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KTH Electrical Engineering

Predictive control over wireless multi-hop networks

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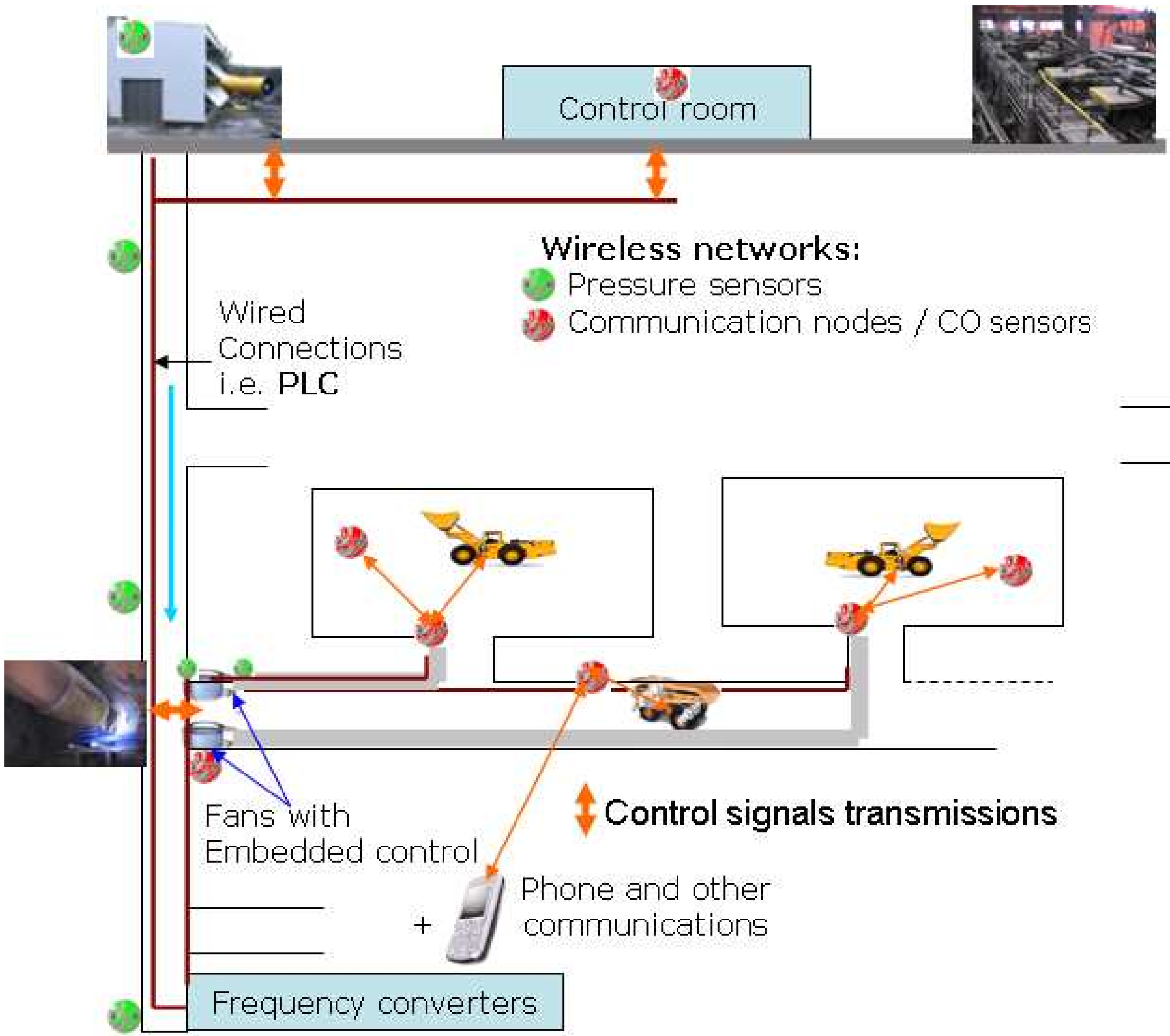
Illustrative example: mining ventilation problem (HYCON)



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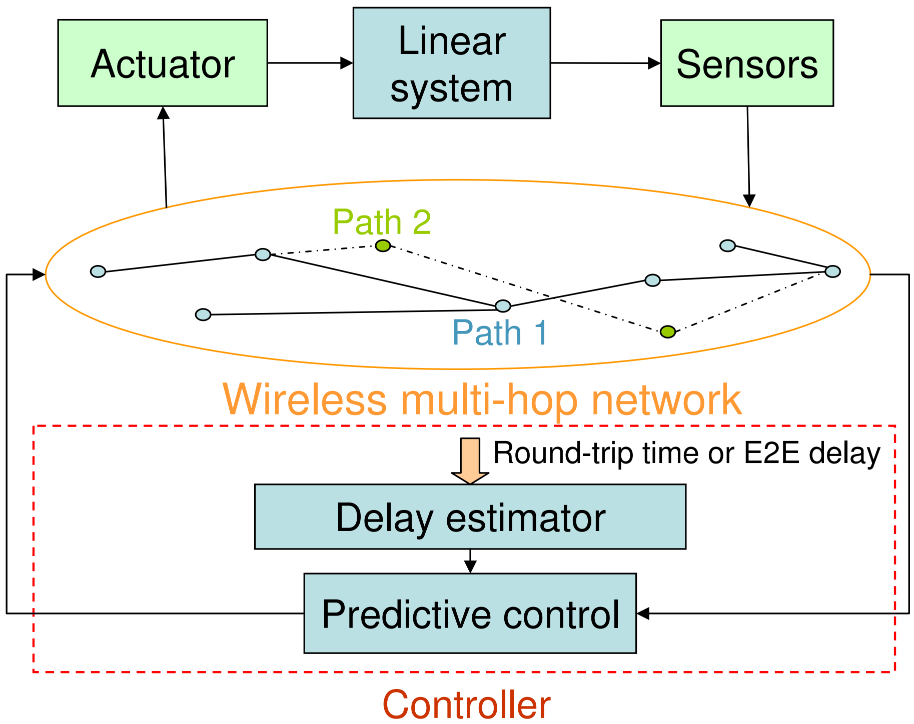
I. Problem description



- Time delays:
  - $\tau_1(t)$: sensors $\rightarrow$ controller;
  - $\tau_2(t)$: controller $\rightarrow$ actuator.

- Packet losses:

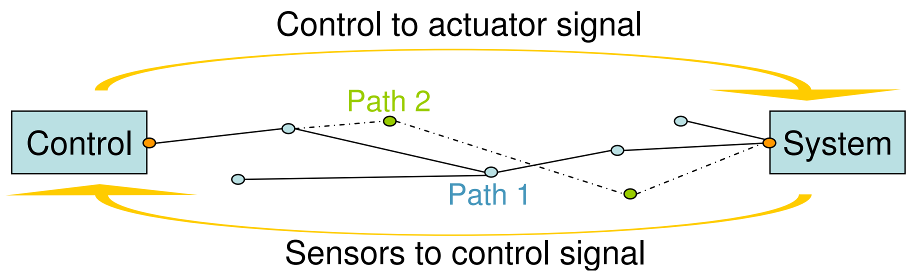
$$\begin{cases} w(t_k) = u(t_k - \tau_2(t_k)) & \textit{if received,} \\ w(t_k) = w(t_{k-1}) & else. \\ v(t) = w(t_k) & \forall\, t \in [t_k, t_{k+1}) \end{cases}$$

- Bandwidth limitation.

- Linear system:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bv(t), \quad x(0) = x_0 \\ y(t) &= Cx(t) \end{aligned}$$

II. Wireless multi-hop networks



- Network characteristics:
  - time-varying channel and network topology;
  - dynamic selection of $h(t)$ hops;
  - next node ensures progress toward destination;
  - i.e. random sleep to save energy.

- Delay associated with each node i:

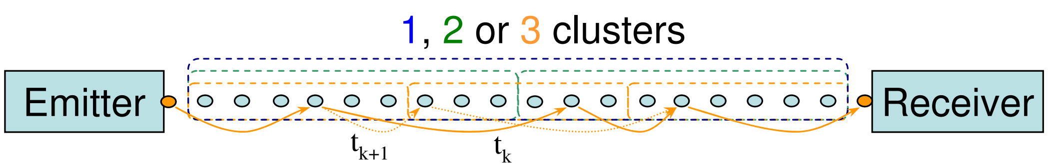
α_i: time to wait before sending a packet (random);

F: propagation and transmission;

β_i: Automatic Repeat reQuest (ARQ): retransmission;

$$\Rightarrow \tau(t) = h(t)F + \sum_{i=1}^{h(t)} (\alpha_i + \beta_i)$$

i.e. clusters and dynamic selection of hops

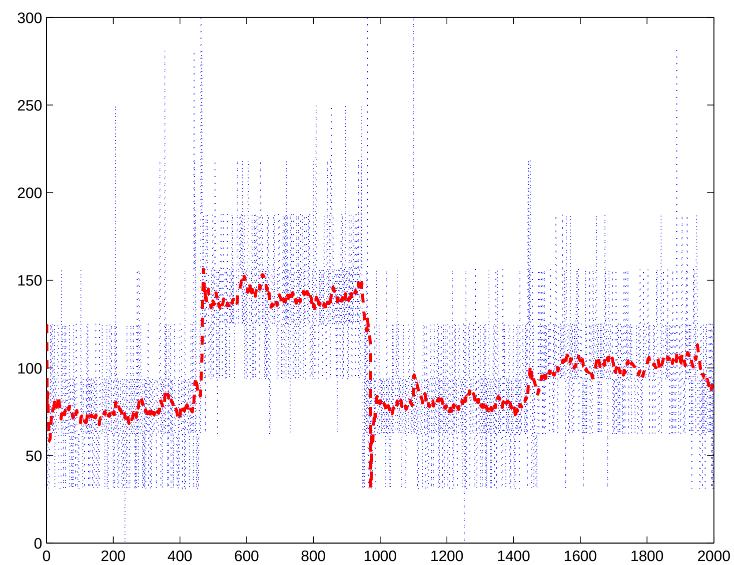


- sleeping strategy depending on traffic and network conditions,
- optimize energy consumption in a clustered environment [Bonivento & al.'06],

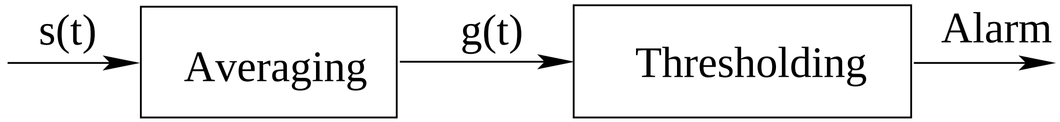
⇒ Dynamical organization in clusters [P.G. Park MS'07].

Three possible behaviors:

- Low freq. = changes in the number of clusters,
- Medium freq. = node selection within a cluster,
- High freq. = transmissions between nodes.



III. Time-delay estimator.



Kalman filter + CUMSUM change-detection [Gustafsson'00, Jacobsson & al.'04]:

- measure RTT or 1-channel delay = large variability and persistent changes in the mean,
- smooth out fast variations and react quickly to changes in mean,

$\Rightarrow$ sets the time-varying horizon of the predictive controller.

The signal model is

$$\begin{aligned} \tau_{k+1} &= \tau_k + v_k \\ r_k &= c\tau_k + w_k \end{aligned} \quad , \quad v_k = \begin{cases} 0 & \text{with probability } 1-q, \\ \nu & \text{with probability } q, \ \text{where } \mathrm{Cov}(\nu_k) = q^{-1} Q_k \end{cases}$$

$$
\begin{aligned}
&\textit{Kalman filter updates} && \textit{CUMSUM detector} \\
\hat{\tau}_{k+1} &= \hat{\tau}_k + K_k(r_k - c\hat{\tau}_k) \\
K_k &= \frac{cP_{k-1}}{R_k + c^2 P_{k-1}} & \varepsilon_k &= r_k - c\hat{\tau}_k \\
& & g_k^+ &= \max(g_{k-1} + \varepsilon_k - \kappa, 0) \\
& & g_k^- &= \max(g_{k-1} - \varepsilon_k - \kappa, 0) \\
P_k &= P_{k-1} - \frac{c^2 P_{k-1}^2}{R_k + c^2 P_{k-1}} + Q_k
\end{aligned}
$$

if g_k^+ or $g_k^- >$ threshold h, P_k set to a large value, g^+ and g^- reset to zero, and $\hat{\tau}_k$ set to $c^{-1} r_k$.

IV. Predictive conrol approach.

Ideal case: known delay [Witrant & al.'05...]

$$\dot{x}(t) = Ax(t) + Bu(t - \tau_2), \quad x(0) = x_0$$

with (A, B) a controllable pair, $\tau_{1,2} \in C(R^+, [0, \tau_{max}])$, $\dot{\tau}_{1,2} < 1$, $\forall t \geq t_0$.
Then the feedback control law

$$\begin{aligned} u(t) &= -Ke^{A\delta}\left[e^{A\tau_1}x(t-\tau_1) + e^{At}\int_{t-\tau_1}^{t+\delta} e^{-A\theta}Bu(\theta - \tau_2(\theta))d\theta\right] \\ \dot{\delta}(t) &= -\frac{\lambda}{1 - d\tau_2(\zeta)/d\zeta}\delta + \frac{d\tau_2(\zeta)/d\zeta + \lambda\tau_2(\zeta)}{1 - d\tau_2(\zeta)/d\zeta} \end{aligned}$$

with $\zeta = t + \delta$, $\lambda > 0$, $\delta(0) = \delta_0$, ensures that the system trajectories $x(t)$ converge exponentially to zero.

⇒ *time-shifted* dynamics

$$\frac{dx}{d\zeta} = (A - BK)x(\zeta)$$

and *equivalent system* [Artstein'82] spectrum

$$A - \left|\frac{1 + \dot{\delta}}{\dot{\tau}(t + \delta)}\right| e^{-A\delta} BK,$$

Worst case (for design):

$$A - \left|\frac{1}{1 - \dot{\tau}_{max}}\right| e^{-A\tau_{max}} BK$$

is Hurwitz.

Use of the delay estimator

i.e. transmission over the same network: $\hat{\tau} \approx \hat{\tau}_1 \approx \hat{\tau}_2$

$$u(t) \quad = \quad -Ke^{A\delta}\left[e^{A\hat{\tau}}x(t-\tau_1) + e^{At}\int_{t-\hat{\tau}}^{t+\hat{\delta}} e^{-A\theta}Bu(\theta - \hat{\tau}(\theta))d\theta\right]$$

$$\dot{\hat{\delta}}(t) \quad = \quad \frac{\dot{\hat{\tau}}(t) + \lambda(\hat{\tau}(t) - \hat{\delta}(t))}{1 - \dot{\hat{\tau}}(t)}$$

and causal integral approximation:

$$\mathcal{I}_k \quad = \quad e^{At_k}\int_{t_k-\hat{\tau}_k}^{t_k+\hat{\delta}_k} e^{-A\theta}Bu(\theta - \hat{\tau}(\theta))d\theta = e^{A\hat{\tau}_k}\int_0^{\hat{\tau}_k+\hat{\delta}_k} f(\mu)d\mu, \quad with$$

$$f(\mu) \quad = \quad e^{-A\mu}Bu(min(\mu + t_k - \hat{\tau}_k - \hat{\tau}(min(\mu + t_k - \hat{\tau}_k, t_k)), t_k))$$

$\Rightarrow$ High frequency noise on control signal when real wireless network data.

Discretization (safe implementation [Mondié & al.’03]):

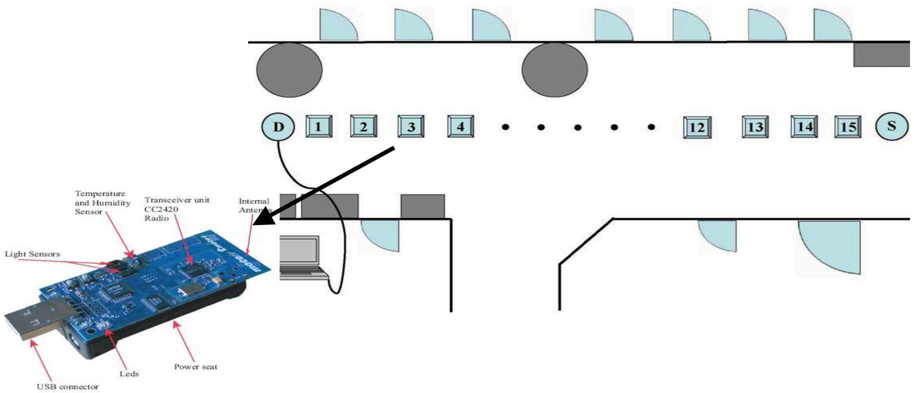
$$\begin{cases} \dot{z}(t) = -az(t) + bKe^{A(\hat{\delta}_k+\hat{\tau}_k)}\left[x(t-\tau_1) + \frac{t_s}{2}\sum_{i=1}^{n_k} f((i-1)t_s) + f(it_s)\right] \\ u_k = z(t_k), \quad n_k \doteq \frac{\hat{\delta}_k+\hat{\tau}_k}{ts} \end{cases}$$

and filter the triggers with the horizon dynamics

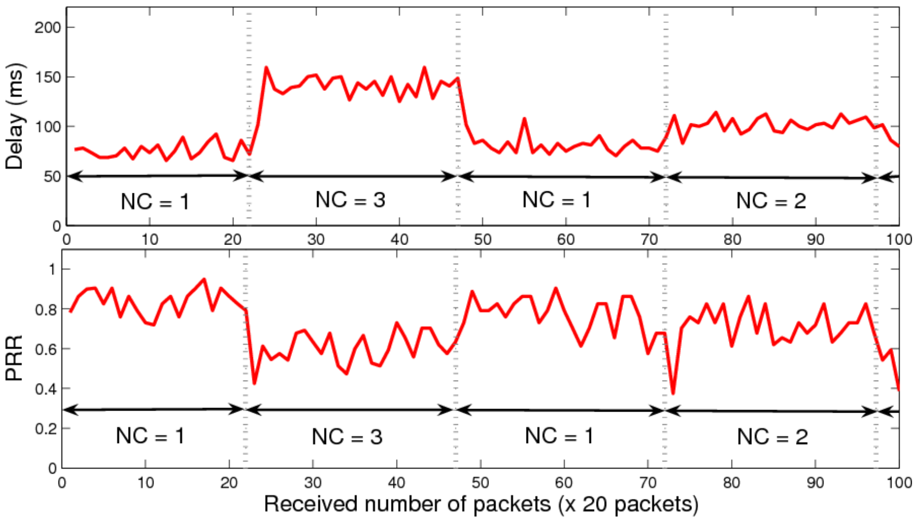
$$\dot{\hat{\delta}}(t) = \frac{\bar{\tau}_k + \lambda(\hat{\tau}_k - \hat{\delta}(t))}{1-\bar{\tau}_k}, \quad where \quad \begin{cases} \bar{\tau}_k = \frac{\hat{\tau}_k - \hat{\tau}_{k-1}}{t_s} & if \quad \bar{\tau} < \bar{\tau}_{thr} \\ \bar{\tau}_k = \bar{\tau}_{thr} & else \end{cases}$$

V. Experimental and simulation results

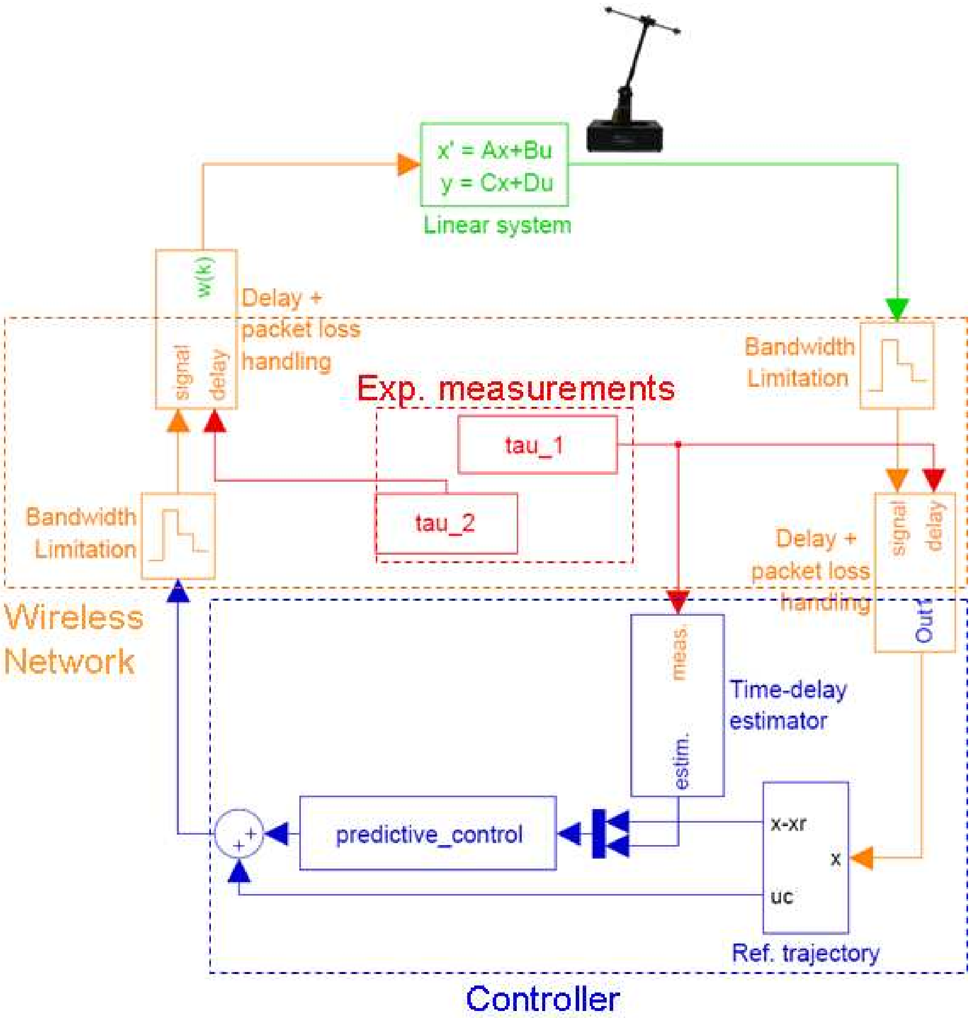
Network setup and experimental measurements



- Tmotes Sky nodes
  - radio controller Chipcon CC2420 (2.4GHz)
  - IEEE 802.15.4
  - 20 packets/s
- Randomized Protocol



Control setup



Linear system:

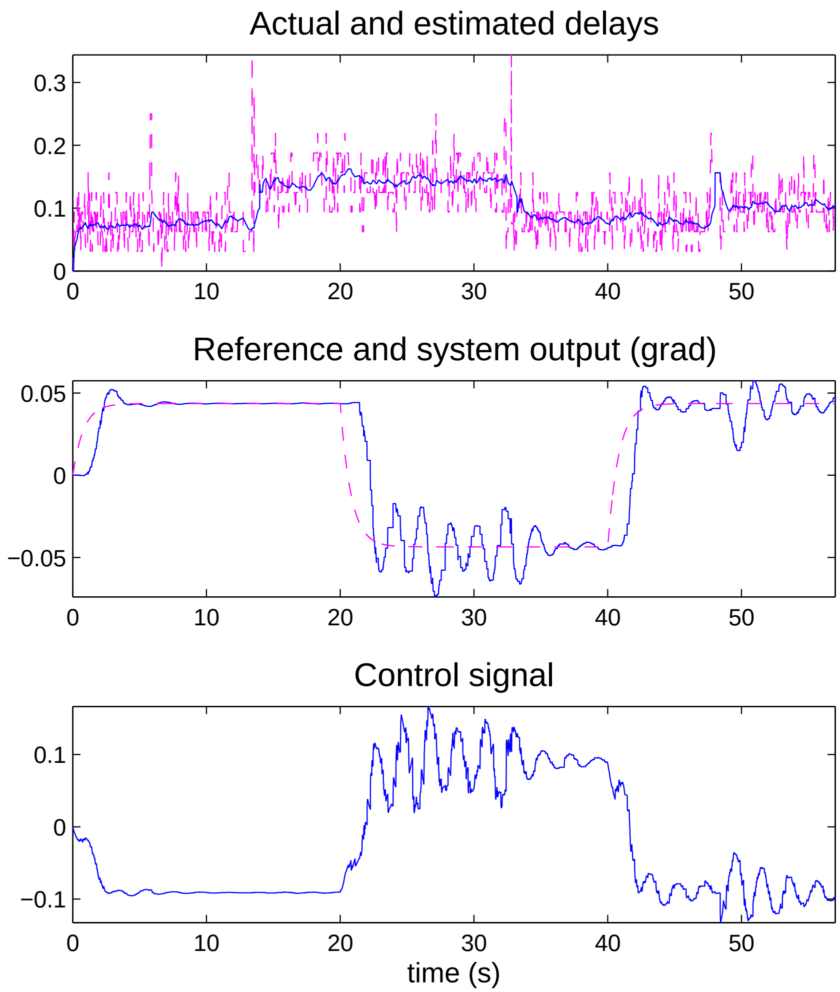
$$\dot{x}(t) = A\,x(t) + B\,u(t)$$
$$y(t) = x(t)$$

with

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0.80 & 0 & 12.56 & 0 \\ 0 & 0 & 0 & 1 \\ -2.42 & 0 & -8.33 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 4.57 \\ 0 \\ 0.38 \end{bmatrix}$$

$\lambda_{1,2} = 0.74 \pm 2.08i$, $\lambda_{3,4} = -0.74 \pm 2.08i$, K sets the poles to $(-3 + 0.1i, -3 - 0.1i, -3.5, -6)$.

Simulation results



- Proportional and *non-safe* predictor do not stabilize this system (divergence when 3 clusters)
- Proper delay estimation and compensation
- Stabilization even with 30% of lost packets, bounded oscillation when 40%
- Still one degree of freedom with K

Conclusions

- Control strategy:
  - multi-hop wireless network = time-varying delays, packet losses and bandwidth limitations,
  - delay from CUMSUM Kalman filter: fast mean tracking + HF rejection,

  $\Rightarrow$ causal dynamic predictive control compensating the estimated delay.

- Simulation results:
  - experimental delay and packet loss data,
  - efficiency to control over noisy communication channels,
  - limitations: effect of packet losses and protocol.

- Perspectives:
  - wireless multi-hop network design for large-scale applications,
  - optimal design to set K depending on packet losses.

Thanks!

Minimized cost function: [P.G. Park'07]

$$Arg\ min_h\ T\Big(\underbrace{\lambda(\underbrace{h\,E_{delay}}_{\text{Rand. contension}} + \underbrace{h\,E_{Re}\,l_m}_{\text{reception}} + \underbrace{h\,E_{Te}\,l_m + \epsilon_m\,l_m\,D^{\beta}h^{1-\beta}}_{\text{send data}})}_{\text{transmission/reception}}$$

$$+ \underbrace{\underbrace{\frac{max\,\{D_c(h), E_c(h)\}}{p}}_{\text{wake up}}(\underbrace{hE_{ac}}_{\text{active time}} + \underbrace{\epsilon_b\,l_b\,D^{\beta}h^{1-\beta}}_{\text{send beacon}})}_{\text{Wake up \& beaconing}}\Big)$$