Profiles Control and Stability in Thermonuclear Fusion: Some Issues for ITER

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Motivation

Abundant fuels:

- Deuterium: from all forms of water, for millions of years.
- Tritium: not natural, bred from Lithium.
- Lithium: from earth's crust, if all the world's electricity provided by fusion, known reserves would last for at least one thousand years.

High energetic gain:

- 10 g of D (500 l of water) and 15 g of T (30 g of L_i) $\Rightarrow$ lifetime electricity needs of an average person in an industrialized country.

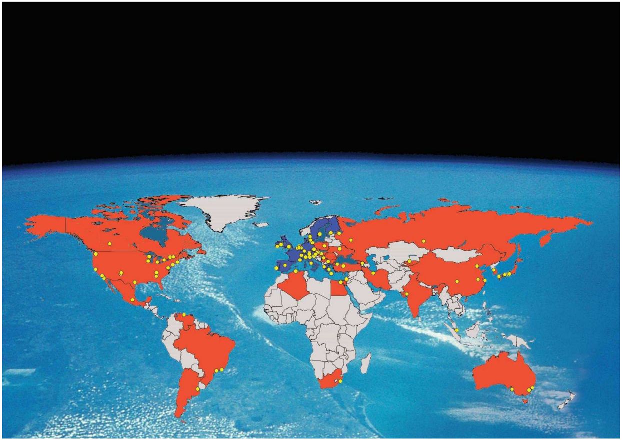
Inherent safety:

- Very small amount of D & T and difficult operating conditions → any deviation results in a rapid cooling of the plasma and its termination.
- No circumstances in which the fusion reaction can proceed into an uncontrollable or critical condition.

Environmental advantages:

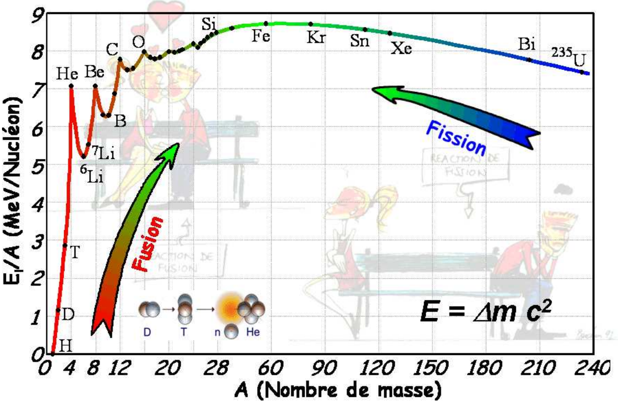
- No 'greenhouse' gases (no global warming contribution).
- Rapid decay of the structure radioactivity and the time span before it can be re-used and handled can be minimised (to around 50 years).
- No radioactive 'waste' from the fusion reaction itself.
- The byproduct is H_e - an inert and harmless gas.

A Short Introduction to Fusion

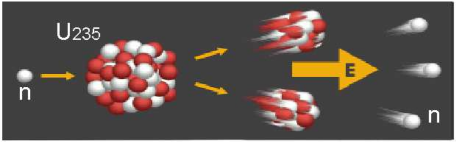


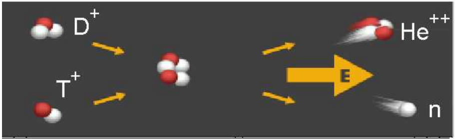
An energy from the stars

Fusion and fission: exo-energetic processes

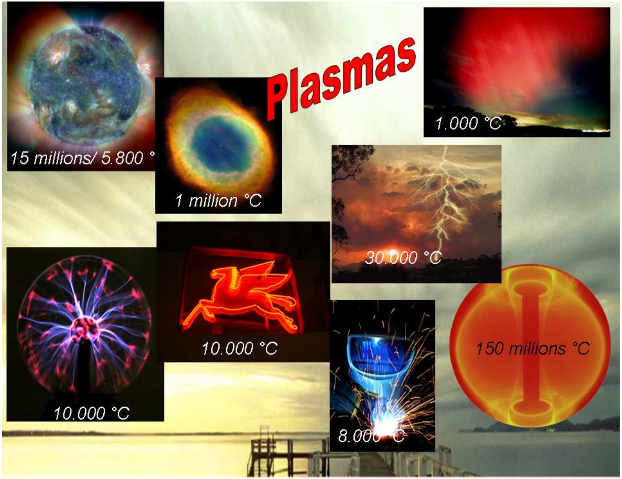


Fusion vs. fission

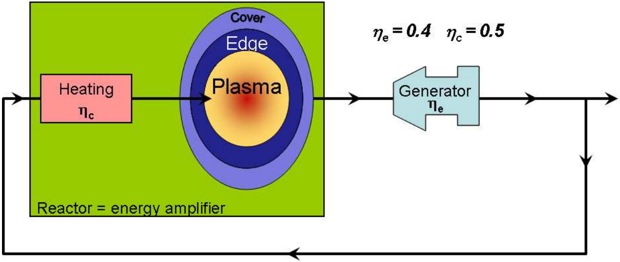




Controlled fusion: Tokamaks



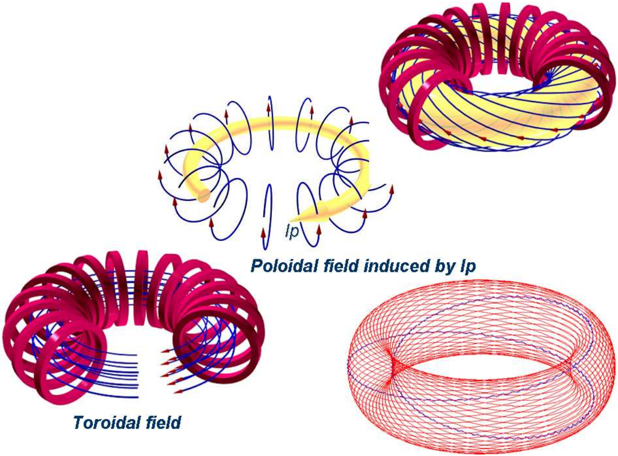
Principle of a fusion reactor



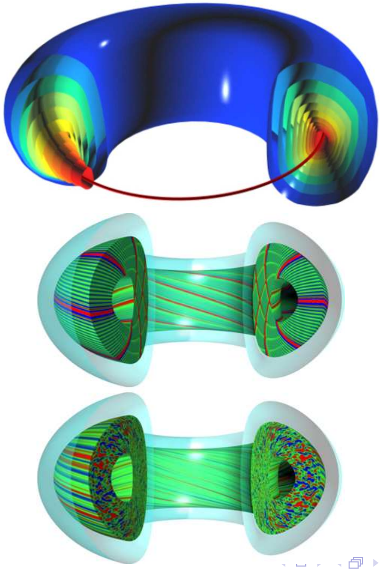
Performances:

- Energy confinement time
- Plasma energy
- Fraction of output power used to heat the plasma

Magnetic confinement: close the field lines to form a torus



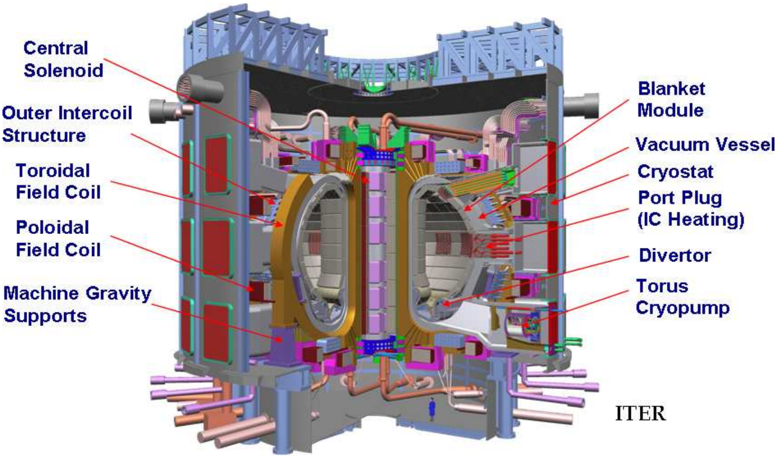
Plasma dynamics: isoflux surfaces . . . and turbulences!



Plasma Physics Issues:

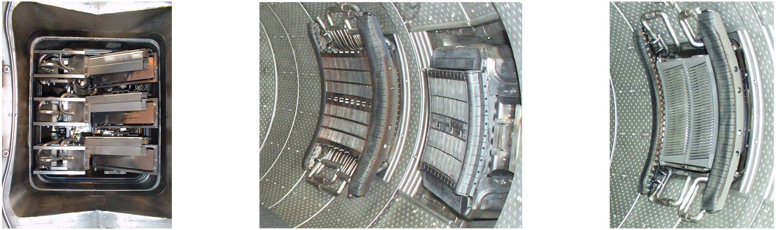
- MHD Stability
- Heat Confinement
- Steady State Operation
- Control of Plasma Purity
- Exploration of the new physics with a dominant a-particles plasma self heating

Tokamak automation

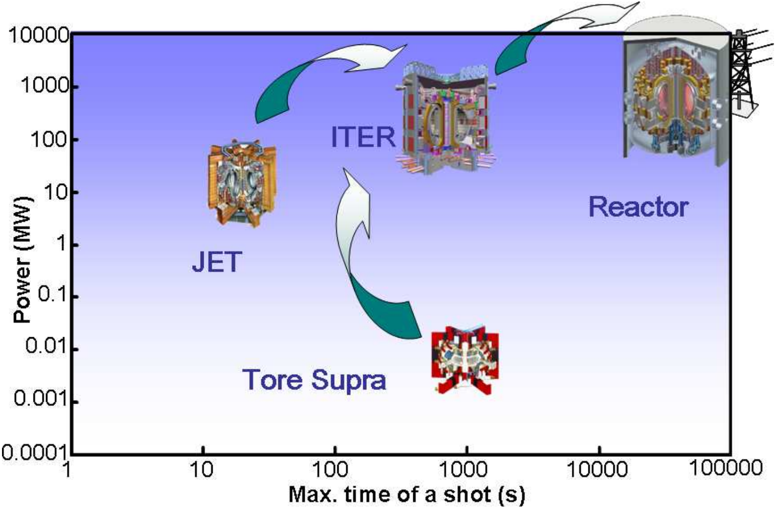


Radio frequency antennas (i.e. Tore Supra):

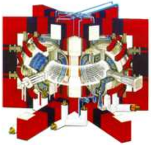
- Ion cyclotron radio heating (ICRH) - 40-80 Mhz (2 antennas, 10 MW)
- Lower hybrid (LH) 3.7 GHz (2 antennas, 16 klystrons, 6.2 MW)
- Electron cyclotron current drive (ECCD) - 118 Ghz (1 antenna, 2 gyrotrons, 0.8 MW)



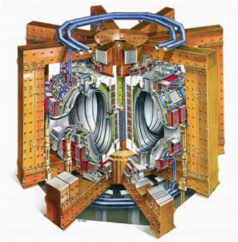
Why ITER?


10000
1000
100
10
1
0.1
0.01
0.001
0.0001
Power (MW)
1
10
100
1000
10000
100000
Max. time of a shot (s)
JET
ITER
Tore Supra
Reactor

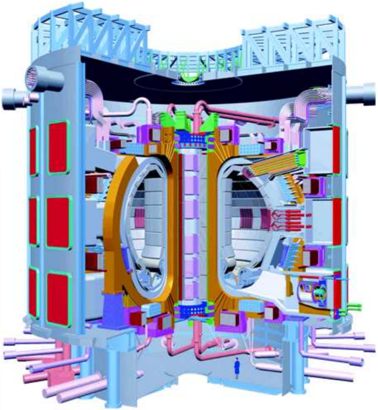
Some numbers . . .



V_{plasma} 25 m^3
P_{fusion} ~0
T_{plasma} ~400 s



V_{plasma} 80 m^3
P_{fusion} ~16 MW
T_{plasma} ~30 s



V_{plasma} 830 m^3
P_{fusion} ~500 MW
T_{plasma} ~400 s

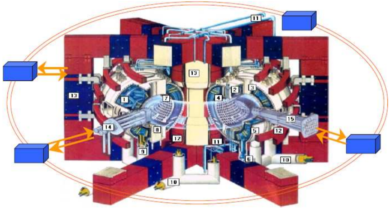
Programmatic:

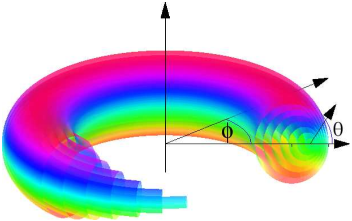
- Demonstrate the scientific and technological feasibility of fusion energy for peaceful purposes.

Technical:

- Demonstrate extended burn of DT plasmas, with steady state as the ultimate goal.
- Integrate and test all essential fusion power reactor technologies and components.
- Demonstrate safety and environmental acceptability of fusion.

Profiles control: modeling and feedback in Tore Supra

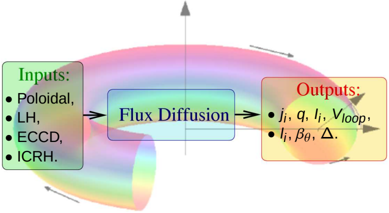




- Advanced plasma confinement schemes ⇒ current density and temperature profiles control;
- Focus on the magnetic flux dynamics and use experimental measurements;
- Include bootstrap effect, poloidal, ECCD, ICRH, LH;

⇒ representative RT model of plasma physics for control synthesis on Tore Supra.

Flux diffusion dynamics



Inputs:
- Poloidal,
- LH,
- ECCD,
- ICRH.

Flux Diffusion

Outputs:
- j_i, q, l_i, V_{loop},
- l_i, β_θ, Δ.

Hypotheses:

- cylindrical coordinates (neglect GSS),
- neglect diamagnetic effect,

System dynamics [Blum'89, Brégeon & al'98]:

$$\begin{aligned}
\frac{\partial \psi}{\partial t}(x,t) &= \eta_{/\!/}(x,t)\left[\frac{1}{\mu_0 a^2}\frac{\partial^2 \psi}{\partial x^2} + \frac{1}{\mu_0 a^2 x}\frac{\partial \psi}{\partial x} + R_0 j_{bs}(x,t) + R_0 j_{ni}(x,t)\right] \\
j_\phi(x,t) &= -\frac{1}{\mu_0 R_0 a^2 x}\frac{\partial}{\partial x}\left[x\frac{\partial \psi}{\partial x}\right]
\end{aligned}$$

with $\psi'(0,t) = 0$, $\psi'(1,t) = f(I_p)$ or $\dot{\psi}(1,t) = f(V_{loop})$ and IC.

Time constant

Resistivity, temperature and density profiles

- Resistivity [Hirshman'77]:

$$\eta_{/\!/}(x,t) = f(T_e, n_e, \bar{Z}) \propto T_e{}^{-3/2}(n_e, \bar{Z})$$

- Temperature:
  - Profile shape determination → parameter dependant identification of nonlinear distributed systems:
    - Grey-box modeling,
    - 3-hidden layers approach: spatial distribution, steady-state and transient behavior,
    - Stochastic descent method with direct differentiation.
  - Predictive mode: identified shape parameters + amplitude/[ITER'99]
  - Direct measurement
- Density - average or interferometer:
  $n_e(x,t) = n_{e0}(t)(1 - x^{\gamma_n})$ and $n_{e0}(t) = \frac{\gamma_n + 1}{\gamma_n}\,\bar{n}_e(t)$.

Nonlinear component

$$\frac{\partial \psi}{\partial t}(x,t) = \eta_{/\!/}(x,t)\left[\frac{1}{\mu_0 a^2}\frac{\partial^2 \psi}{\partial x^2} + \frac{1}{\mu_0 a^2 x}\frac{\partial \psi}{\partial x} + R_0 j_{bs}(x,t) + R_0 j_{ni}(x,t)\right]$$

Bootstrap effect:

- Self induced current,
- Nonlinear component [Hirshman'88]:

$$\begin{aligned} j_{bs}(x,t) &= \frac{eR_0}{\partial\psi/\partial x}\left\{(A_1 - A_2)n_e\frac{\partial T_e}{\partial x} + A_1 T_e\frac{\partial n_e}{\partial x}\right. \\ &\quad \left. + A_1(1-\alpha_i)n_i\frac{\partial T_i}{\partial x} + A_1 T_i\frac{\partial n_i}{\partial x}\right\} \end{aligned}$$

with $n_i(x,t) \approx \frac{7-\bar{Z}(t)}{6} n_e(x,t)$ and $T_i(x,t) \approx \alpha_{Ti}(t) T_e(x,t)$.

Controlled inputs

- BC input from the poloidal coil:

$$\begin{aligned} V_c &= R_c I_c + L_c \dot{I}_c + M \dot{I}_p \\ V_{loop} &= M \dot{I}_c - \frac{1}{I_p} \frac{\partial}{\partial t} \left[\frac{L_p I_p^2}{2} \right] = \dot{\psi}_a \end{aligned}$$

  Local control loop: set V_c according to $I_{p,ref}$ or $V_{loop,ref}$.

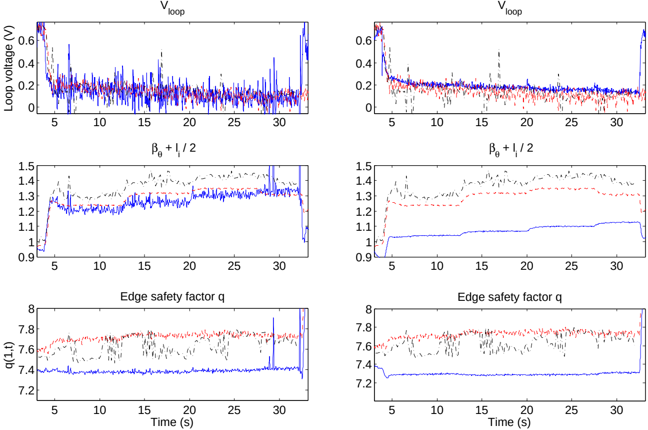
- Distributed current deposit from the antennas (LH, ECCD) - Engineering approach with gaussian approximation:

$$j_{ni}(x,t) = \vartheta(t) e^{-(\mu(t)-x)^2/2\sigma(t)}$$

- Temperature profile modification (ICRH, LH) ⇒ Hybrid dynamics (scaling laws).

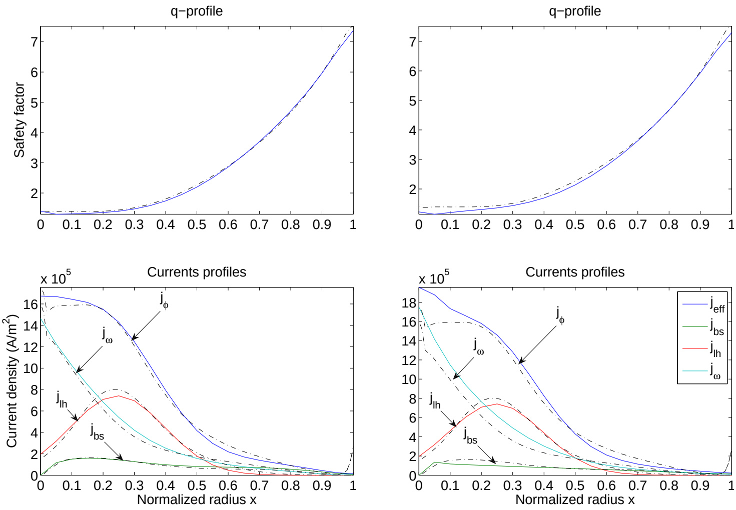
Experimental results

Lower Hybrid effect: shot TS 35109 - variations in $N_{//}$, constant I_p (0.6 MA) and power input (1.8 MW).



(a) Measured T_e profile (b) Estimated T_e profile

Figure: ψ_{sim} (—) vs. measurements (– –) and CRONOS (– · –): loop voltage (top), $\beta_\theta + l_i/2$ (middle) and edge safety factor (bottom).



(a) Measured T_e profile (b) Estimated T_e profile

Figure: ψ_{sim} (—) vs. CRONOS (– · –) at $t = 7$ s: safety factor (top) and current densities (effective j_ϕ, LH j_{lh}, ohmic j_ω and bootstrap j_{bs}) profiles (bottom).

NL Model Predictive Control

$$\min_{LH,ECRF} J \doteq \int_0^1 \left(j_{ref} - (j_{bs} + j_{ohm} + j_{ecrf} + j_{lh})\right)^2 dx$$

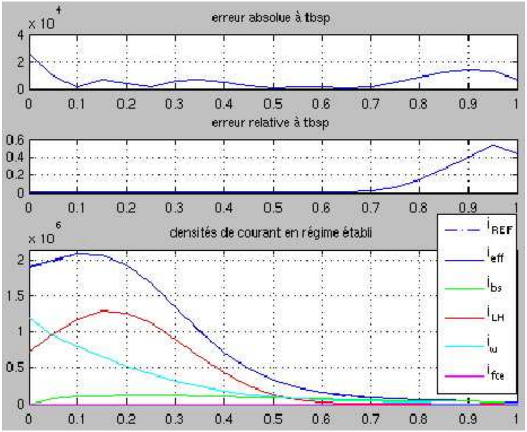


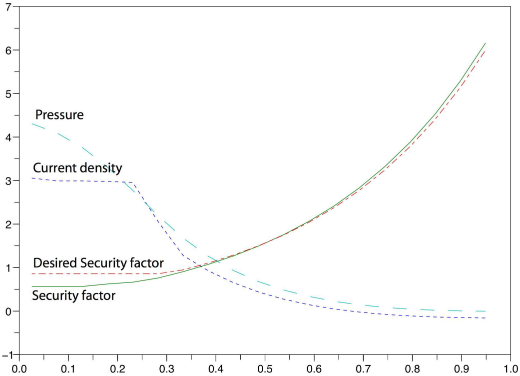
Figure: Error and current profiles with NLMPC

Tracking control

Security factor profile tracking q^*

$$j_{lh} = -j_{bs}\left(\partial_x P_e, \frac{-c_q x}{q^*}\right) - c_j \frac{1}{x}\partial_x\left[x\partial_x\left[\frac{-c_q x}{q^*}\right]\right] + \frac{V}{R\eta}$$

$\Rightarrow$ Easy to implement (P_e & T_e as inputs), high sensibility to q^*



Summary

- Efficient model to represent the main control-relevant profiles: 3 coupled PDE + wave/particule interactions → 1 PDE + identified shapes;
- Specific care of the discretization scheme and numerical integration;
- Fast computation ($\approx 1.7s$ for shot TS 33632 with 21 points and $t_s = 10\,ms$), suitable for RT NLMPC;
- Variable step spatial discretization $\Rightarrow$ reduce the number of points (i.e. dynamically);
- Robust wrt. measurement noises, discretization steps (spatial and temporal);
- Simplified model and first steps toward output feedback control.

Stability analysis and model-based control in EXTRAP-T2R with time-delay compensation (Alfvén-lab, KTH, Stockholm)

- Magnetohydrodynamic instabilities (non-symmetric electric currents) = crucial issue for fusion plasmas & ITER

⇒ Dedicated experiment: Reverse Field Pinch EXTRAP-T2R, with Intelligent-Shell feedback (4×32 actuator, 4×32 sensor saddle coils)

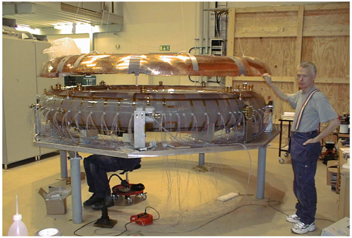
- MHD control implies to consider
  - the aliasing of spatial harmonics
  - actuation dynamics and control latencies

⇒ New model with experimental constraints and model-based control

EXTRAP T2R fusion plasma experiment

Machine parameters

- major radius R_0=1.24 m
- plasma minor radius a=18 cm
- shell time constant τ_{ver}=6 ms
- plasma current I_p=80 kA
- electron temp. T_e=250 eV
- pulse length $\tau_{pulse} < 60$ ms



EXTRAP T2R vessel and shell during assembly

Pulse lengths $\tau_{pulse} >> \tau_{ver}$ allow studies of MHD instabilities of the resistive wall mode (RWM) type (growing on the time scale of the shell)

Magnetohydrodynamic unstable modes model

Resistive-wall mode physics in RFP:

from MHD to perturbed ODE

- Linear stability investigated by periodic spectral decomposition

$$\mathbf{b}(r,t) = \sum_{mn} \mathbf{b}_{mn}(r) e^{j(t\omega + m\theta + n\phi)}$$

Fourier eigenmodes $\mathbf{b}_{mn}(r)$ with growth-rate $\gamma_{mn} = j\omega_{mn}$,

- Ideal MHD modes:

$$\tau_{mn}\dot{b}^r_{mn} - \tau_{mn}\gamma_{mn}b^r_{mn} = b^{r,ext}_{mn}$$

b^r_{mn}: radial component of perturbed field, $b^{r,ext}_{mn}$: external active coil, τ_{mn}: penetration time.

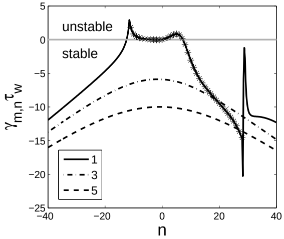


Figure: Growth-rates $\tau_w\gamma_{mn}$. *: Integer-n non-resonant positions (RWMs) for $m = 1$.

MIMO plant modeling by geometric coupling of SISO dynamics

Standard state-space form

- Faraday and Biot-Savart laws + ideal integrator on the sensor coil output voltage (hyp.)

$$\begin{cases} \dot{\mathbf{x}} & = & A\mathbf{x} + B\mathbf{u} + N\mathbf{v}_1 & \textit{MHD-modes vs. active coil cur. \& exog. signal} \\ \mathbf{z} & = & M\mathbf{x} & \textit{optional performance vector} \\ \mathbf{y} & = & C\mathbf{x} + \mathbf{v}_2 & \textit{time-integrated sensor voltages \& white noise} \end{cases}$$

- with state matrix elements (instantiated for T2R geometry and routing)

$$\begin{aligned} A_{mn,m'n'} &\sim \gamma_{mn}\delta_{mn,m'n'} \\ B_{mn,ij} &\sim \tau_{mn}^{-1}\int_{\Omega} e^{-\iota(m\theta+n\phi)}\left(\hat{\mathbf{r}}\cdot\oint_{l_{ij}}\frac{d\mathbf{l}_{ij}\times\left(\mathbf{r}-\mathbf{r}_{ij}\right)}{|\mathbf{r}-\mathbf{r}_{ij}|^3}\right)d\Omega \\ C_{pq,mn} &\sim \int_{\Omega} e^{+\iota(m\theta+n\phi)} f_{pq}A_{pq}\,d\Omega \end{aligned}$$

Modes coupling and aliasing of spatial frequencies:

- sensors and actuators: single-mode to multiple-mode model affected by *aliasing*;
- the traditional *IS-regulator* (output convergence) does not counteract the disturbances influence on MHD-modes;

⇒ Precise *mode-control* to address the problem of internal state stability.

Actuators dynamics, latencies and PID control

- Characteristic times (T2R) → actuation

$$\mathbf{u}_{sys}(t) \approx \frac{1}{\tau_c s + 1}\frac{\kappa}{\tau_a s + 1}\mathbf{u}_{DAC}(t - \tau_h)$$

- PID implementation with time-delay

$$\mathbf{u}_{DAC}(t) = K_p\,\mathbf{e}(t) + K_i\,\mathbf{q}(t) + {\tau_d}^{-1}K_d\,(\mathbf{e}(t) - \mathbf{e}(t - \tau_d))$$

⇒ Closed-loop dynamics

$$\dot{\tilde{\mathbf{x}}}(t) = \mathcal{A}_0\tilde{\mathbf{x}}(t) + \mathcal{A}_1(\theta)\tilde{\mathbf{x}}(t - \tau_h) + \mathcal{A}_2(\theta)\tilde{\mathbf{x}}(t - \tau_h - \tau_d) + \mathcal{E}\mathbf{v}_1(t)$$

Time constants and delays

Symbol	Value/order	Description/comment
$\tau_{w,N}$	$13.8\,\mathrm{ms}$	*Nominal* resistive wall time
τ_w	$\approx 10\,\mathrm{ms}$	*Experimental* resistive wall time
τ_{mn}	$\lesssim \frac{1}{2}\tau_w$	Actual model mode time
τ_{MHD}	$\sim 1\mu\mathrm{s}$	Internal MHD activity/fluctuations
τ_d	$100\,\mu\mathrm{s}$	Digital sampling time, controller cycle
τ_h	$\sim 100\,\mu\mathrm{s}$	Control latency, dead time
τ_{CPU}	$< 100\,\mu\mathrm{s}$	Algorithm-dependent part of τ_h
τ_a	$8\,\mu\mathrm{s}$	Active amplifier first-order time
τ_c	$1\,\mathrm{ms}$	Active coil L/R-time
$\tau_{A\&D}$	$\sim 1\mu\mathrm{s}$	ADC/DAC settle, $\mathrm{ns}/\mu\mathrm{s}$ respectively

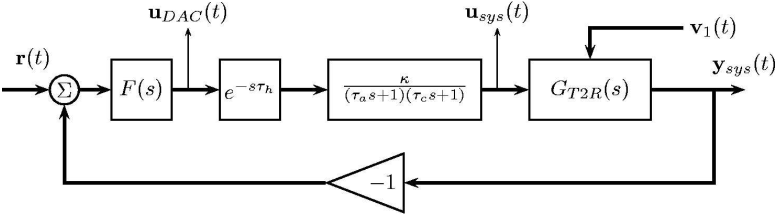
Open-loop error estimation and parameter identification

- Actuators dynamics identification $\rho^{ij} \doteq \{\tau_c^{ij}, \tau_h^{ij}, \kappa^{ij}\}$ from (PRBS).
- Error-field estimation and filtering

$$\begin{cases} \dot{\tilde{\mathbf{x}}} & = \begin{pmatrix} A & N \\ 0 & -\tau_s^{-1} I \end{pmatrix} \tilde{\mathbf{x}} + \begin{pmatrix} B \\ 0 \end{pmatrix} \mathbf{u} + \mathbf{v}_1' \\ \mathbf{y} & = \begin{pmatrix} C & 0 \end{pmatrix} \tilde{\mathbf{x}} + \mathbf{v}_2 \end{cases}$$

$\tilde{\mathbf{x}}(t) \doteq (\mathbf{x}(t)^T \, \mathbf{x}_s(t)^T)^T$ estimated by Kalman filter, $\mathbf{x}_s(t)$ is *inter alia* RWM-instabilities

Model summary



Stability analysis and delay effects

- Infinite spectrum of the Delay Differential Equation

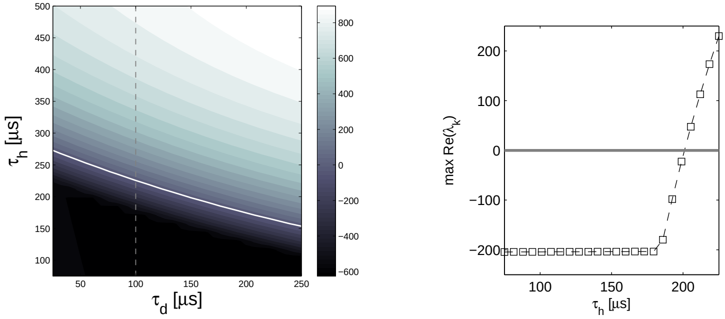
$$\det \Delta(s) = \det \left(sI - \mathcal{A}_0 - \sum_{i=1}^{n} \mathcal{A}_i e^{-s\tau_i} \right) = 0$$

- Mode-control and perfect decoupling: SISO dynamics (fixed gains)

$$G_{mn}(s) = \frac{1}{\tau_{mn}s - \tau_{mn}\gamma_{mn}} \frac{1}{\tau_c s + 1} \frac{1}{\tau_a s + 1} e^{-s\tau_h}$$

→ fictitious but usefull for disturbance rejection and resonant-field amplification analysis

- Spectrum dependence on τ_h: MIMO (rightmost roots) and SISO cases



SISO-set stability in (τ_h, τ_d)-space and MIMO-plant stability w.r.t. τ_h.

$\Rightarrow$ Infinite spectrum model with key periferal (automation) dynamics.

Model-based control and delay compensation

- Objective: ensure MHD stability and minimize closed-loop spectral abscissa under PI-TD structure constraints
- Advantages of TD approach:
  - varying computational complexity implies varying τ_h
  - free τ_h = fitting parameter to mimic experimental instability onset

Direct Eigenvalue Optimization

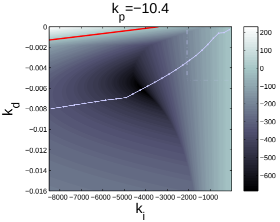
- direct MIMO approach generally nonconvex and nonsmooth

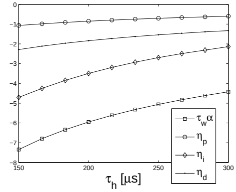
$\Rightarrow$ hybrid SISO/MIMO method: minimize the maximum spectral abscissas of the SISO set

- *gradient-sampling* method: *robustified* steepest-descent method suitable for nonsmooth optimization

Optimization results

- Two different parameterizations, implicitly assigning the closed-loop performance and control-input norm:
  - *a)* varying k_p and searching for the optimal $\tilde{\theta}^* = (k_i^*, k_d^*)$ for a nominal $\tau_h \rightarrow$ Max CL spectrum in (k_i, k_d)-space,
  - *b)* varying τ_h and determining the full optimal PID $\tilde{\theta}^* = (k_p^*, k_i^*, k_d^*)$
- Robust on the given problems: convergence within 10 – 30 iterations





Experimental results

Setup

- New T2R experiments: $\#20743 - \#20755$ and $\#20824 - \#20838$ - $I_p \approx 85\,\mathrm{kA}$, shot length $\tau_p = \sim 50 - 70\,\mathrm{ms}$ and *reversal* and *pinch* $(F, \Theta) \approx (-0.27, 1.72)$.
- Experiment vs. simulation model: global shape is conserved = suitable for control
- Initial horizontal motion compensated by an external system stronger than IS controller and needs $\sim 10\,\mathrm{ms}$ to become stationary
- Transient or steady-state generic performance index

$$J_\nu(\theta) \equiv \frac{1}{t_1 - t_0} \int_{t_0}^{t_1} \sigma_\tau \, \nu^T(\tau, \theta) \nu(\tau, \theta)$$

Performance improvements

Shot#	K_p	K_i	K_d	J_y	J_u	Comment
20743	150	16000	0.05	0.464	1.66	old setting 1
20744	160	16000	0.04	0.509	1.80	old setting 2
20746	106	37500	0.061	0.259	2.12	series *a)*
20747	126	47500	0.073	0.304	1.94	*a)*
20827	150	16000	0.05	0.501	1.60	old setting 1
20833	119.6	46800	0.065	0.304	1.77	*b)*
20835	106.8	39860	0.058	0.288	1.64	*b)*

$\Rightarrow$ Optimized controllers = 44% reduction of average field energy at the sensors during steady-state period, at the expense of higher input power (+28%).

Summary

- New model for MHD instabilities in T2R, explicitly including important geometrical and engineering aspects
- Direct closed-loop PID gain optimization for the corresponding DDE model

⇒ experimental intelligent-shell feedback in a RFP fusion research device

- Applicability of the model to real experimental conditions
- Strongly encourage future work, theoretically and experimentally, in both physical modeling and multivariable control for MHD.

General conclusions

- Thermonuclear fusion motivates key advances in many fields of research.
- Advanced control methodologies definitely make a difference.
- Most problems are associated with transport phenomena and the system complexity.
- Need to "cross the bridge" between physicists and control engineers.
- Controlled fusion aims to become a national field of expertise with ITER.

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