Stabilisation of network controlled systems with a predictive approach

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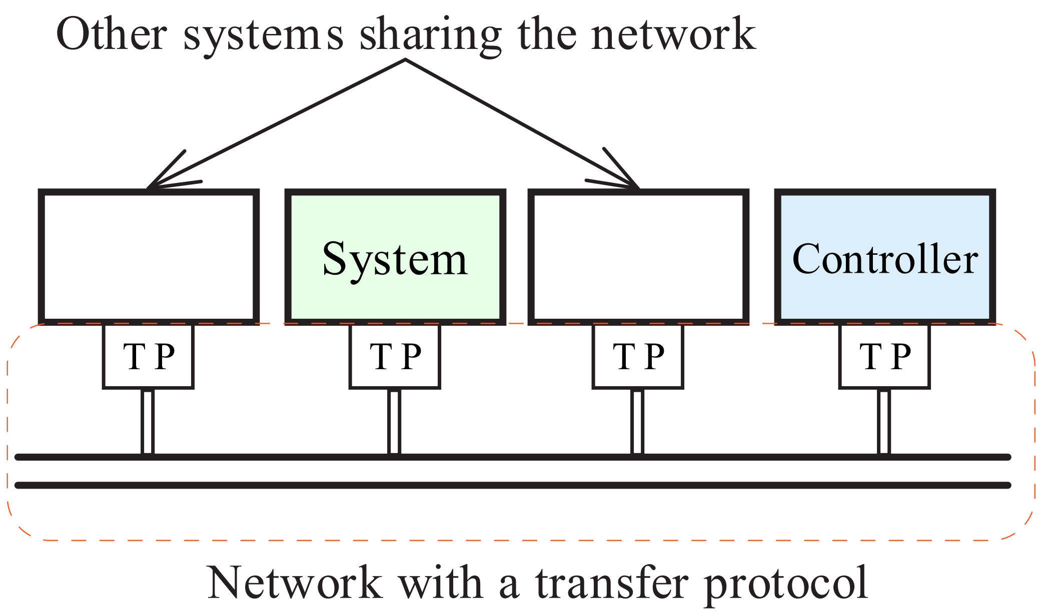


- Deterministic model of the network
  - allow for non-deterministic behavior: robustness

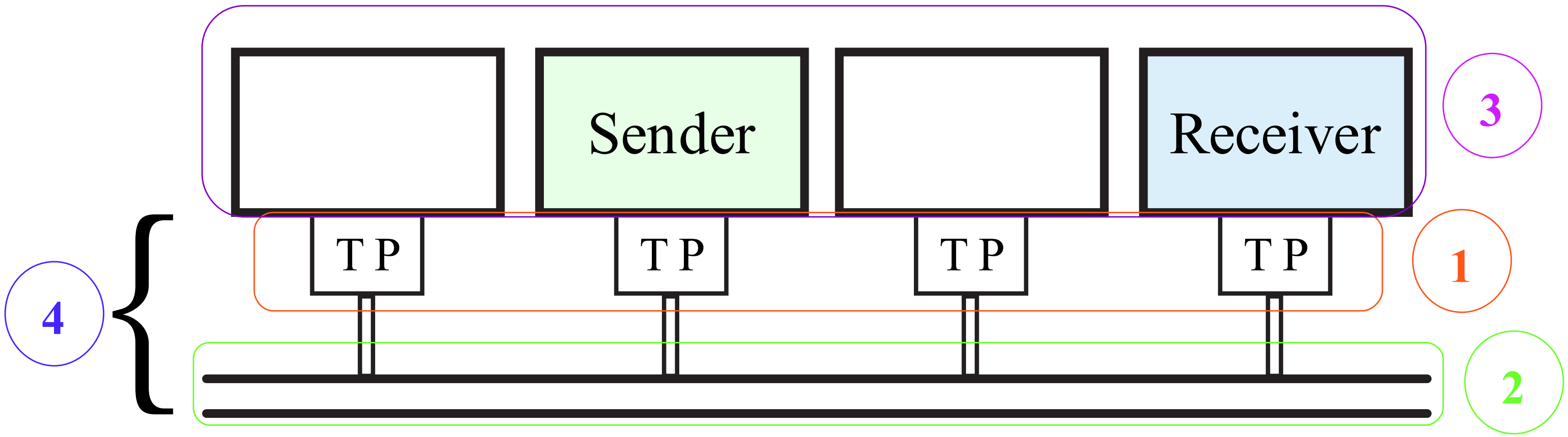
$\Rightarrow$ use system information to increase performance

- Application to secure networks (TCP-SPX-LAN)

- Open-loop unstable system

Contents

I. Overview of Network Problems



1. Quantization, encoding/decoding:
   - related to information theory,
   - control with limited information,
   - time-varying sampling,
   - differential coding - Δ-Modulation.
2. Congestion and packet loss:
   - congestion control,
   - discrete analysis and game theory.
3. Link and bandwidth allocation
   - distributed systems,
   - quality of service,
   - control under communication constraints.
4. Time-delays
   - Lyapunov-Krasovskii/Razumikhin approaches: constant time-delays/upper bound.
   - Passivity: teleoperation,
   - Stability: robustness,
   - Stochastic approach: LQG control.

$\Rightarrow$ Pole-placement: state predictor.

Modern cars:

- multiple safety/comfort devices,
- VAN/CAN,
- high jitter.

SX-29:

- open-loop unstable,
- LAN,
- high performance control.

Global Hawk (UAV):

- local + remote control,
- wideband satellite and Line-Of-Sight data link communications.

ITER:

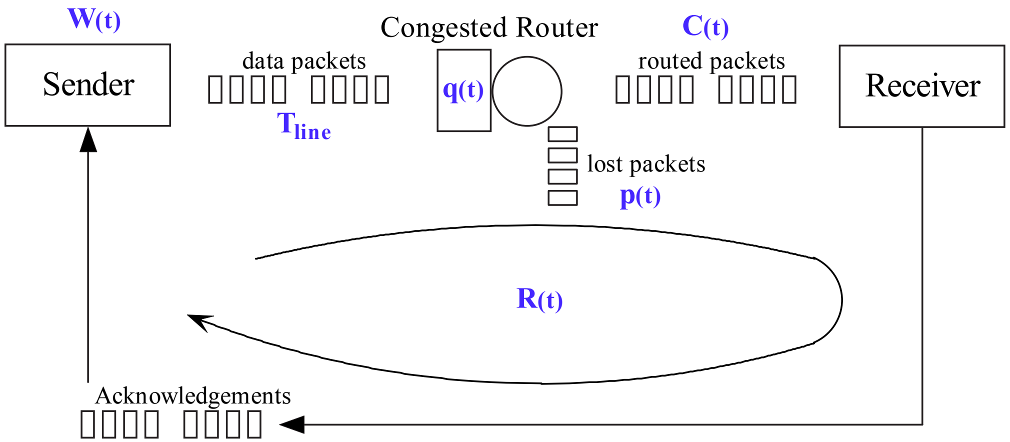
- large multi-systems device,
- LAN: control and data signals,
- scheduled tasks.

II. Problem Formulation

- The network dynamics is described by a dynamical model,

$$
\begin{aligned}
\dot{z}(t) &= f(z(t), u_d(t)), \quad z(t_0) = z_0 \\
\tau(t) &= h(z(t), u_d(t))
\end{aligned}
$$

i.e. for secure networks (one flow)*[Misra & all 00]*: TCP with AQM



$$
\begin{aligned}
\frac{dW_i(t)}{dt} &= \frac{1}{R_i(q)} - \frac{W_i(t)W_i(t-R_i(q))}{2R_i(q(t-R_i(q)))}p(t) \\
\frac{dq(t)}{dt} &\approx -C + \sum_{i=1}^{N} \frac{W_i(t)}{R_i(q)} \\
R_i(q) &= \frac{q}{C} + T_{pi}
\end{aligned}
$$

- The remotely controlled system has the form

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t - \tau(t)) \\ y(t) &= Cx(t) \end{aligned}$$

- Hypotheses
  - (A, B) and (A, C) controllable and observable
  - the network dynamics is such that (secure network)

$$\begin{aligned} 0 \leq \tau(t) \leq \tau_{max}, &\quad \forall t \geq 0 \\ \dot{\tau}(t) < 1, &\quad for\ almost\ all\ t \geq 0 \end{aligned}$$

NB: $\tau(t)$ is the delay *experienced* by the signal, i.e. $\dot{\tau}(t) = 1 \Leftrightarrow$ the data never gets to its destination.

III. Background on the State Predictor

- based on a state predictor with a time-varying horizon $\delta(t)$ *[Artstein 82, Nihtilä 89, Uchida & all. 03]* *[Springer 2005]*

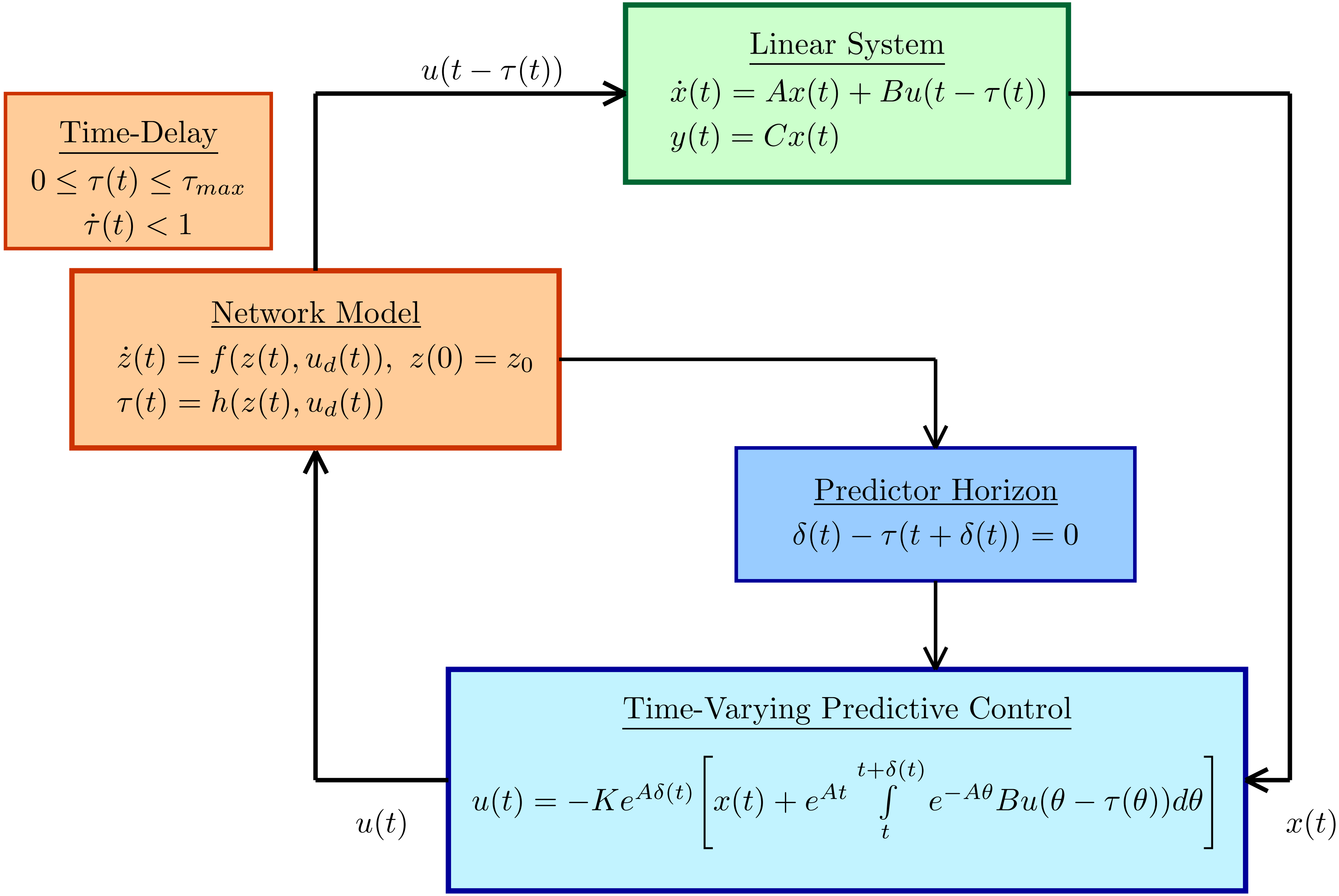
$$\begin{aligned} x(t+\delta(t)) &= e^{A\delta(t)}x(t) + e^{A(t+\delta(t))}\int_t^{t+\delta(t)} e^{-A\theta}Bu(\theta-\tau(\theta))d\theta \\ u(t) &= -Kx(t+\delta(t)) \end{aligned}$$

- results in the pole placement of the *time-shifted* closed-loop system

$$\frac{dx(t+\delta(t))}{d(t+\delta(t))} = (A-BK)x(t+\delta(t)) = A_{cl}\ x(t+\delta(t))$$

$\Rightarrow$ Non-linear time transformation $t \mapsto t+\delta(t)$ but exponential convergence if A_{cl} Hurwitz $\&$ hyp. on $\tau(t)$ are satisfied.

- explicit use of the network dynamics: $\delta(t) = \tau(t+\delta(t))$


Time-Delay
$0 \leq \tau(t) \leq \tau_{max}$
$\dot{\tau}(t) < 1$
$u(t-\tau(t))$
Linear System
$\dot{x}(t) = Ax(t) + Bu(t-\tau(t))$
$y(t) = Cx(t)$
Network Model
$\dot{z}(t) = f(z(t), u_d(t)), \; z(0) = z_0$
$\tau(t) = h(z(t), u_d(t))$
Predictor Horizon
$\delta(t) - \tau(t + \delta(t)) = 0$
Time-Varying Predictive Control
$u(t) = -Ke^{A\delta(t)}\left[x(t) + e^{At}\int_{t}^{t+\delta(t)} e^{-A\theta}Bu(\theta - \tau(\theta))d\theta\right]$
$u(t)$
$x(t)$

IV. Computation of the Predictor's Horizon

$\delta(t) = \tau(t + \delta(t))$ ***[IEEE CCA 2004]***

- Let

$$S(t) \doteq \hat{\delta}(t) - \tau(t + \hat{\delta}(t))$$

with

$$\dot{S}(t) + \sigma S(t) = 0$$

and $\sigma > 0$, to prevent for the numerical instabilities,

$\Rightarrow$ find $\dot{\hat{\delta}}(t)$ such that $\hat{\delta}(t)$ reaches asymptotically the manifold $S(t) = 0$.

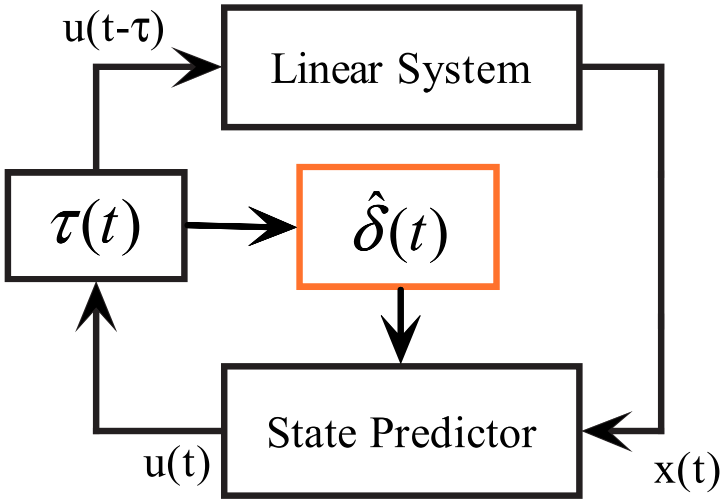
Using the assumption $\dot{\tau} \neq 1$, $\hat{\delta}(t)$ has the following dynamics

$$\dot{\hat{\delta}}(t) = -\frac{\sigma}{1 - d\tau(\hat{\zeta})/d\hat{\zeta}}\hat{\delta} + \frac{d\tau(\hat{\zeta})/d\hat{\zeta} + \sigma\tau(\hat{\zeta})}{1 - d\tau(\hat{\zeta})/d\hat{\zeta}}$$

where $\hat{\zeta} = t + \hat{\delta}$ and $|\epsilon(t)| = |\delta(t) - \hat{\delta}(t)| \leq \dfrac{|\hat{\delta}_0 - \tau(\hat{\delta}_0)|e^{-\sigma t}}{1 - \nu}$

V. Predictor with an Estimated Horizon

$$u(t) = -Ke^{A\hat{\delta}(t)}\left[x(t) + e^{At}\int_{t}^{t+\hat{\delta}(t)} e^{-A\theta}Bu(\theta - \tau(\theta))d\theta\right]$$



The control law writes equivalently

$$u(t) = -Kx(t + \hat{\delta}(t))$$

Using $t \mapsto t + \delta(t)$:

$$\begin{aligned} x'(t+\delta) &= Ax(t+\delta) + Bu(t) \\ &= Ax(t+\delta) - BKx(t+\hat{\delta}) \end{aligned}$$

which is analyzed from

$$\begin{aligned} \Sigma_t : x'(\zeta) &= (A - BK)x(\zeta) + BKA\int_{-\epsilon}^{0} x(\zeta+\theta)d\theta \\ &\quad -(BK)^2\int_{-2\epsilon}^{-\epsilon} x(\zeta+\theta)d\theta \end{aligned}$$

Lemma 1. *Consider the system Σ_t with appropriate distributed initial conditions. If the following conditions hold*

i) A_{cl} *is Hurwitz,*

ii) $\epsilon(t)$ *converges exponentially and is such that*

$$0 < \dot{\epsilon}_M \doteq \sup_t \dot{\epsilon}(t) < \frac{1}{2}$$

then the trajectories of $x(\zeta(t))$ *are asymptotically bounded.*

$\Rightarrow$ σ must be selected such that

$$\sigma < \frac{1-\nu}{2|\hat{\delta}_0 - \tau(\hat{\delta}_0)|}$$

Remark: $\dot{\epsilon}_M$ is given by the precision of the network model or can be set with the transfer algorithm.

V. Explicit use of the Network Model [video]

Theorem 1. *Consider the system described by*

$$\dot{x}(t) = Ax(t) + Bu(t - \tau(t))$$

where (A, B) *is a controllable pair. Suppose that the delay dynamics and* σ *are such that*

$A1)$ $0 \le \tau(t) \le \tau_{max}$,

$A2)$ $\dot{\tau}(t) \le \nu < 1$,

$A3)$ $0 < \dot{\epsilon}_M \doteq \sup_t \dot{\epsilon}(t) < \frac{1}{2}$

then the state feedback control law

$$\begin{aligned}
u(t) &= -Kx(t + \hat{\delta}(t)) \\
\dot{\hat{\delta}}(t) &= -\frac{\sigma}{1 - d\tau(\hat{\zeta})/d\hat{\zeta}}\hat{\delta} + \frac{d\tau(\hat{\zeta})/d\hat{\zeta} + \sigma\tau(\hat{\zeta})}{1 - d\tau(\hat{\zeta})/d\hat{\zeta}} \\
\frac{d\tau}{d\hat{\zeta}}(\hat{\zeta}) &= \frac{dh}{d\hat{\zeta}}(z(\hat{\zeta}), u_d(\hat{\zeta})) \\
\frac{dz}{d\hat{\zeta}}(\hat{\zeta}) &= f(z(\hat{\zeta}), u_d(\hat{\zeta})), \quad z(0) = z_0
\end{aligned}$$

with $\hat{\zeta} = \hat{\zeta}(t) = 1 + \hat{\delta}(t)$ *and* $\hat{\delta}(0) = \hat{\delta}_0 \in [0, \tau_{max}]$, *ensures that the closed-loop system trajectories are asymptotically stable.*

Conclusions and Perspectives

- Remote stabilization via communication networks
  $\Rightarrow$ stabilizing an open-loop unstable system with a time-varying delay.

- The proposed controller:
  - based on a $\delta(t)$-step ahead predictor,
  - results in an exponentially converging (non uniform) closed-loop system and pole placement on the time-shifted system,
  - applied to remote output stabilization and observer-based control,
  - robust with respect to time-delay uncertainties.

- Perspectives:
  - feedback/observer gain co-design,
  - extension to the nonlinear case,
  - investigate the network delay estimation and the dedicated network control *[Briat05]*,
  - coupling between the system controller and the dedicated network controller.