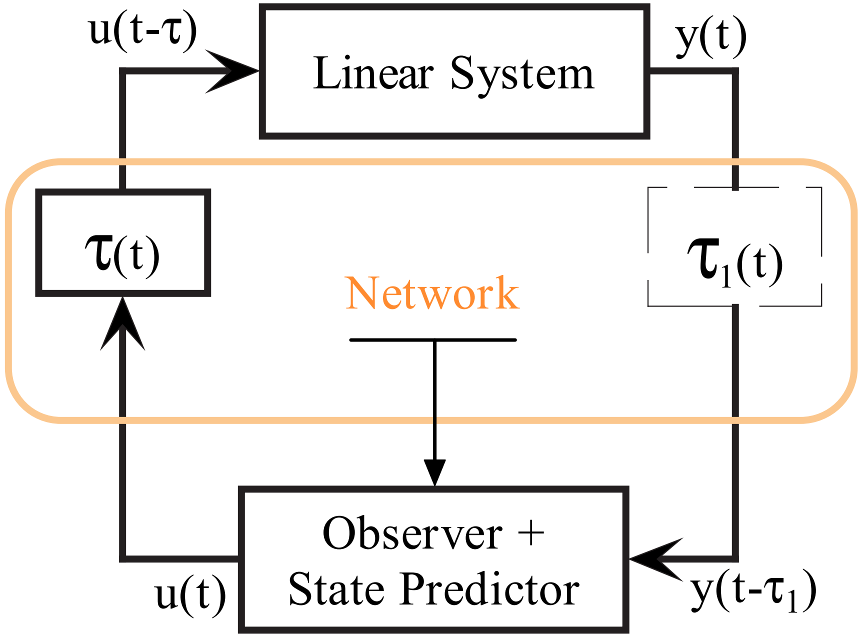
$\Rightarrow$ find $\dot{\hat{\delta}}(t)$ such that $\hat{\delta}(t)$ reaches asymptotically the manifold $S(t) = 0$.

Using the assumption $\dot{\tau} \neq 1$, $\hat{\delta}(t)$ has the following dynamics

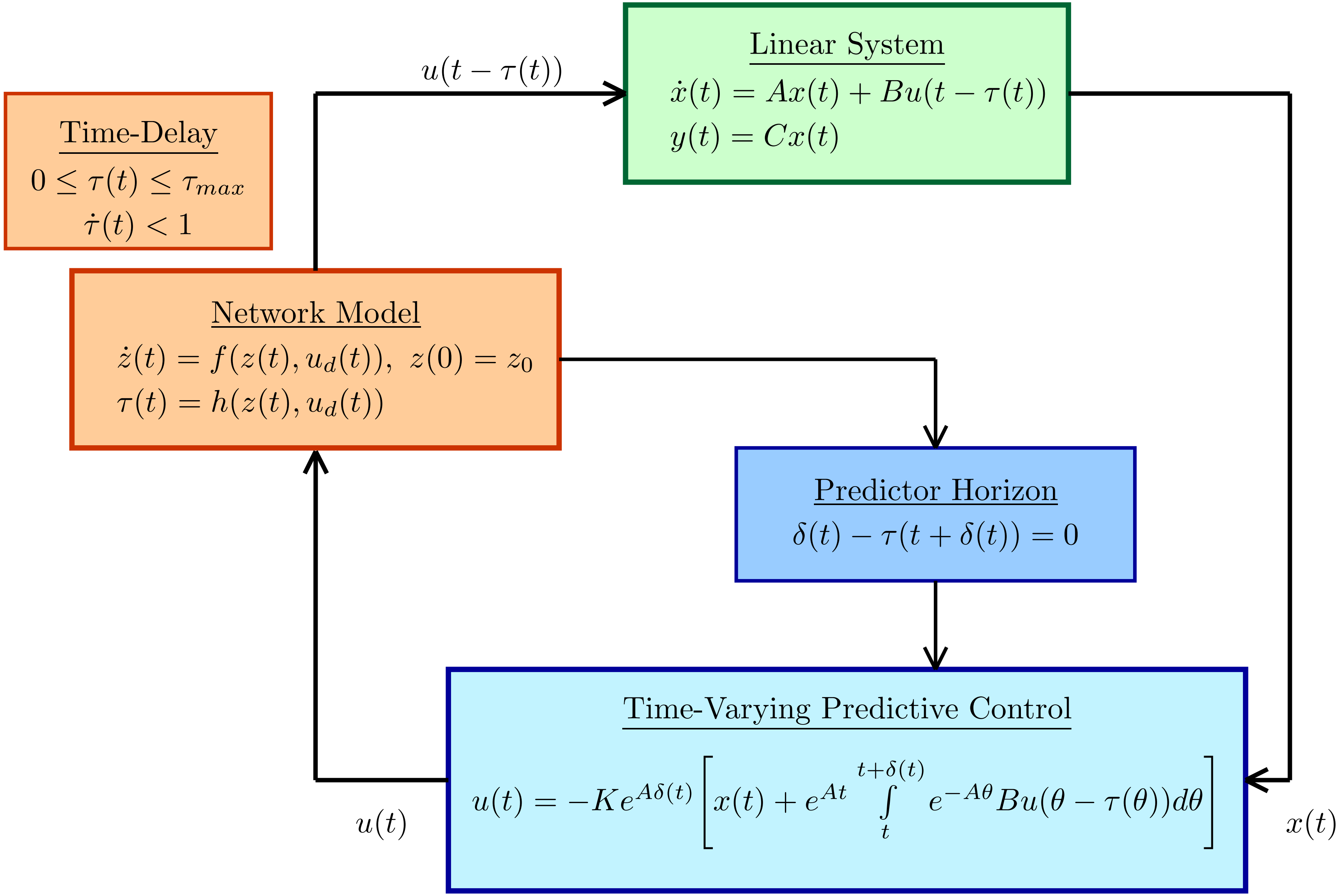
$$\dot{\hat{\delta}}(t) = -\frac{\lambda}{1 - d\tau(\zeta)/d\zeta}\hat{\delta} + \frac{d\tau(\zeta)/d\zeta + \lambda\tau(\zeta)}{1 - d\tau(\zeta)/d\zeta}$$

where $\zeta = t + \hat{\delta}$.

Output feedback and two-channels delays *[IFAC TdS 2003]*



$$
\begin{aligned}
u(t) &= -Ke^{A(\delta+\tau_1)}\hat{\hat{x}}(t) - Ke^{A(t+\delta)}\int_{t-\tau_1}^{t+\delta} e^{-A\theta}Bu(\theta-\tau(\theta))d\theta \\
\dot{\hat{\hat{x}}}(t) &= A\hat{\hat{x}}(t) + Bu(t-\tau_1-\tau(t-\tau_1)) + H\{y(t-\tau_1) - C\hat{\hat{x}}(t)\}, \quad \hat{\hat{x}}(t) \doteq \hat{x}(t-\tau_1(t))
\end{aligned}
$$



Time-Delay
$0 \leq \tau(t) \leq \tau_{max}$
$\dot{\tau}(t) < 1$
$u(t-\tau(t))$
Linear System
$\dot{x}(t) = Ax(t) + Bu(t-\tau(t))$
$y(t) = Cx(t)$
Network Model
$\dot{z}(t) = f(z(t), u_d(t)), \; z(0) = z_0$
$\tau(t) = h(z(t), u_d(t))$
Predictor Horizon
$\delta(t) - \tau(t+\delta(t)) = 0$
Time-Varying Predictive Control
$u(t) = -Ke^{A\delta(t)}\left[x(t) + e^{At}\int_t^{t+\delta(t)} e^{-A\theta}Bu(\theta-\tau(\theta))d\theta\right]$
$u(t)$
$x(t)$

IV. Robustness with respect to delay uncertainties

Problem description: $\epsilon(t) = \tau(t) - \bar{\tau}(t)$, $\{\epsilon_M, \dot{\epsilon}_M\} = \sup_t\{\epsilon(t), \dot{\epsilon}(t)\}$?

The estimated delay $\bar{\tau}(t)$ is modelled by

$$\begin{aligned}\dot{\bar{z}}(t) &= f_e(\bar{z}(t), u_{meas.}(t), u_{de}(t)), \quad \bar{z}(0) = \bar{z}_0 \\ \bar{\tau}(t) &= h_e(\bar{z}(t), u_{meas.}(t), u_{de}(t))\end{aligned}$$

The closed-loop system writes as

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t - \tau(t)) \\ u(t) &= -Ke^{A\bar{\delta}(t)}\left[x(t) + e^{At}\int_t^{t+\bar{\delta}(t)} e^{-A\theta}Bu(\theta - \bar{\tau}(\theta))d\theta\right] \\ \bar{\delta}(t) &= \bar{\tau}(t + \bar{\delta}(t)) \quad (\bar{\tau} \neq \tau \Rightarrow \bar{\delta} \neq \delta)\end{aligned}$$

$\Rightarrow$ *algebro-differential* system with a delayed state i.e. differentiating the integral term leads to

$$\begin{aligned}
\dot{\mathcal{X}}(t) &= \begin{bmatrix} A & 0 \\ -(1+\dot{\bar{\delta}})e^{-A\bar{\delta}}BKe^{A\bar{\delta}} & A-(1+\dot{\bar{\delta}})e^{-A\bar{\delta}}BKe^{A\bar{\delta}} \end{bmatrix}\mathcal{X}(t) \\
&+ \begin{bmatrix} -BKe^{A\bar{\delta}(t-\tau)} & -BKe^{A\bar{\delta}(t-\tau)} \\ 0 & 0 \end{bmatrix}\mathcal{X}(t-\tau) \\
&+ \begin{bmatrix} 0 & 0 \\ BKe^{A\bar{\delta}(t-\bar{\tau})} & BKe^{A\bar{\delta}(t-\bar{\tau})} \end{bmatrix}\mathcal{X}(t-\bar{\tau}) \\
&= \begin{bmatrix} A-BK & -BK \\ 0 & A \end{bmatrix}\mathcal{X}(t) \quad when\ \tau(t)=\bar{\tau}=0
\end{aligned}$$

$\Rightarrow$ Artstein's equivalence principle does not work.

Exact method:

- System dynamics

$$\begin{cases} \dot{x}(t) & = & Ax(t) - BKe^{A\bar{\delta}(t-\tau)}x(t-\tau) - BK\mathcal{I}(t-\tau) \\ \mathcal{I}(t) & = & \int_0^{\bar{\delta}(t)} e^{-A\theta}Bu(t+\theta-\bar{\tau}(t+\theta))d\theta \end{cases}$$

with the control law $u(t) = -Ke^{A\bar{\delta}(t)}\left[x(t) + \mathcal{I}(t)\right]$

- Proposed analysis

1. consider $\mathcal{I}(t)$ as a norm-bounded disturbance on the state :

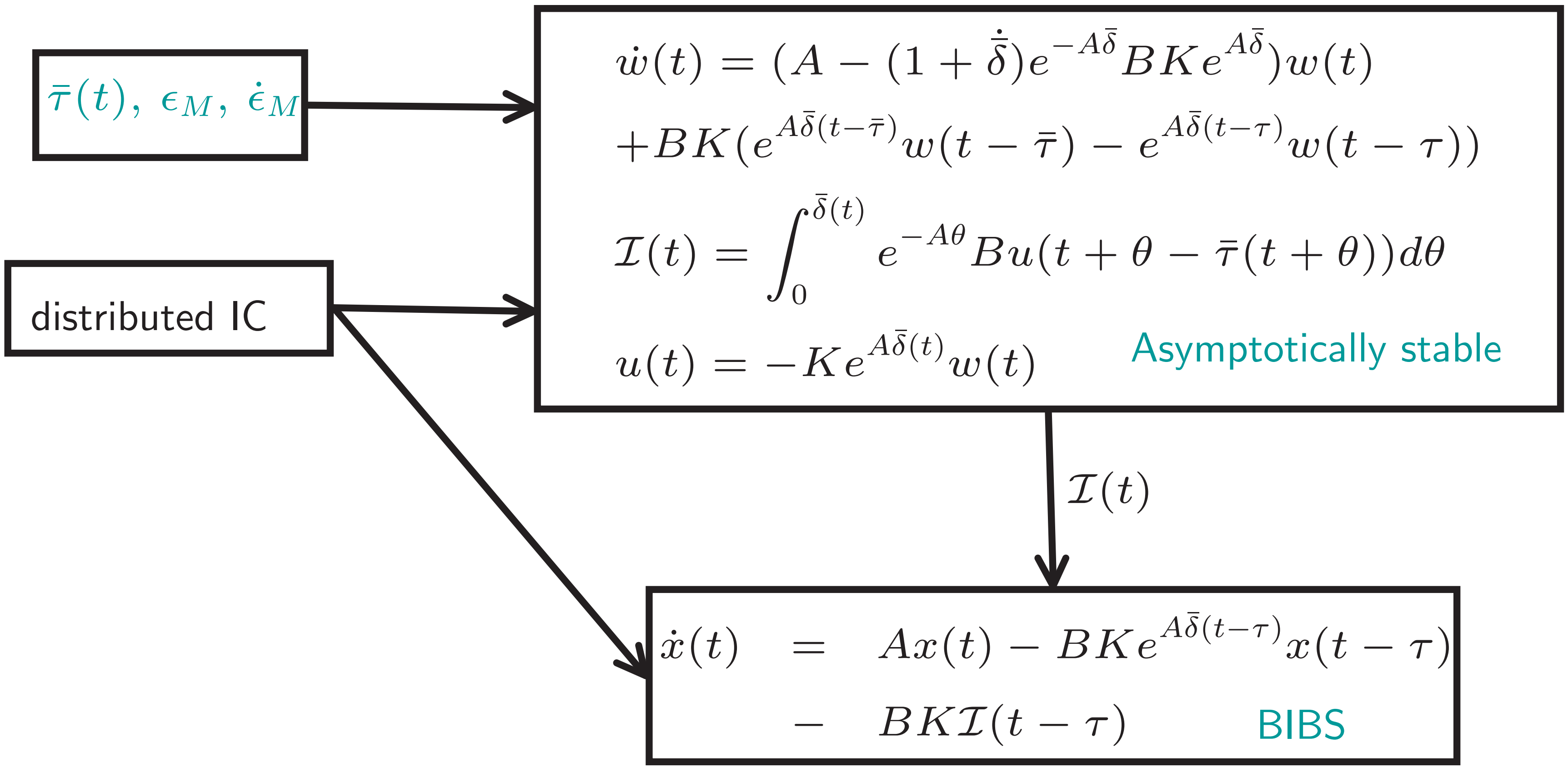
$$\Delta(t) \doteq -BK\mathcal{I}(t-\tau) \quad with \quad |\Delta(t)| < \infty \quad and \quad \lim_{t\to\infty} \Delta(t) \neq 0$$

$\longrightarrow$ the state remains bounded,

2. show that this perturbation vanishes (stability of the control law).

- Solution
  - LMI: stability of the state & control law,
  - heavy tools inducing conservatism,

$\Rightarrow$ formal approach but far from physical reality:



Approximated method: *[IEEE TAC 2005]*

- Fundamental facts

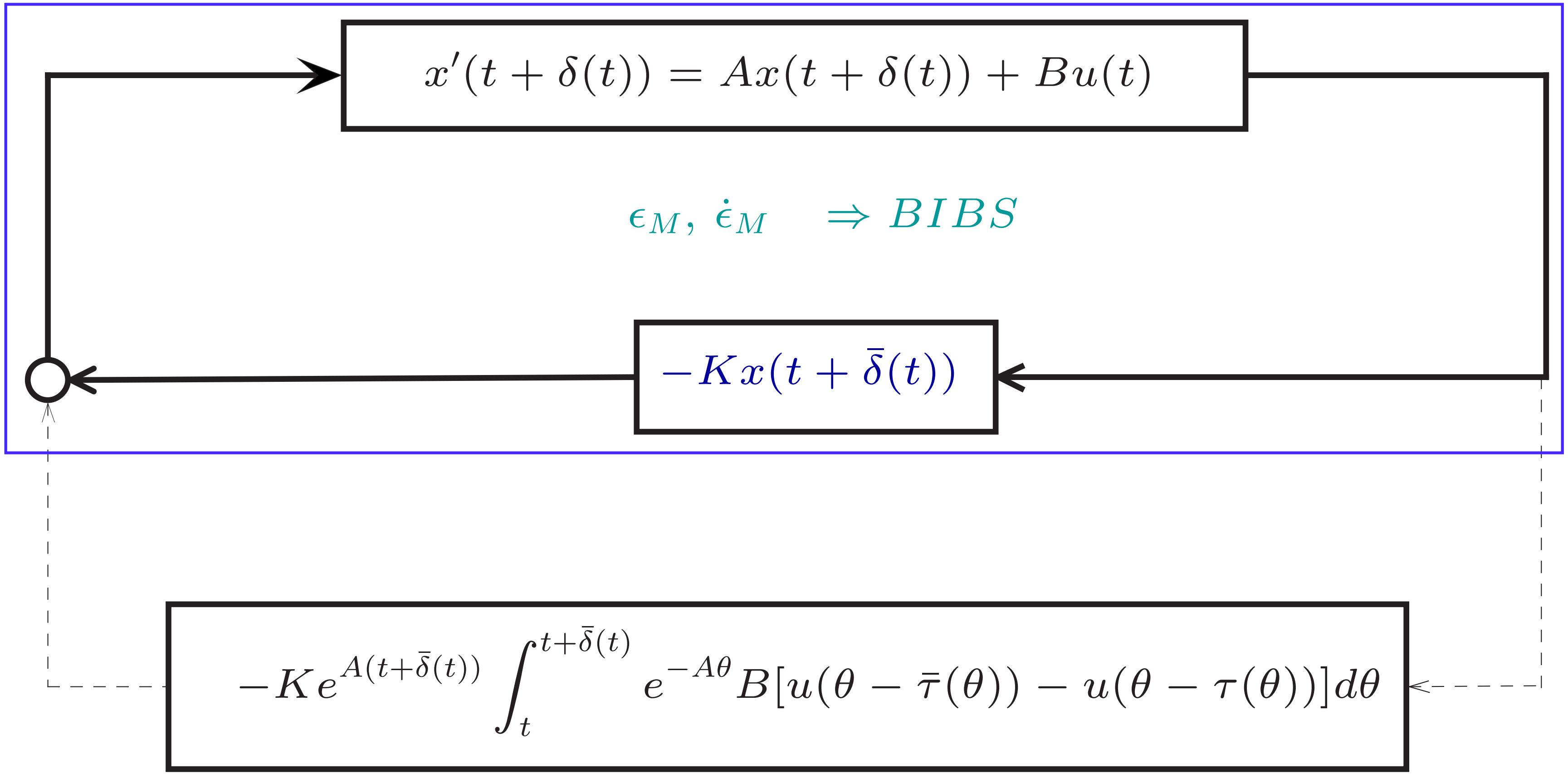
  – a norm-bounded disturbance on a nominally stable time-delayed system leads to a norm-bounded state,

  → the control law quickly exhibits slow variations,

  – the main disturbing effect comes from the fact that we have $x(t+\bar{\delta}(t))$ instead of $x(t+\delta(t))$.

- Problem formulation

$$\begin{aligned}
u(t) &= -Ke^{A\bar{\delta}(t)}\left[x(t) + e^{At}\int_t^{t+\bar{\delta}(t)} e^{-A\theta}Bu(\theta-\bar{\tau}(\theta))d\theta\right] \\
&= -Ke^{A\bar{\delta}(t)}\left[x(t) + e^{At}\int_t^{t+\bar{\delta}(t)} e^{-A\theta}Bu(\theta-\tau(\theta))d\theta\right. \\
&+ \left. e^{At}\int_t^{t+\bar{\delta}(t)} e^{-A\theta}B[u(\theta-\bar{\tau}(\theta)) - u(\theta-\tau(\theta))]d\theta\right] \\
&\simeq -Kx(t+\bar{\delta}(t))
\end{aligned}$$



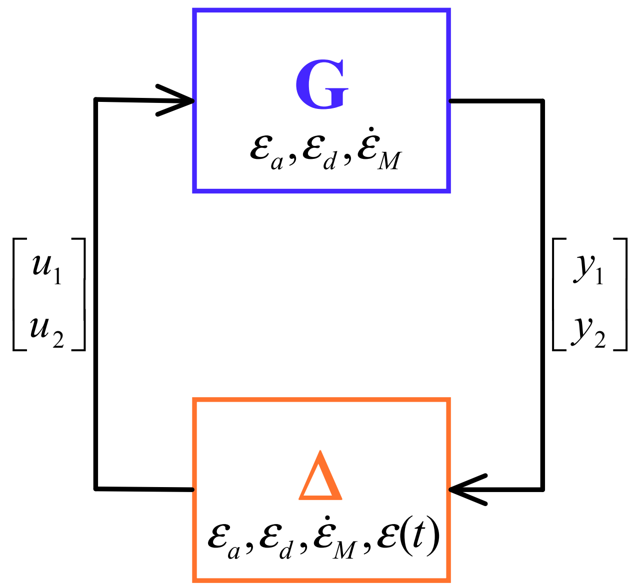
$$\Rightarrow \quad x'(t+\delta(t)) = (A-BK)x(t+\delta(t)) + BK\left(x(t+\delta(t)) - x(t+\bar{\delta}(t))\right)$$

- Proposed solution *[Gu, Kharitonov and Chen 03]*
  - Approximation of the time-varying delay: $\epsilon(t)$ is characterized by its average ϵ_a, max. deviation ϵ_d and max. variation $\dot{\epsilon}_M$

$$x'(\zeta) = Ax(\zeta) - BKx(\zeta - \epsilon_a) + BK \int_{\zeta-\epsilon(t)}^{\zeta-\epsilon_a} [Ax(\theta) - BKx(\theta - \epsilon(\theta))]\, d\theta$$

$$G \quad : \quad \begin{cases} x'(\zeta) &= Ax(\zeta) - BKx(\zeta - \epsilon_a) + \epsilon_d BK u_2(\zeta) \\ y_1(t) &= \dfrac{1}{\sqrt{1 - \dot{\epsilon}_M}} x(t) \\ y_2(t) &= Ax(t) - BKu_1(t) \end{cases}$$

$$\Delta \quad : \quad \begin{cases} u_1(t) &= \Delta_1 y_1(t) = \sqrt{1 - \dot{\epsilon}_M}\, y_1(t - \epsilon(t)) \\ u_2(t) &= \Delta_2 y_2(t) = \dfrac{1}{\epsilon_d} \displaystyle\int_{t-\epsilon(t)}^{t-\epsilon_a} y_2(\theta) d\theta \end{cases}$$



[Gu, Kharitonov and Chen 03]: $\gamma(\Delta) < 1$

– Scaled small gain: show that $\gamma_0(G_X) < 1$ for $X = diag(X_1\ X_2)$, X_1,$X_2 \in \mathbb{R}^{n\times n}$ non-singular, with

$$G \quad \subset \quad \left\{ \begin{array}{rcl} \dot{x}(t) & = & A_0 x(t) + A_1 x(t-r) + Eu(t) \\ y(t) & = & G_0 x(t) + G_1 x(t-r) + Du(t), \end{array} \right.$$

analyzed with a parameterized Lyapunov-Krasovskii functional

$$\begin{array}{rcl} V(t,\phi) & = & \phi^T(0)P\phi(0) + 2\phi^T(0)\int_{-r}^{0} Q(\xi)\phi(\xi)d\xi \\ & + & \int_{-r}^{0}\left[\int_{-r}^{0}\phi^T(\xi)R(\xi,\eta)\phi(\eta)d\eta\right]d\xi + \int_{-r}^{0}\phi^T(\xi)S(\xi)\phi(\xi)d\xi \end{array}$$

which is then discretized for LMI synthesis.

$\Rightarrow$ more realistic result:

$*$ full use of the predictor's properties,

$*$ Gu's approach is far less conservative.

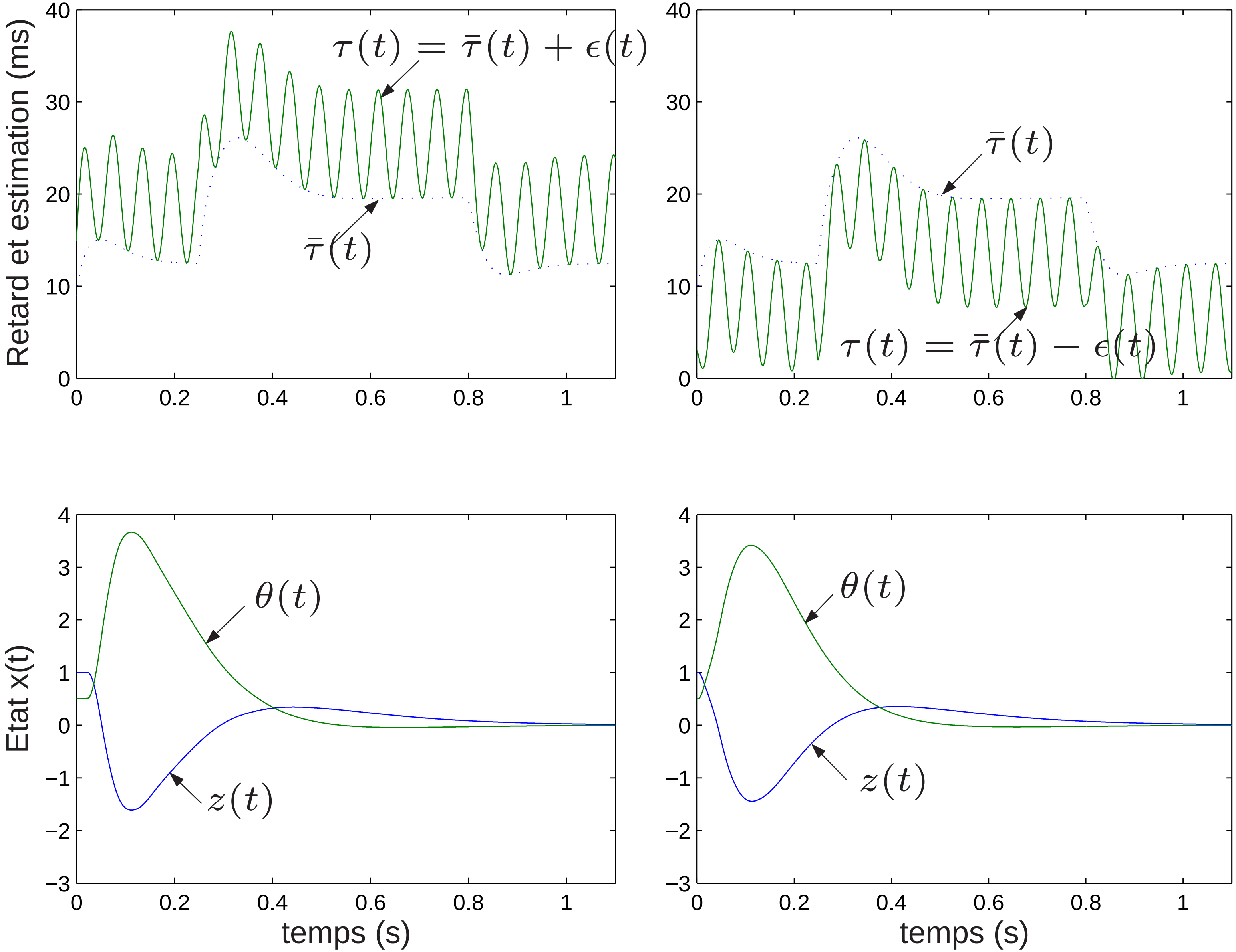
Example

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -21.54 & 0 & 14.96 & 0 \\ 0 & 0 & 0 & 1 \\ 65.28 & 0 & -15.59 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 8.10 \\ 0 \\ -10.31 \end{bmatrix} u(t-\tau)$$

$\dot{\epsilon}_M$	0.00	0.09	0.18	0.27	0.36	0.45	0.54	0.63	0.72	0.81	0.90
$\epsilon_a(ms)$	8.1	7.8	7.6	7.3	7.0	6.7	6.3	5.8	5.2	4.4	3.4

i.e. $\dot{\epsilon}_M = 0.6167, \epsilon_d = 2\epsilon_a \Rightarrow \epsilon_a = 5.9ms$

$$\epsilon(t) = \epsilon_a + \epsilon_a \sin\left(\frac{\dot{\epsilon}_M}{\epsilon_a} t\right)$$


Retard et estimation (ms)
$\tau(t) = \bar{\tau}(t) + \epsilon(t)$
$\bar{\tau}(t)$
$\bar{\tau}(t)$
$\tau(t) = \bar{\tau}(t) - \epsilon(t)$
Etat x(t)
$\theta(t)$
$z(t)$
$\theta(t)$
$z(t)$
temps (s)
temps (s)

V. Application: control of an inverted pendulum through a TCP network

T-shape ECP Inverted Pendulum:

- Dynamics: 4^{th} order, OL unstable, nonminimum phase, coupled nonlinearities...
- Linearized model $\longrightarrow A, B$
- LQR synthesis $\longrightarrow K$

TCP network:

From the fluid flow model developed by *[Misra & all 00]* and assuming that $N(\zeta)$ is known at t, $\delta(t)$ is obtained from

$$\tau(\zeta) = \frac{1}{2}\left[\frac{q(\zeta)}{C_r} + T_{pcs}\right], \quad \frac{d\tau}{d\zeta}(\zeta) = \frac{1}{2C_r}\left[\sum_{i=1}^{N(\zeta)} \frac{W_i(\zeta)}{R_i(\zeta)} - C_r\right] \longrightarrow \delta(t)$$

Experimental setup

Network model (simulated):

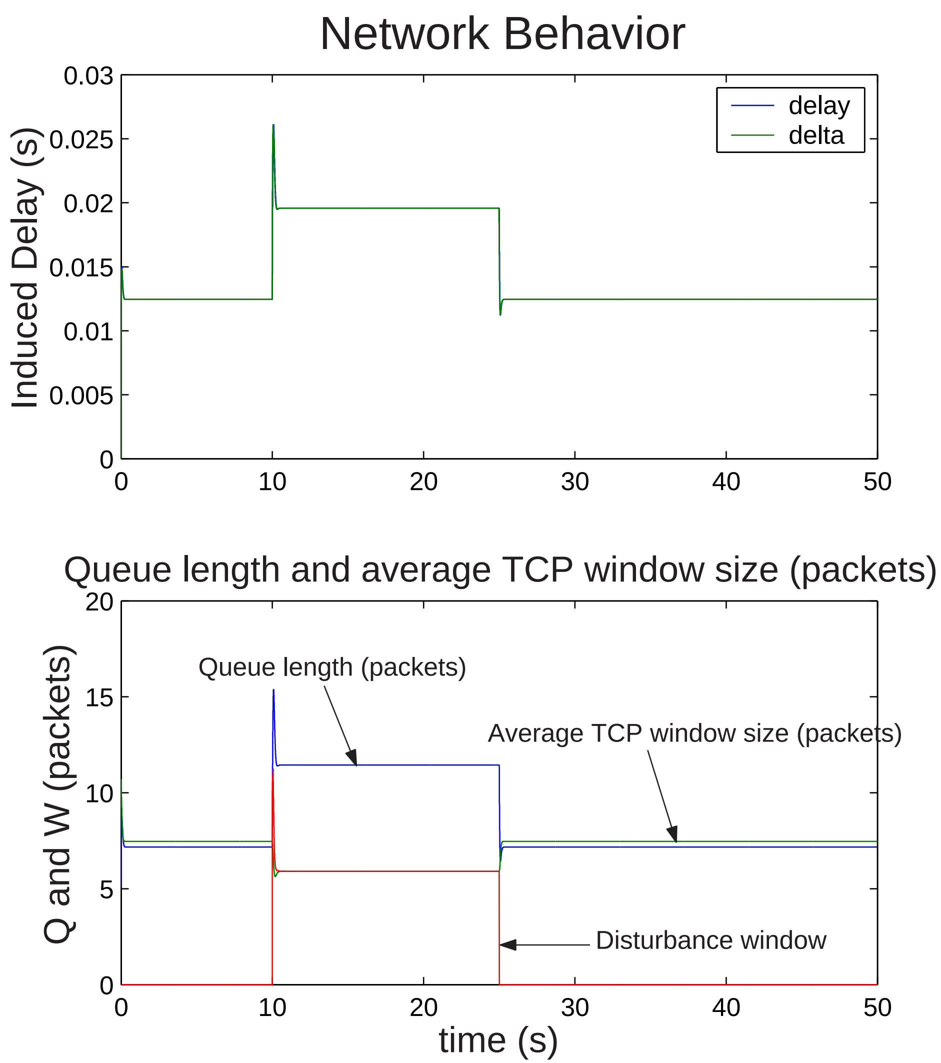
$$
\begin{aligned}
\frac{dW_1(t)}{dt} &= \frac{1}{R_1(t)} - \frac{W_1(t)}{2}\frac{W_1(t-R_1(t))}{R_1(t-R_1(t))}p_1(t),\\
\frac{dW_2(t)}{dt} &= \frac{1}{R_2(t)} - \frac{W_2(t)}{2}\frac{W_2(t-R_2(t))}{R_2(t-R_2(t))}p_2(t),\\
\frac{dq(t)}{dt} &= -300 + \sum_{i=1}^{2}\frac{W_i(t)}{R_i(t)}, \quad q(0)=5\\
\tau(t) &= R_1(t)/2
\end{aligned}
$$

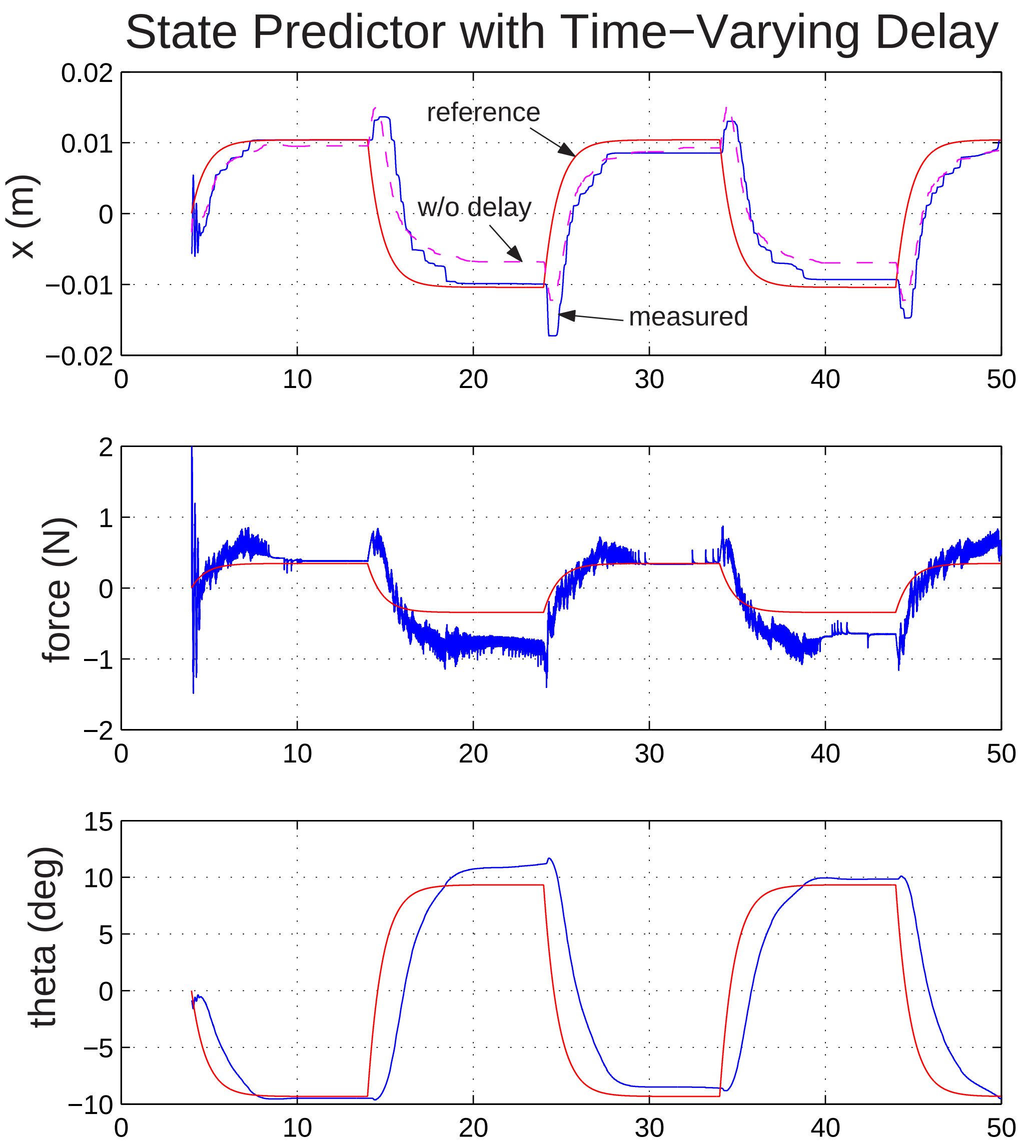
with

$$
\begin{aligned}
R_1(t) &\doteq \frac{q(t)}{300} + 0.001\\
R_2(t) &\doteq \frac{q(t)}{300} + 0.0015\\
p_{1,2}(t) &= 0.005q(t-R_{1,2}(t))\\
W_1(0) &= W_2(10) = 10packets.
\end{aligned}
$$

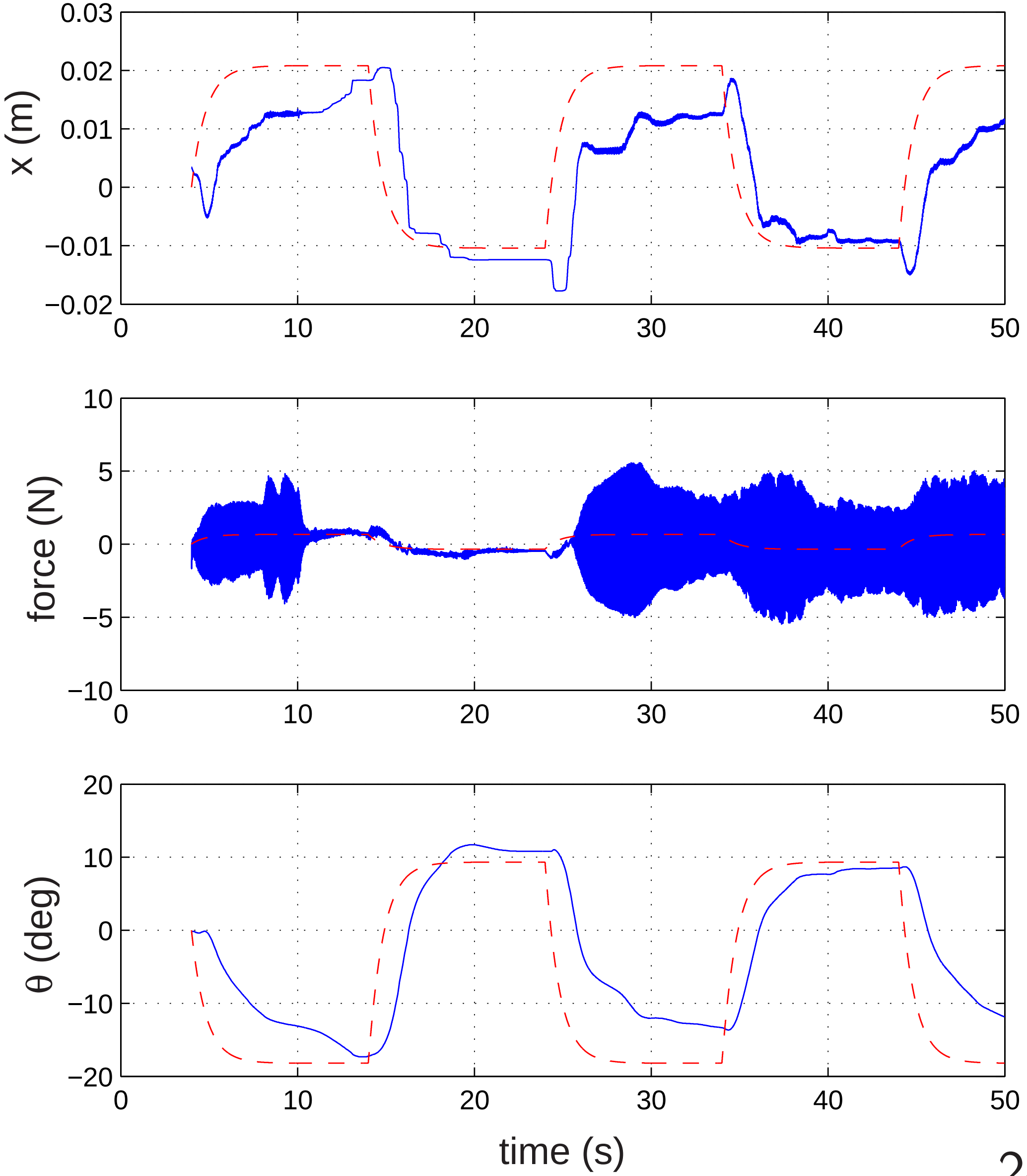
Inverted Pendulum:

Experimental results [video]





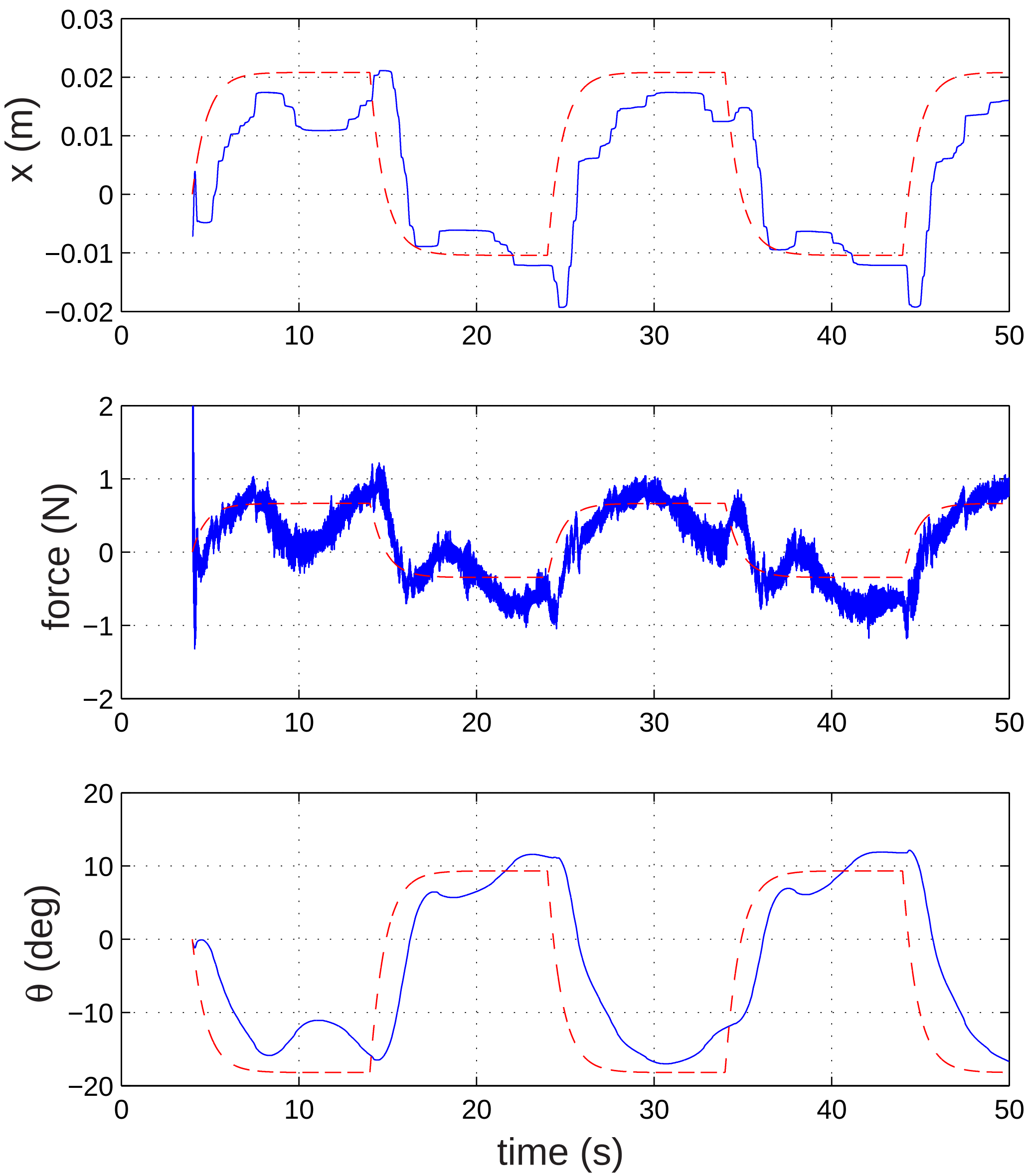
Comparison with other methods



Fixed horizon predictor:

$$u(t) = -K\left[e^{A\tau_{max}}x(t) + e^{A(t+\tau_{max})}\int_t^{t+\tau_{max}} e^{-A\theta}Bu(\theta-\tau_{max})d\theta\right]$$

- better when $\tau(t)$ close to τ_{max},
- HF disturbance on the control signal,
- deteriorated system response.

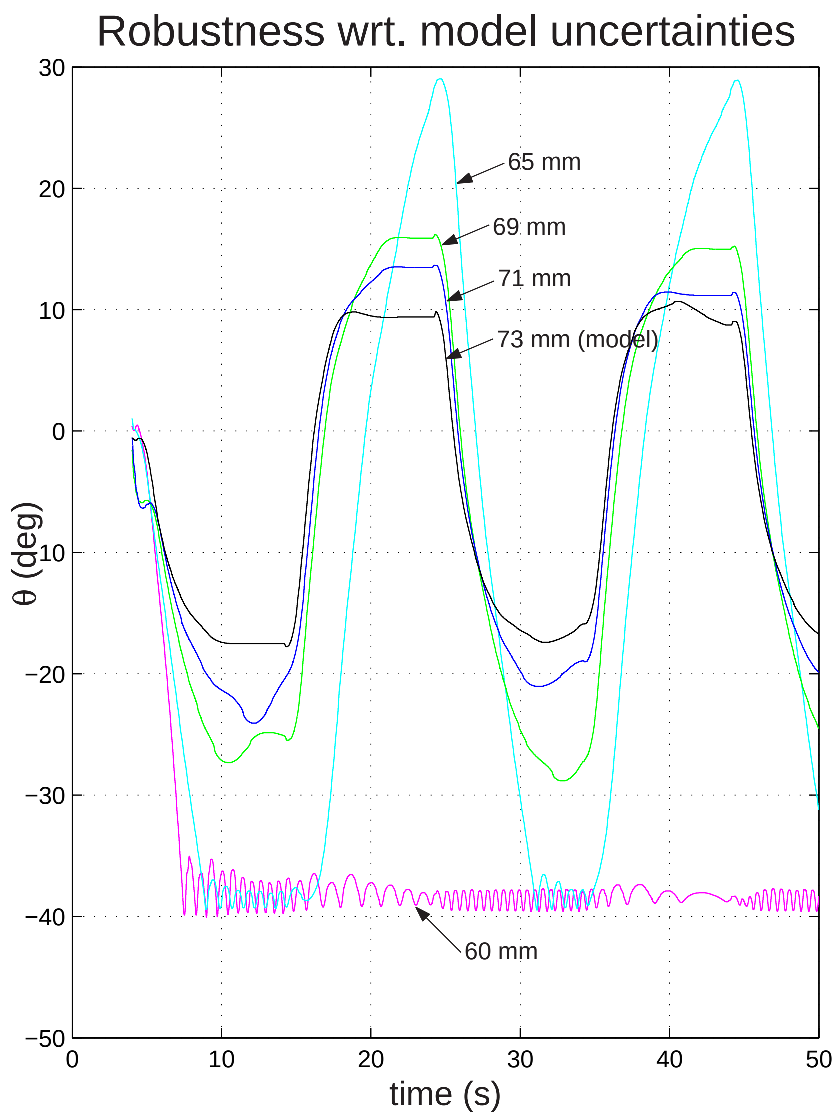


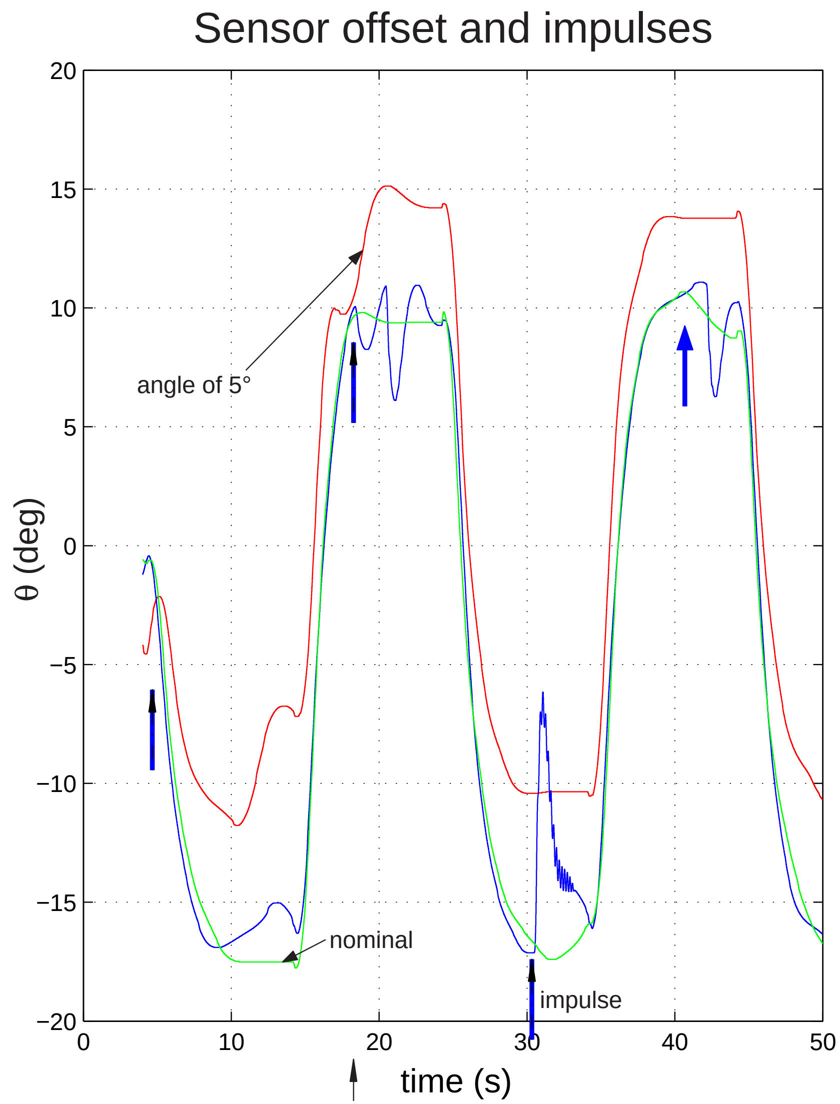
Buffer strategy:

$$u(t) = -K\left[e^{A\tau_{max}}x(t) + e^{A(t+\tau_{max})}\int_t^{t+\tau_{max}} e^{-A\theta}Bu(\theta-\tau_{max})d\theta\right]$$

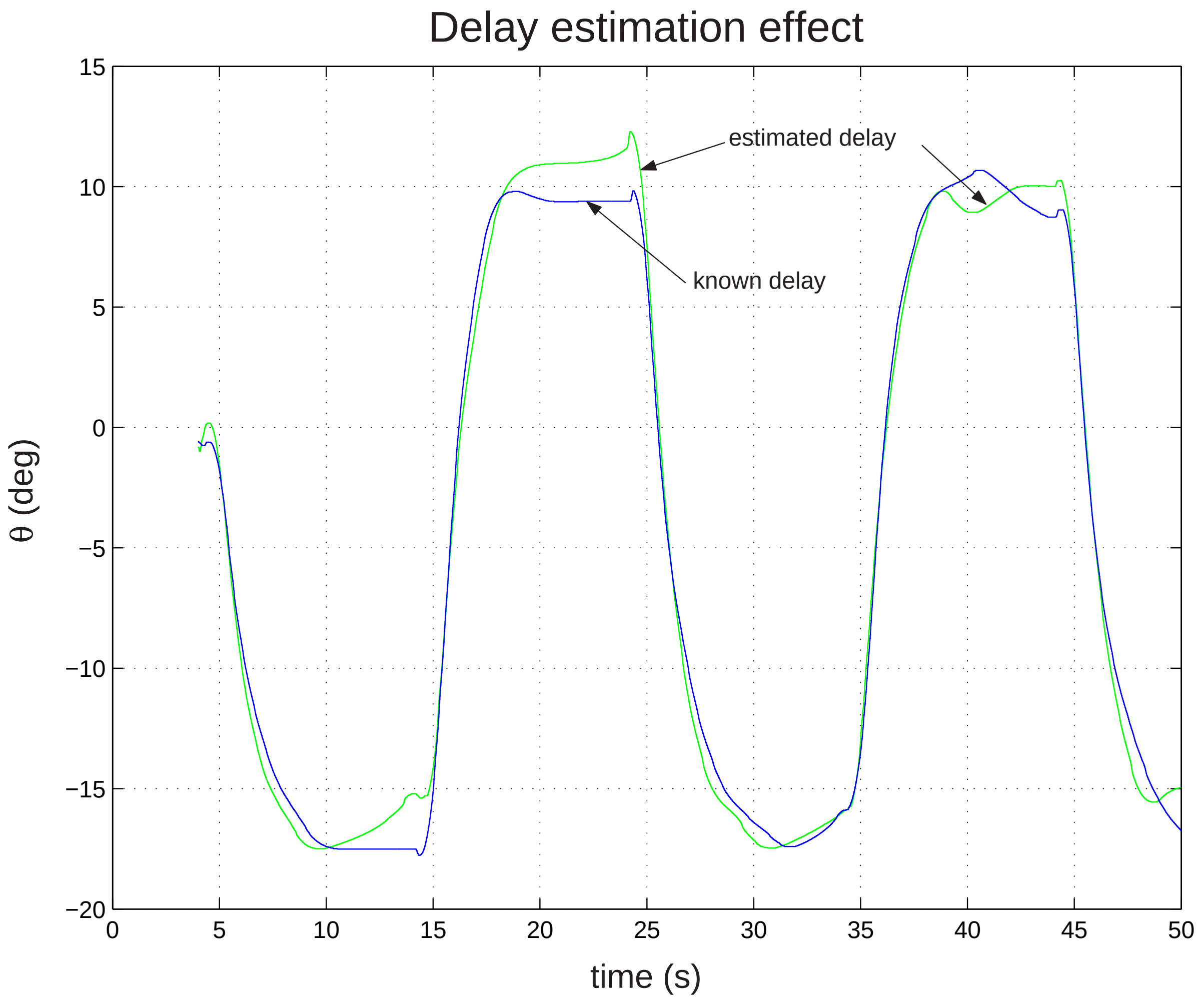
- a buffer with delay $\tau_{max} - \tau(t)$ is set at the system's input,
- HF disturbance on the control signal,
- deteriorated system response.

Robustness issues





Robustness issues (2)



Conclusions and Perspectives

- Remote stabilization via communication networks
  $\Rightarrow$ stabilizing an open-loop unstable system with a time-varying delay.

- The proposed controller:

  - based on a $\delta(t)$-step ahead predictor,
  - results in an exponentially converging (non uniform) closed-loop system and pole placement on the time-shifted system,
  - applied to remote output stabilization and observer-based control,
  - robust with respect to time-delay uncertainties.

- Perspectives:

  - experiments: faster system or longer delay (wireless),
  - extension to the nonlinear case,
  - investigate the network delay estimation and the dedicated network control *[Briat05]*,
  - coupling between the system controller and the dedicated network controller.