FR-FCM 2010 Final report:
Optimal control of safety factor profile in tokamaks

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Abstract

The aim of this project is to develop dedicated control methodologies for the tracking of a desired safety factor or current profile in Tore Supra tokamak. Such topic is of immediate interest for ITER and the EFDA Feedback Control group. The approaches considered necessitate advanced technical skills in automatic control and systems theory, which motivate a collaboration between CEA/IRFM and GIPSA-lab. They are based on optimization control theory applied first to a discretized (ODE) and then to a distributed (PDE) model of the plasma magnetic flux. Several issues, such as the nonhomogenous properties of the transport phenomena, actuation limitations, and control robustness are addressed.

I. Project objectives

A. Scientific framework

In the coming years the main challenge in the fusion community will be the development of experimental scenarios for the International Tokamak Experimental Reactor (ITER). Amongst them, the so-called advanced tokamak steady-state ones will play a significant role, since they will allow reproduction and study (on a smaller scale) of the conditions that are expected to be obtained in a fusion plant of reduced size and costs [1]. In these scenarios particular emphasis is given to the current density profile and to the way of producing the plasma current IP: due to the intrinsic limited availability of magnetic flux in the fusion devices, needed to sustain a purely inductive current, IP will have to be generated mainly by non-inductive sources. In particular, the importance of the real-time safety factor profile (q-profile) control has been emphasized in several works.

The control of so-called “advanced” plasma regimes for steady-state high performance tokamak operation is a challenge [2], in particular because of the non-linear coupling between the current density and the pressure profiles. In a burning plasma, the alpha-particle power will also be a strong function of these profiles, and, through its effect on the bootstrap current, will be at the origin of a large (though ultra-slow) redistribution of the current density. The possible destabilization of adverse toroidal Alfvèn eigenmodes (TAEs)-such as the drift kinetic modes that are anticipated to appear at high values of the central safety factor -as well as potential thermal instabilities due to the ITB dynamics will further complicate the issue. This provides additional arguments on the need for further investigation of plasma profiles regulation to ensure steady-state operation of the plasmas.

Previous control approaches have shown the interest of appropriate control methods to improve the plasma performances. Nevertheless, they are based on identified linear models of the plasma and/or semi-empirical tuning of the gains of a proportional-integrator controller, rendering the real-time control particularly sensitive to the operating conditions. The aim of our work is then to propose a new, model-based control approach focused on the dynamics of the magnetic flux profiles. More specifically, we will use the control-oriented model described in [3], where the current profile dynamics is derived based on a consistent set of simplified relationships, in particular for the microwave current drive sources, rather than exact physical modelling. This model has been compared with experimental results and has shown its efficiency to represent key issues for profile control on Tore Supra, while fulfilling tight real-time computation constraints. The core of this distributed model (described by a 1-D Partial Differential Equation) is composed of a non-homogeneous transport coefficient (resistivity), a nonlinear term (bootstrap current), distributed non-inductive sources (LH and ECCD inputs) and time-varying boundary conditions (magnetic coils).

B. *Specific objectives*

Considering that the magnetic coils are dedicated to the plasma shape stabilization and inductive current generation, they are considered as exogenous inputs and not explicitly considered in the regulation scheme. Supposing a consistent desired safety factor profile q_{ref}, the control objective is then to regulate the non-inductive sources such that q_{ref} is tracked efficiently according to dynamical and actuation constraints. Recent results [4] on the applicability of optimal and quasi-steady-state strategies to the modeling of particle transport in Tore Supra (also described by a PDE with non-homogeneous transport coefficients and with similar time-scales) motivated the following workplan for 2010:

- Formulate the control problem based on a discretized version of the model (set of ordinary differential equations) and investigate the efficiency and limitations of an optimal (LQR) approach combined with a state-linearizing feedback control design.
- Extend the optimal design approach in the PDE framework thanks to a Lagrangian approach and adjoint state computation.
- Investigate the robustness of the proposed controller with respect to model uncertainties in the transport coefficients, in the antennas coupling models and depending on different operating points. These robustness issues will be illustrated by simulation results.

II. Work description

A. *Context, background, positioning*

This work is in the direct continuation of the post-doc done by E. Witrant at CEA-Cadarache under the guidance of S. Brémond in 2006, when the control-oriented model [3] was derived. The aim of the present project is to use this model in an optimal model-based profile control approach for long pulses scenarios, a topic that has been classified as “very high priority” during the EFDA Feedback Control group meeting in July 2009. Integrated plasma control, and more particularly the control of profiles peaking during the current ramp-up phase, is recognized as a key issue for ITER and can be considered as a direct outcome of the proposed control approach. A collaboration with the Universities of Nice, Anger and Grenoble, managed by CEA-IRFM, has been set to address several aspects of model-based profile control.

B. *Scientific methods and main results*

We considered the simplified diffusion equation describing the dynamics of the poloidal flux, as stated in [3]:

$$\frac{\partial \psi}{\partial t}(\rho,t) = \frac{\eta_{\|}}{\mu_0}\frac{\partial^2 \psi}{\partial \rho^2} + \frac{\eta_{\|}}{\mu_0 \rho}\frac{\partial \psi}{\partial \rho} + \eta_{\|} R_0 j_{ni} \tag{1}$$

where $\eta_{\|}(\rho,t)$ is the plasma resistivity, $\mu_0 = 4\pi \times 10^{-7} Hm^{-1}$ is the permeability of free space, R_0 is the geometric center of the plasma torus and j_{ni} is the source term due to non inductive current sources (bootstrap effect and microwave current drives). The spatial index ρ is replaced with the normalized variable $x = \rho/a$, where a is the minor radius corresponding to the last closed magnetic surface (is considered constant). The inclusion of peripheral dynamics (temperature, density, bootstrap computation, antennas coupling with the plasma, etc.) is done according to Tore Supra configuration and scaling laws, as detailed in [3]. The control objective was formulated as a tracking

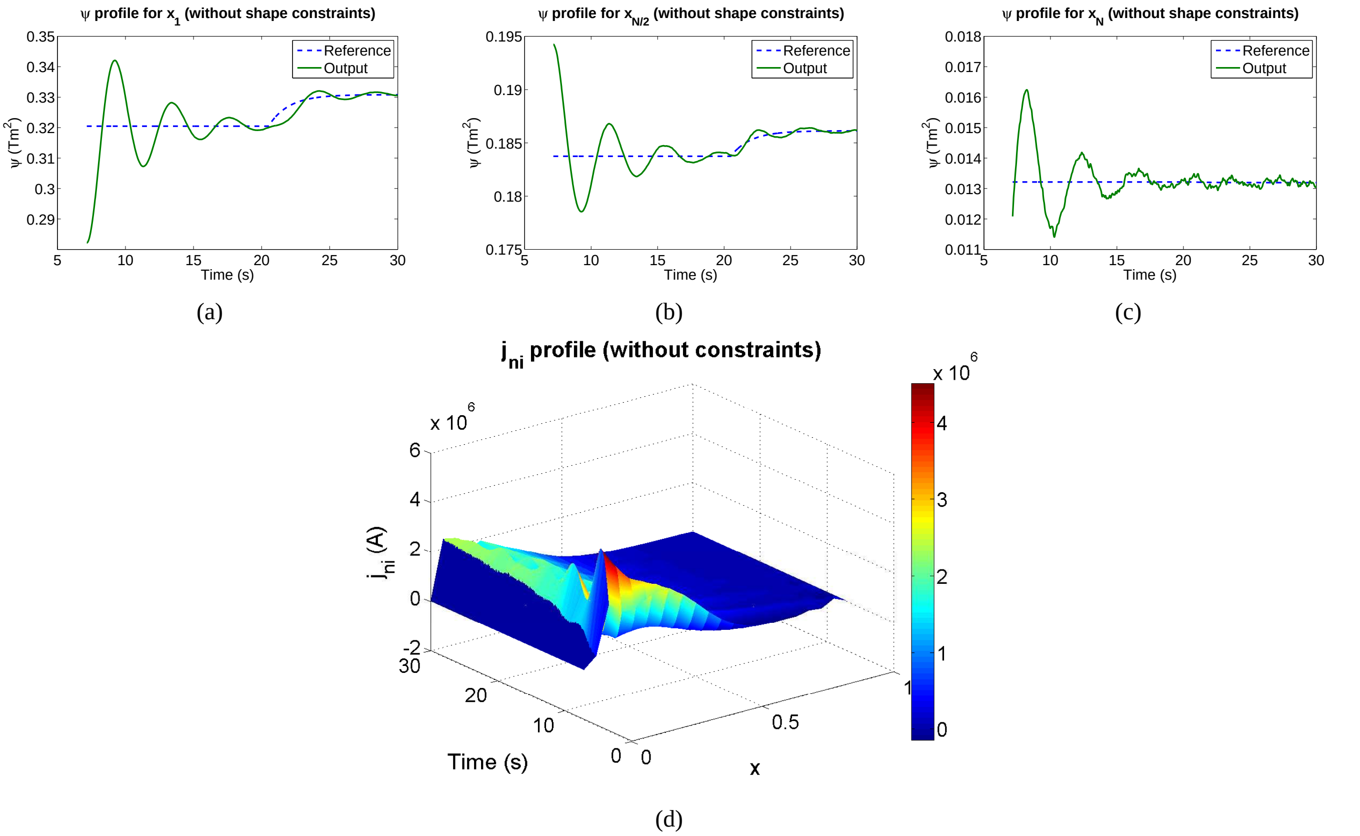


Figure 1. Regulation around $\overline{\psi}$ with the unconstrained controller (plain line: numerical simulation, dashed line: the reference). (a) Evolution and reference of the state ψ_1 (plasma center); (b) evolution and reference of the state $\psi_{N/2}$ (mid radius $a/2$); (c) evolution and reference of the state ψ_N (plasma edge); (d) applied control signal u.

problem, namely to regulate the profile $\psi(t)$ around a reference operating point profile $\overline{\psi}$. The controlled input was the non-inductive current deposit and the loop voltage was considered as an exogenous time-varying input (not controlled).

a) Model-based Control of the Magnetic Flux Profile in a Tokamak Plasma [5]: The first approach was carried by considering (1) discretized at specific locations and linearized around $\overline{\psi}$. Introducing an integrator to remove the steady-state error, this lead to the lumped dynamics:

$$\begin{bmatrix} \dot{\psi} \\ \dot{E} \end{bmatrix} = \begin{bmatrix} A(t) & 0 \\ -\mathbb{I} & -\lambda(t) \end{bmatrix} \begin{bmatrix} \psi \\ E \end{bmatrix} + \begin{bmatrix} B(t) \\ 0 \end{bmatrix} j_{ni} + \begin{bmatrix} W(t) \\ \overline{\psi} \end{bmatrix} \tag{2}$$

where E is the integral of the error. A new parameter $\lambda_{max} \geq \lambda(t) \geq 0$ has been introduced as a "forgetting factor" for the integrator. The purpose of this term is to avoid high overshoots when changing the operating point by weighting down past accumulated errors. It is clear that, to avoid steady-state errors, we must have $\lambda(t) \to 0$ as $t \to \infty$. This parameter is designed to vanish in finite time. The state-space matrices A and B are provided by the model, and W includes the boundary conditions. Optimal and pseudo-optimal profile regulation (input minimizing a quadratic cost over an infinite horizon) were then achieved for two different cases:

1) unconstrained input: $j_{ni}(x,t)$ is allowed to take any value to solve the optimization problem;
2) constrained input: $j_{ni}(x,t)$ is constrained to have a Gaussian shape distribution, which is motivated by the modeling of plasma-wave interaction proposed in [3].

The solution of the optimal control problem led to the online resolution of an agebraic Riccati equation (ARE), thus updating the feedback gain computation according to the present plasma state (trmerature, density, etc.).

The simulation results are presented in Figures 1 and 2, for the unconstrained and constrained case, respectively. The robustness of the feedback and the sensitivity to modeling errors was considered by introducing disturbances on the value of $\eta_{\parallel}$ used in the feedback design, which led to the simulation results presented in Figure 3. Note that

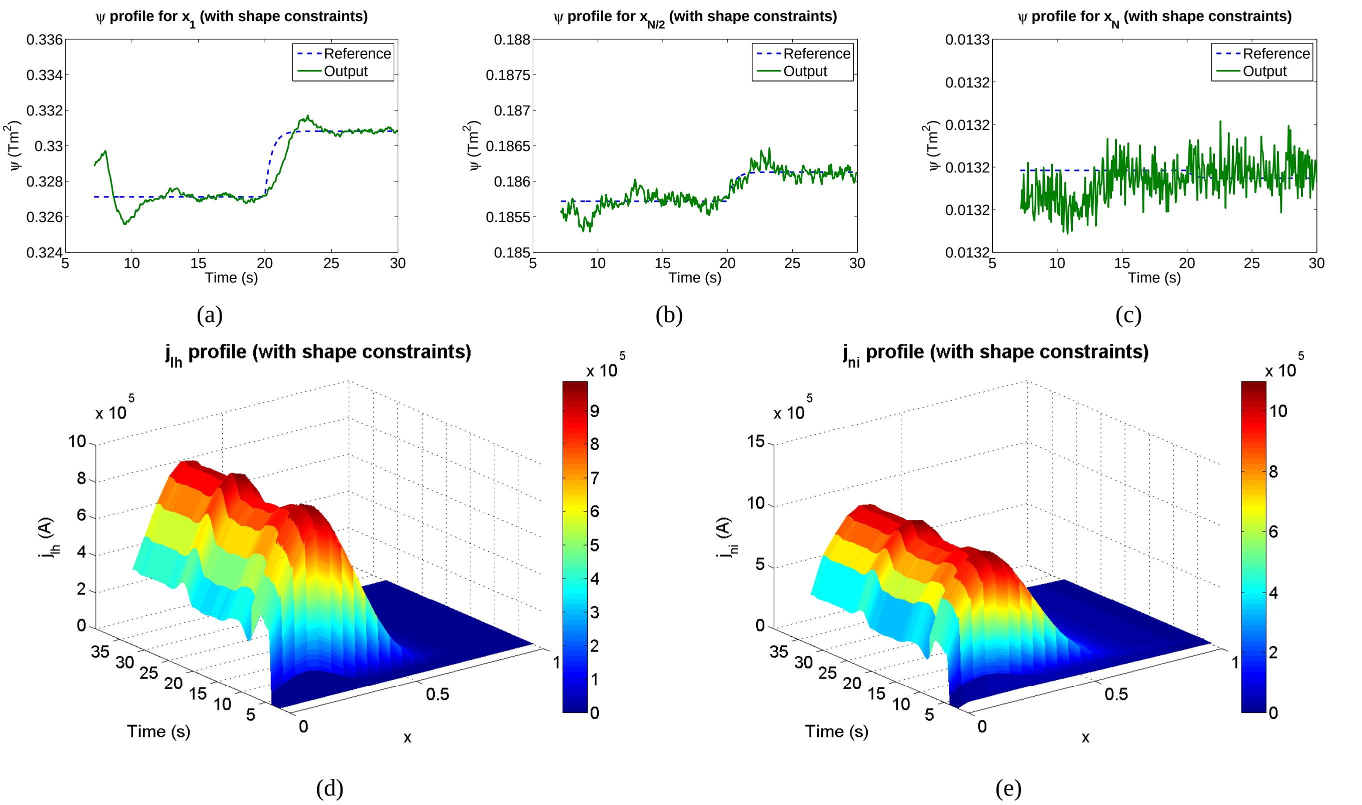


Figure 2. Regulation around $\overline{\psi}$ with the shape-constrained controller (plain line: numerical simulation, dashed line: the reference). (a) Evolution and reference of the state ψ_1; (b) evolution and reference of the state $\psi_{N/2}$; (c) evolution and reference of the state ψ_N; (d) applied control signal u; (e) resulting j_{ni} (for comparison with the unconstrained case).

the simulations are focused on the use of a single antenna (Lower Hybrid) but the proposed results can easily be extended to the multiple-antennas case.

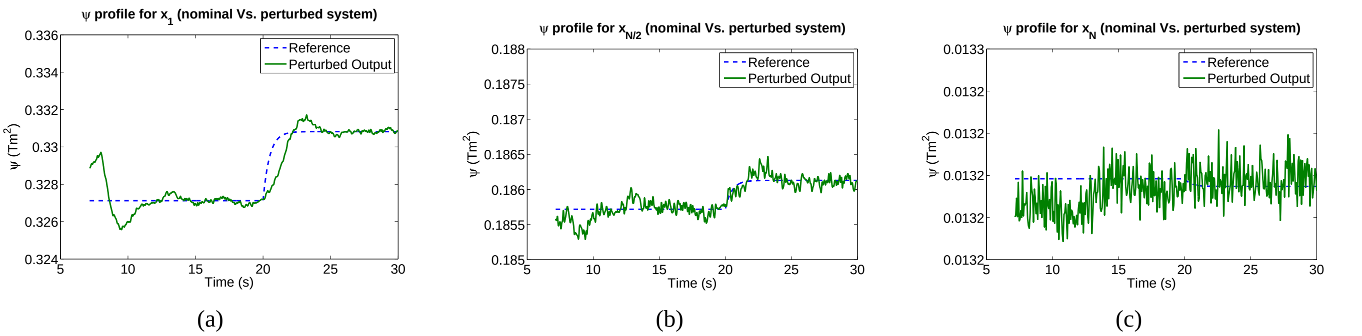


Figure 3. *Reference and perturbed output caused by an error in the estimation of* $\eta_{\|}$ *(dashed line: reference, solid line: numerical simulation).*

b) Polytopic Control of the Magnetic Flux Profile in a Tokamak Plasma [6]: The previous approach was then refined by parametrizing the time-varying state matrices in a linear parameter varying (LPV) formulation. More precisely, the discretized state matrix was expressed as:

$$A(t) = A_0 + \sum_{i=1}^{n_p} \lambda_i(t) A_i$$

where $\{A_0,\ A_1, \ldots,\ A_{n_p}\}$ is a nonempty base, λ_i are scalar parameters and n_p denotes the number of free parameter that is necessary to describe the system behavior. Based on this formulation, new theoretical results were obtained in the field of LPV control and a constructive feedback design methodology was proposed, guaranteeing the system

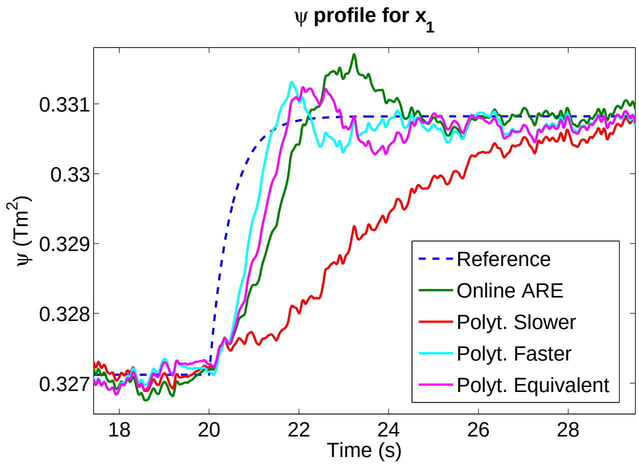
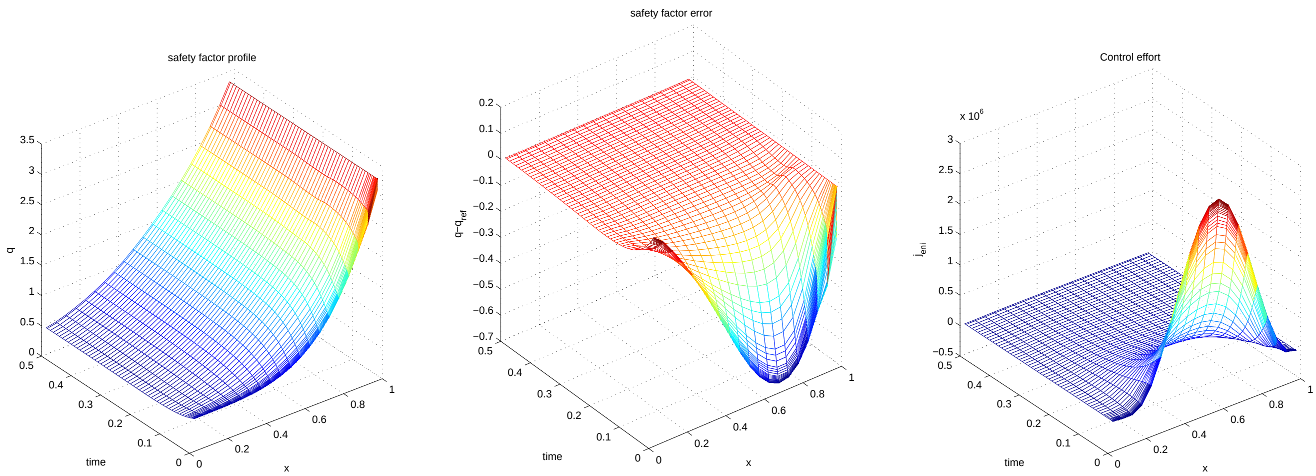


Figure 4. Regulation around $\bar{\psi}$ for ψ_1 with the LPV approach. The thin dashed line is the reference, the solid line is the ARE-based simulation (first approach) and the others are polytopic (LPV) regulators with different γ values.

convergence for all λ_is within given bounds and for a maximal maximum amplitude of the feedback gain (L_2 norm $||K||_2 \leq \gamma$).

The main advantage of this approach is to guarantee a proper tracking for a given range of resistivity bounds and to limit the user choice in the feedback design to a single scalar parameter related to the maximum feedback energy. Similar simulation tests as those presented in the previous section performed were carried, and the comparison between the two approaches is presented in Figure 4. The main advantage of this approach is to allow a simple tuning for the transient behavior of the magnetic flux (trade-off between fast response and energy consumption).



(a) Safety factor profile or the q-profile

(b) q-profile tracking error, $q(x,t) - q_{ref}(x,t)$

(c) External non-inductive current deposit, i.e. LH: $j_{lh}(x,t)$

Figure 5. Safety factor regulation using the SOS method (PDE feedback synthesis)

c) Control and verification of the safety-factor profile in tokamaks using Sum-of-Squares polynomials [7]: In this work, the magnetic flux dynamics was expressed in terms of the spatial derivative of ψ as:

$$\frac{\partial \psi_x(x,t)}{\partial t} = \frac{1}{\mu_0 a^2} \frac{\partial}{\partial x} \left(\frac{\eta_\parallel(x,t)}{x} \frac{\partial}{\partial x} \left(x \psi_x(x,t) \right) \right) + R_0 \frac{\partial}{\partial x} \left(\eta_\parallel(x,t) j_{ni}(x,t) \right).$$

The advantage of this formulation is to allow a direct computation for the safety factor profile q with:

$$q(x,t) \doteq \frac{\partial \phi / \partial x}{\partial \psi / \partial x} = \frac{-B_{\phi_0} a^2 x}{\partial \psi / \partial x},$$

where B_{ϕ_0} is the toroidal magnetic field at the plasma center and ϕ is the magnetic flux of the toroidal field.

Our goal was to design a controller for the dynamics of $\psi_x(x,t)$ without discretization of the PDE. In this work we used sum-of-squares optimization techniques to solve a dual version of the Lyapunov inequality. These methods are an extension of the duality theory developed for time-delay systems in [8]. The application of sum-of-squares to stability of PDEs was first investigated in [9]. We used the Matlab package SOSTOOLS [10] to set up and solve the SOS-based synthesis conditions.

Assuming that $\eta_{\|}$ is in quasi-steady-state (depending on x only) and using a controller of the following form:

$$j_{lh}(x,t) = K_1(x)\psi_x + \frac{d}{dx}(K_2(x)\psi_x), \tag{3}$$

where $K_1(x)$ and $K_2(x)$ are polynomial gains, new theoretical results were established to guarantee the closed-loop stability in the PDE framework. Additional constraints, such as the input shape (Gaussian) distribution or maximum amplitude, were implemented. The bootstrap effect was considered by removing its effect on the current distribution from the reference safety factor distribution. The simulation results presented in Figure 5, even if obtained with a constant resistivity profile, are very promising in terms of convergence speed and smooth tracking (no oscillating behavior as the ones obtained with the previous methods.

III. Conclusions

The main objectives of this project, concerning the *Optimal control of safety factor profile in tokamaks*, were met satisfyingly. More precisely:

- optimal control of the discretized dynamics was obtained with constrained and unconstrained inputs, including a forgetting factor to update the feedback when the operation point is modified;
- a linear parameter varying (LPV) approach was developed in order to guarantee the tracking efficiency for bounded resistivity profiles;
- a sum-of-squares method provided a PDE-control approach to this problem.

Each of these approaches motivated novel theoretical results in the field of automatic control and were applied in simulation to Tore Supra shot 35 109.

References

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[5] F. Bribiesca Argomedo, E. Witrant, C. Prieur, D. Georges, and S. Brémond, “Model-based control of the magnetic flux profile in a tokamak plasma,” in *Proc. of 49th IEEE Conference on Decision and Control*.

[6] F. Bribiesca Argomedo, C. Prieur, E. Witrant, and S. Brémond, “Polytopic control of the magnetic flux profile in a tokamak plasma,” in submitted to *IFAC World Congress*.

[7] A. Gahlawat, M. Peet, and E. Witrant, “Control and verification of the safety-factor profile in tokamaks using sum-of-squares polynomials,” in submitted to *IFAC World Congress*.

[8] M. M. Peet and A. Papachristodoulou, “Inverses of positive linear operators and state feedback design for time-delay systems,” in *8th IFAC Workshop on Time-Delay Systems*, 2009.

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[10] S. Prajna, A. Papachristodoulou, and P. Parrilo, “Introducing SOSTOOLS: A general purpose sum of squares programming solver,” in *IEEE conference on decision and control*, vol. 1. Citeseer, 2001, pp. 741–746.

The Papers and Conference contributions

Peer-reviewed conferences with papers published in the proceedings (full papers provided in the Appendix):

- F. Bribiesca Argomedo, E. Witrant, C. Prieur, D. Georges and S. Brémond, "Model-based Control of the Magnetic Flux Profile in a Tokamak Plasma", Proc. of 49th IEEE Conference on Decision and Control, Atlanta, USA, December 15-17, 2010.
- F. Bribiesca Argomedo, C. Prieur, E. Witrant and S. Brémond, "Polytopic Control of the Magnetic Flux Profile in a Tokamak Plasma", *submitted to* IFAC World Congress, 2011.*
- A. Gahlawat, M.M. Peet and E. Witrant, "Control and verification of the safety-factor profile in tokamaks using Sum-of-Squares polynomials", *submitted to* IFAC World Congress, 2011. Invited paper.*

Conferences based on abstract submissions:

- H. Ouarit, S. Brémond, R. Nouailletas, J.-F. Artaud, V. Basiuk, E. Witrant, L. Autrique, "Model based predictive control of tokamak plasma current profile", 26th Symposium on Fusion Technology (SOFT), Porto, Portugal, Sept. 27 - Oct. 1, 2010.
- E. Witrant and S. Brémond, "Parameter Dependant Shape Identification of Temperature Profiles in Tokamak Plasmas", *submitted to* 7th International Congress on Industrial and Applied Mathematics (ICIAM), Vancouver, BC, Canada, July 18-22, 2011. Invited paper.*

*: the publication decision for these papers is still pending, please do not diffuse over the internet.

APPENDIX

Related publications:

- F. Bribiesca Argomedo, E. Witrant, C. Prieur, D. Georges and S. Brémond, "Model-based Control of the Magnetic Flux Profile in a Tokamak Plasma", Proc. of 49th IEEE Conference on Decision and Control, Atlanta, USA, December 15-17, 2010.
- F. Bribiesca Argomedo, C. Prieur, E. Witrant and S. Brémond, "Polytopic Control of the Magnetic Flux Profile in a Tokamak Plasma", *submitted to* IFAC World Congress, 2011.
- A. Gahlawat, M.M. Peet and E. Witrant, "Control and verification of the safety-factor profile in tokamaks using Sum-of-Squares polynomials", *submitted to* IFAC World Congress, 2011.