Adaptive Control

Part 9: Adaptive Control with Multiple Models and Switching

Outline

1. Introduction
2. Multimodel Adaptive Control with Switching
3. Stability of the Adaptive System
4. Stability of the Switching System
5. Stability with Minimum Dwell Time
6. Application to the Flexible Transmission System
7. Effects of the Design Parameters

Introduction

Consider controller design for a system with very large uncertainty

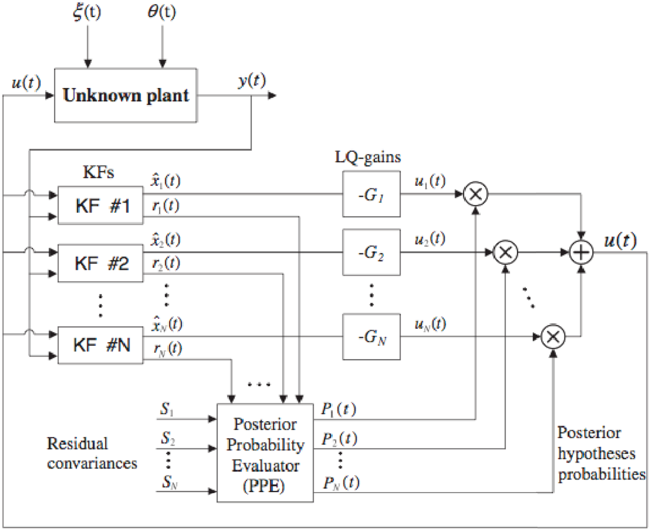
Robust Control : A fixed controller does not necessarily exist that stabilizes the system, or if it exists, it does not give good performances.

Adaptive Control : The classical adaptive control gives unacceptable transient adaptation for large and fast parameter variation.

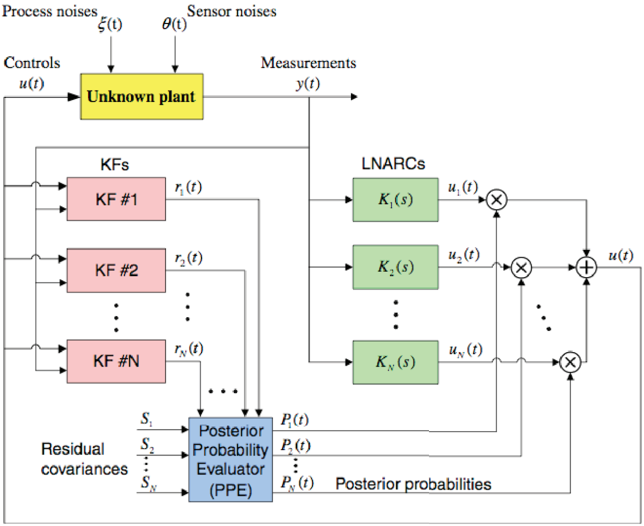
Solution: Multimodel adaptive control

- Classical multimodel adaptive control
- Robust multimodel adaptive control
- Multimodel adaptive control with switching
- Multimodel adaptive control with switching and tuning

Classical multimodel adaptive control



Robust multimodel adaptive control



Robust multimodel adaptive control

- Very similar to the classical multimodel adaptive control.
- The controllers are robust with respect to unmodeled dynamics and uses output feedback instead of state-feedback.
- The control input is the weighted sum of the outputs of the controllers (no switching).

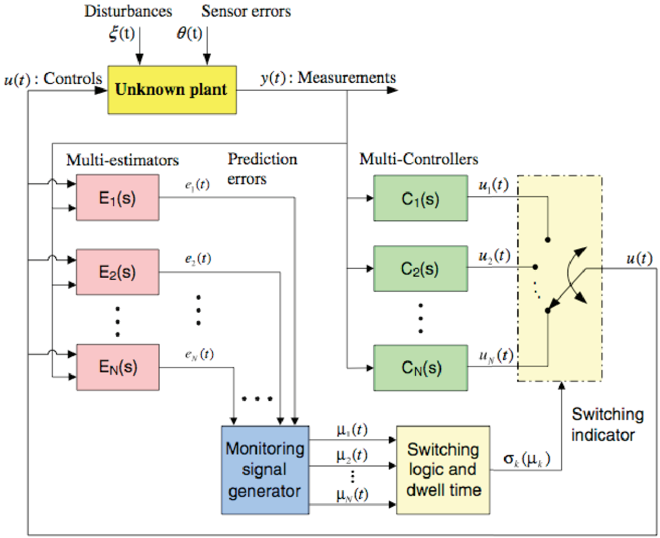
$$u(t) = \sum_{i=1}^{N} P_i(t) u_i(t)$$

where $P_i(t)$ is the posterior probability of i-th estimator.
- The posterior probabilities are computed as:

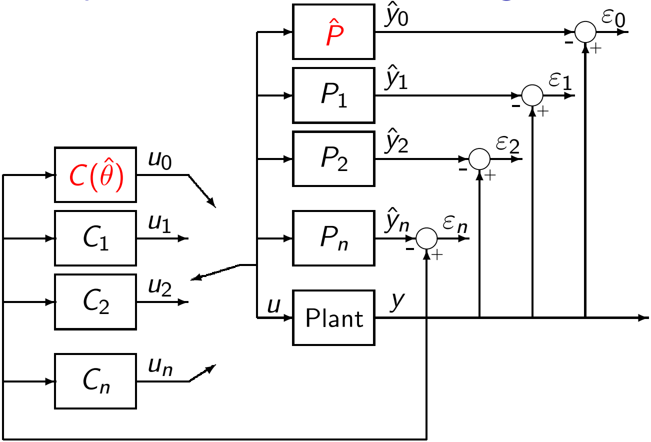
$$P_k(t+1) = \left[\frac{\beta_k e^{-\frac{1}{2} r_k'(t+1) S_k^{-1} r_k(t+1)}}{\sum_{i=1}^{N} \beta_i e^{-\frac{1}{2} r_i'(t+1) S_i^{-1} r_i(t+1)} P_i(t)} \right] P_k(t)$$

- The stability of this scheme is not guaranteed.

Multimodel adaptive control with switching



Multimodel adaptive control with switching and tuning



After a parameter variation (a large estimation error)

- First the controller corresponding to the closest model (fixed model) is chosen (switching).
- Then the adaptive model is initialized with the parameter of this model and will be adapted (tuning).

Structure of Multimodel Adaptive Control with Switching

Plant: LTI-SISO (for analysis) with parametric uncertainty and unmodelled dynamics:

$$\bigcup_{\theta \in \Theta} P(\theta)$$

where $P(\theta) = P_0(\theta)[1 + W_2\Delta]$ with $\|\Delta\|_\infty < 1$.
Other type of uncertainty can also be considered.

Multi-Estimator: Kalman filters, fixed models, adaptive models. If Θ is a finite set of n models, these models can be used as estimators (output-error estimator). If Θ is infinite but compact, a finite set of n models with and adaptive model can be used.

Multi-Controller: We suppose that for each $P(\theta)$ there exists $C(\theta)$ in the multi-controlller set that stabilizes $P(\theta)$ and satisfies the desired performances (the controllers are robust with respect to unmodelled dynamics).

Structure of Multimodel Adaptive Control with Switching

Monitoring Signal: is a function of the estimation error to indicate the best estimator at each time.

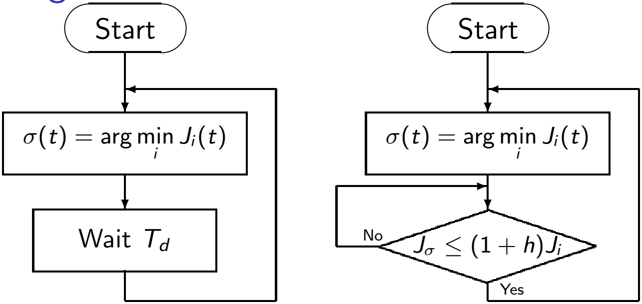
$$J_i(t) = \alpha\varepsilon_i^2(t) + \beta\sum_{j=0}^{t} e^{-\lambda(t-j)}\varepsilon_i^2(j)$$

with $\lambda > 0$ a forgetting factor, $\alpha \geq 0$ and $\beta > 0$ weightings for instantaneous and past errors.

Switching Logic: Based on the monitoring signal, a switching signal $\sigma(t)$ is computed that indicates which control input should be applied to the real plant. To avoid chattering, a minimum dwell-time between two consecutive switchings or a hysteresis is considered.

The dwell-time and hysteresis play an important role on the stability of the switching system.

Switching Logic



Dwell-time

Hysteresis

- A large value for T_d may deteriorate the performance and a small value can lead to instability.
- With hysteresis, large errors are rapidly detected and a better controller is chosen. However, the algorithm does not switch to a better controller in the set if the performance improvement is not significant.

Stability of Adaptive Control with Switching

Trivial case:

- No unmodelled dynamics and no noise,
- the set of models is finite,
- parameters of one of the estimators matches those of the plant model,
- plant is detectable.

Main steps toward stability:

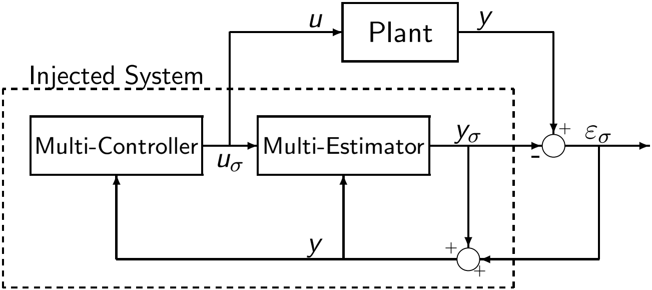
1. One of the estimation errors (say ε_k) goes to zero.
2. $\varepsilon_\sigma(t) = y(t) - y_\sigma(t)$ goes to zero as well.
3. After a finite time τ switching stops $(\sigma(\tau) = k \quad t \geq \tau)$.
4. If ε_k goes to zero, θ_k will be equal to θ and the controller C_k stabilizes the plant $P(\theta)$:

   (*Certainty equivalence stabilization theorem*)

Stability of Adaptive Control with Switching

Assumptions: Presence of unmodelled dynamics and noise.
Existence of some "good" estimators in the multi-estimator block.
The plant P is detectable.

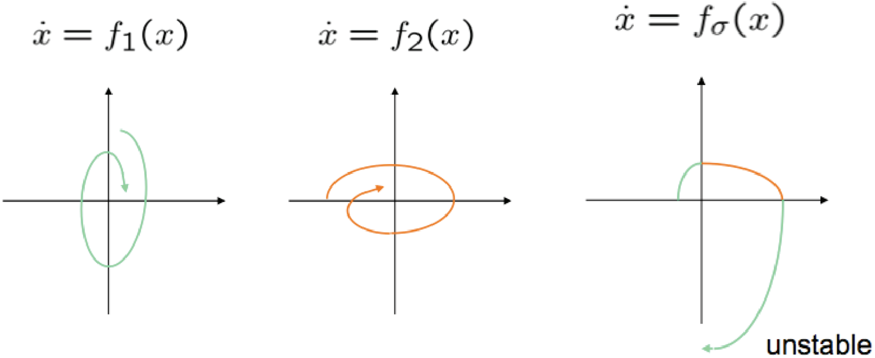
1. ε_k for some k is small.
2. ε_σ is small (because of a "good" monitoring signal).
3. All closed-loop signals and states are bounded if:
   The injected system is stable.



Stability of Switching Systems

Each controller stabilizes the corresponding model in the multi-estimator for frozen σ.

Question: Is the injected system stable for a time-varying switching signal $\sigma(t)$?



Is $f_\sigma(x)$ stable? No

Stability of Switching Systems

Consider a set of stable systems:

$$\dot{x} = A_1 x, \quad \dot{x} = A_2 x, \quad \dots \quad \dot{x} = A_n x$$

then $\dot{x} = A_\sigma x$ is stable if :

- A_1 to A_n have a common Lyapunov matrix (*quadratic stability*).
- If the minimum time between two switchings is greater than T_d (*minimum dwell time*).
- If the number of switching in the interval (t, T) does not grow faster than linearly with T (*average dwell time*).

$$N_\sigma(t, T) \leq N_0 + \frac{T - t}{T_d} \qquad \forall T \geq t \geq 0$$

$N_0 = 1$ implies that σ cannot switch twice on any interval shorter than T_d.

Stability of Switching Systems

Common Lyapunov Matrix : The existence of a common Lyapunov matrix for $A_1, \ldots, A_n$ guarantees the stability of A_σ. This can be verified by a set of Linear Matrix Inequalities (LMI):

Continuous-time

$$\begin{array}{c} A_1^T P + PA_1 < 0 \\ A_2^T P + PA_2 < 0 \\ \vdots \\ A_n^T P + PA_n < 0 \end{array}$$

Discrete-time

$$\begin{array}{c} A_1^T PA_1 - P < 0 \\ A_2^T PA_2 - P < 0 \\ \vdots \\ A_n^T PA_n - P < 0 \end{array}$$

- The stability is guaranteed for arbitrary fast switching.
- The stability condition is too conservative.

Stability of Switching Systems by Minimum Dwell Time

Theorem : Assume that for some $T_d > 0$ there exists a set of positive definite matrix $\{P_1, P_2, \ldots, P_n\}$ such that:

$$A_i^T P_i + P_i A_i < 0 \qquad \forall i = 1, \ldots, N$$

and

$$e^{A_i^T T_d} P_j e^{A_i T_d} - P_i < 0 \qquad \forall i \neq j = 1, \ldots, N$$

Then any switching signal $\sigma(t) \in \{1, 2, \ldots, N\}$ with $t_{k+1} - t_k \geq T_d$ makes the equilibrium solution $x = 0$ of

$$\dot{x}(t) = A_{\sigma(t)} x(t) \qquad x(0) = x_0$$

globally asymptotically stable.

- The first group of LMIs are always feasible because A_1 to A_n are stable.
- The second LMIs are always feasible if T_d is large enough.
- T_d can be minimized using LMIs.

Stability of Switching Systems by Minimum Dwell Time

Proof : Consider the Lyapunov function $V(x(t)) = x^T(t)P_\sigma x(t)$. We should show that for any $t_{k+1} = t_k + T_k$ with $T_k \geq T_d > 0$ we have $V(x(t+k)) < V(x(t))$. Assume that $\sigma(t) = i$ for $t \in [t_k \; t_{k+1})$ and $\sigma(t_{k+1}) = j$. We have :

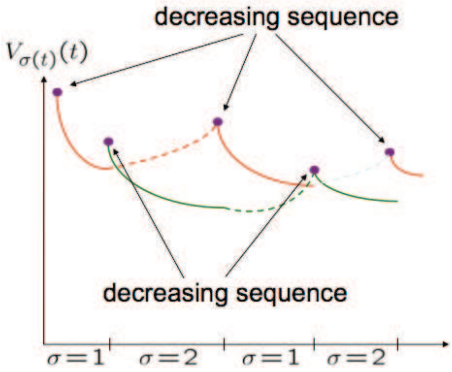
$$
\begin{aligned}
V(x(t+1)) &= x^T(t_{k+1})P_j x(t_{k+1}) \\
&= x^T(t_k)e^{A_i^T T_k} P_j e^{A_i T_k} x(t_k) \\
&< x^T(t_k)e^{A_i^T (T_k - T_d)} P_i e^{A_i (T_k - T_d)} x(t_k) \\
&< x^T(t_k)P_i x(t_k) \\
&< V(x(t_k))
\end{aligned}
$$

- This can be proved for discrete-time systems as well.
- If A_1 to A_n are quadratically stable, i.e. $P = P_1 = P_2 = \ldots = P_n$ then the LMIs are feasible for any $T_d > 0$.

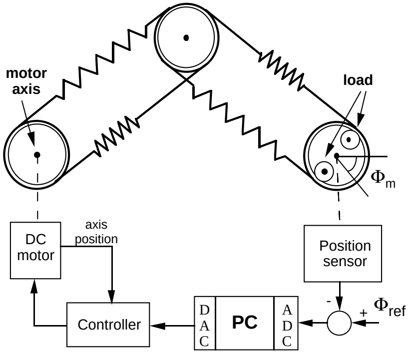
Stability of Switching Systems by Minimum Dwell Time

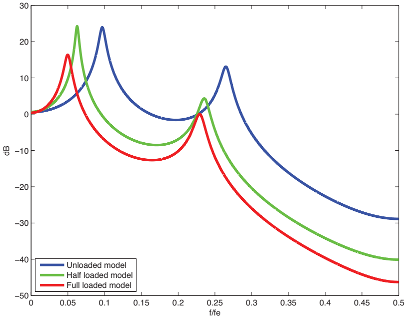
Comments : There are some conservatism in this approach.

1. Minimum of T_d is an upper bound on the minimum dwell time.
2. $V_i(x(t_q)) < V_i(x(t_p))$ where $\sigma_(t_p) = \sigma(t_q) = i$ is sufficient for the stability.

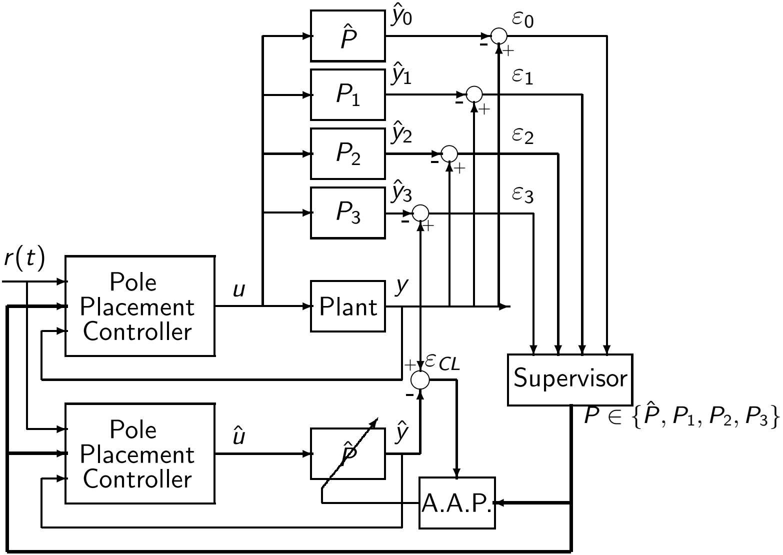


Application to the Flexible Transmission System





Multiple Model with Switching and Tuning



Design Procedure

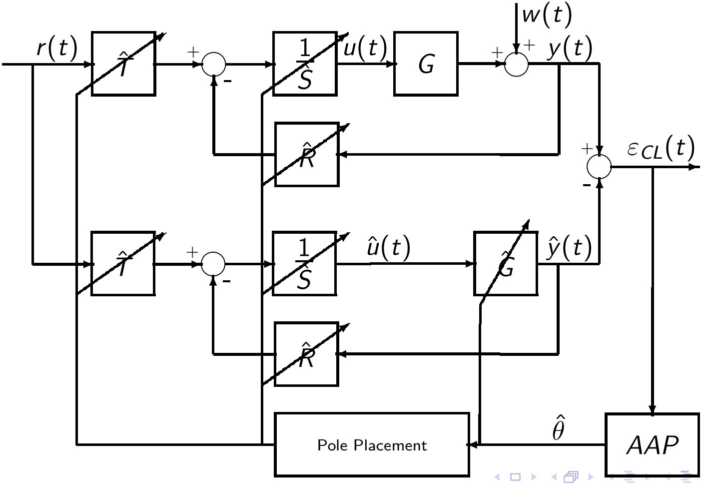
Multi estimator Design:

- Number of estimators: more estimator $\rightarrow$ more accuracy but more complexity.
- Type of estimators: output error, AMAX predictor, Kalman filter, etc.
- Fixed or adaptive: All fixed needs too many models for a desired accuracy. With all adaptive estimators, a persistently excitation signal is needed. A trade-off is a few number of fixed model such that at least one of them stabilizes the plant model and an adaptive model to improve the accuracy.
- Adaptive model: It should be initialized with the parameters of the closest fixed model. It can be a classical RLS or a CLOE adaptive model. For regulation problem adaptive model is not proposed.

For flexible transmission system we chose 3 output error fixed estimators in 0 % 50% and 100% load and one adaptive model with CLOE.

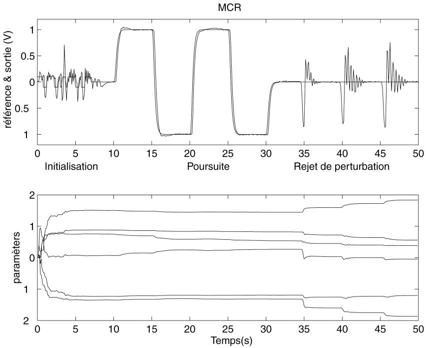
Design Procedure

RLS versus CLOE : Disturbance and noise affect the parameters of RLS estimators (drift problem) but not the parameters of CLOE estimators. Therefore, in the absence of excitation signal, the closed-loop system may become unstable.

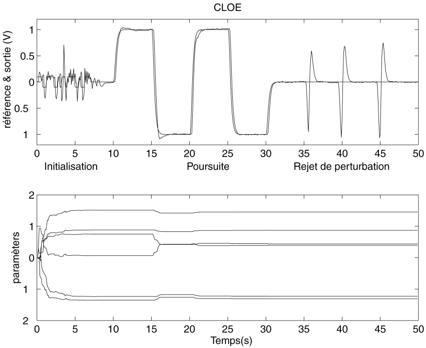


RLS versus CLOE

Experimental results



Classical adaptive control



CLOE adaptive control

Design Procedure

Multi-controller design : Robust pole placement

Desired closed-loop poles: Complex and simple poles are chosen.

- A pair of complex poles with the same frequency as the first resonance mode and a damping factor of 0.8.
- A pair of complex poles with the same frequency as the second resonance mode and a damping factor of 0.2.
- 6 auxiliary poles at 0.2.

Fixed terms in the controller: A fixed pole at 1 (integrator) and a zero at -1 for reducing the input sensitivity function at high frequencies.

Remark : Adaptive model is initialized with the parameters of the last switched estimator and the desired closed-loop poles are chosen based on this fixed model.

Design procedure

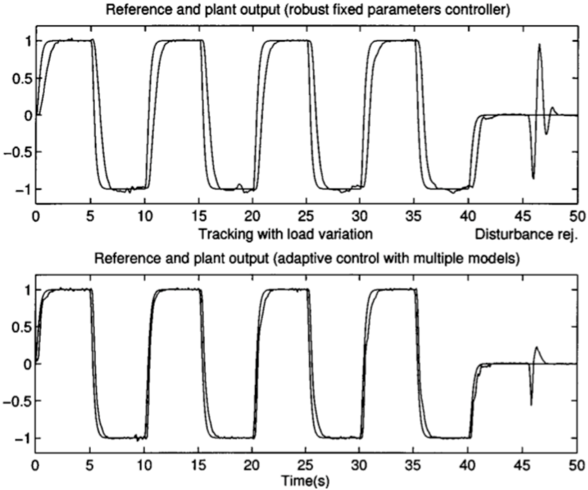
Monitoring signal and switching logic:

$$J_i(t) = \alpha\varepsilon_i^2(t) + \beta\sum_{j=0}^{t} e^{-\lambda(t-j)}\varepsilon_i^2(j) \qquad \alpha \geq 0,\ \beta > 0,\ \lambda > 0$$

- $\alpha \gg \beta$: More weights on instantaneous errors $\rightarrow$ fast reaction. This leads to fast parameter adaptation but poor performance w.r.t. disturbance.
- $\alpha = \lambda = 0$: Monitoring signal is the two-norm of the error for each estimator. The reaction to parameter variation is slow but leads to good performance w.r.t. disturbance.
- The minimum dwell time should be chosen to assure the stability. If the theoretical minimum is too large, a hysteresis cycle with an average dwell time is preferred.

Experimental results on the flexible transmission system

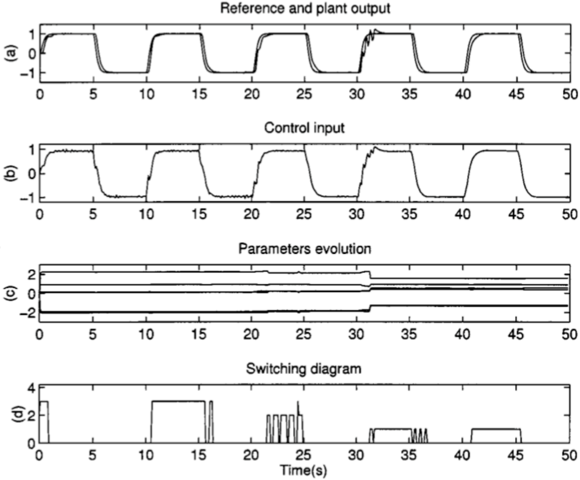
Multimodel adaptive control versus robust control



Load changes from 0 to 100 % in four steps (9,19,29 and 39s)

Experimental results on the flexible transmission system

Multiple model with CLOE

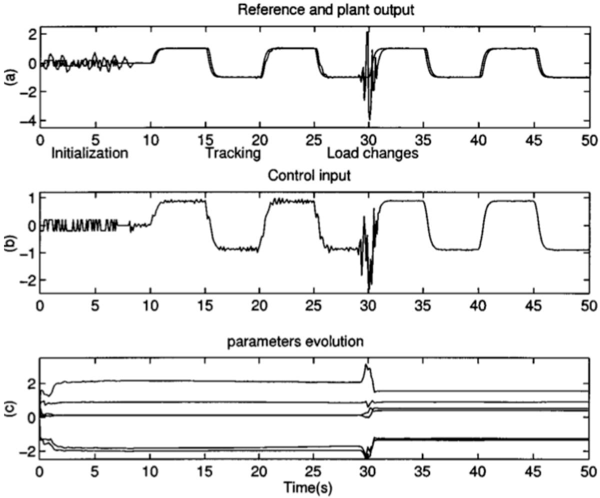


Load changes from 100% to 0% in two stages (19 and 29s)

$$\alpha = 1, \beta = 1, \lambda = 0.1$$

Experimental results on the flexible transmission system

Same case with classical adaptive control (simulation)

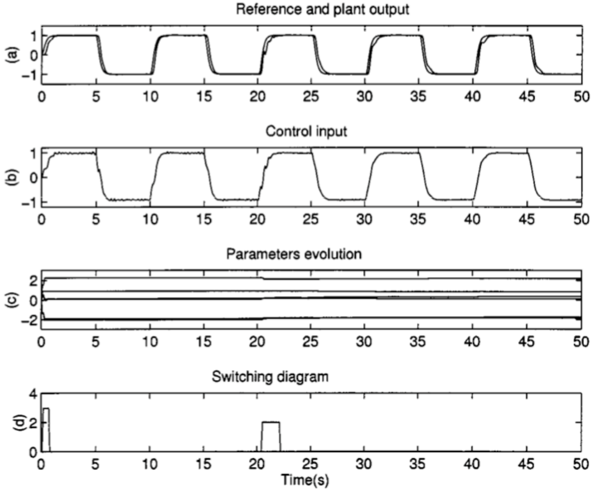


Load changes from 100% to 0% in two stages (19 and 29s)

Unstable in real time experiment

Experimental results on the flexible transmission system

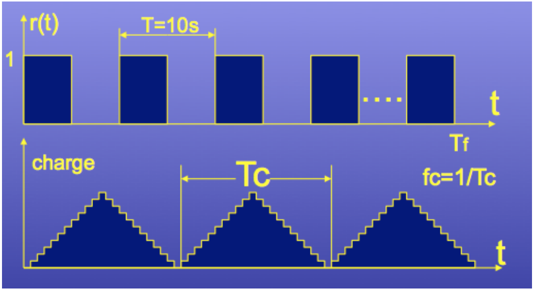
Real system does not belong to the fixed models set



Load changes from 75% to 25% in one stages (19s)

Effects of design parameters

Design parameters: Number of fixed and adaptive models, choice of adaptation algorithm, forgetting factor λ, dwell time.
Test conditions: Spaced parameter variation and frequent parameter variation are simulated using following signals:

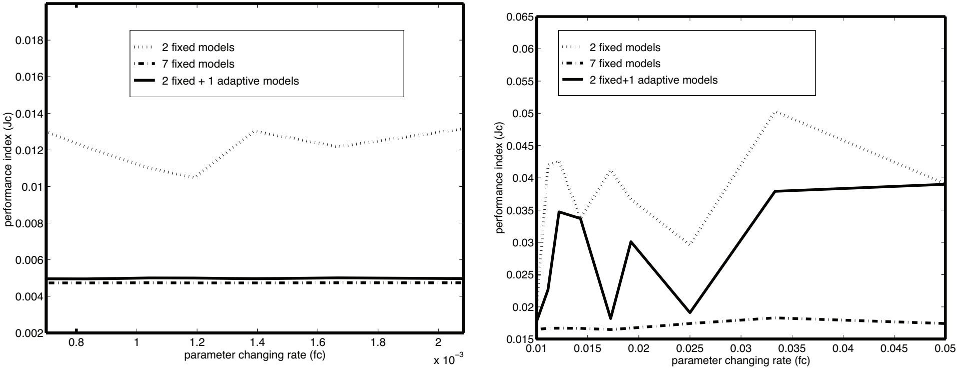


$T_c \gg T$ represents spaced parameter variation and
$2T < T_c < 100T$ frequent parameter variation.

Performance criterion: $J_c(t) = \left(\frac{1}{T_f} \int_0^{T_f} [r(t) - y(t)]^2 \right)^{1/2}$

Effects of design parameters

Number of fixed and adaptive models

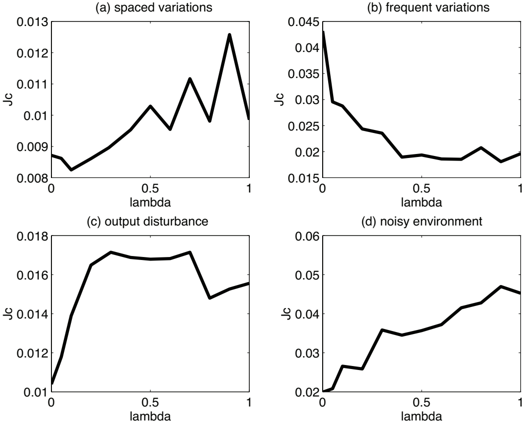


Spaced parameter variation *Frequent parameter variation*

- One adaptive model can reduce the number of fixed models.
- Adaptive models have more effects for spaced parameter variation.

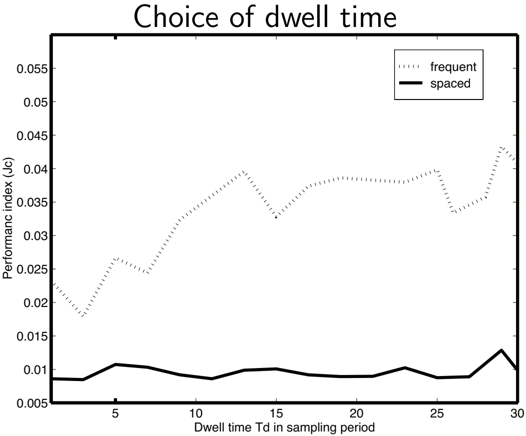
Effects of design parameters

Choice of forgetting factor



- Spaced variation $\rightarrow \lambda$ small. Frequent variation $\rightarrow \lambda$ large.
- Disturbance and noisy at the output $\rightarrow \lambda$ small.
- In this example $\lambda = 0.3 - 0.4$ is a good trade off.

Effects of design parameters

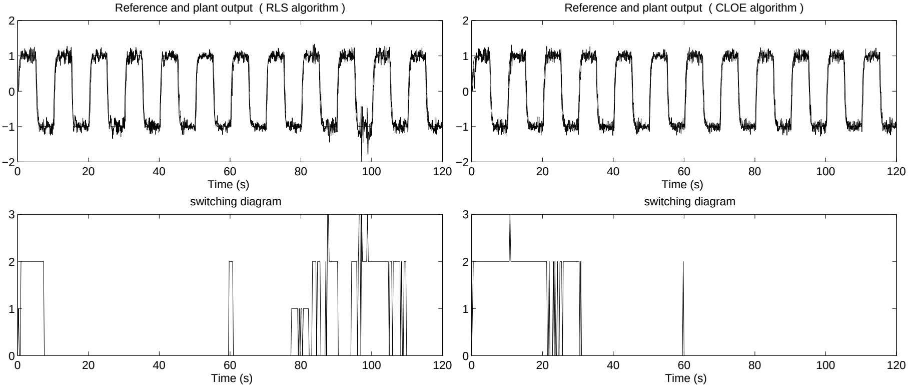


Performance criterion versus dwell time for spaced and frequent parameter variation

The smallest dwell time that assures the stability should be chosen

Effects of design parameters

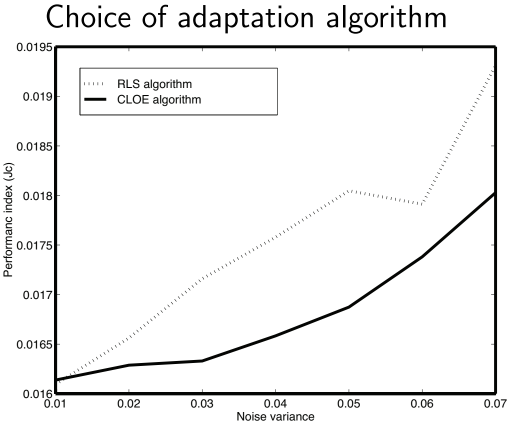
Choice of adaptation algorithm



RLS adaptation algorithm *CLOE adaptation algorithm*

- Three fixed models (no-load, half-load, full-load) in multi-estimator.
- Fixed plant model (25% load).

Effects of design parameters



Performance criterion versus noise variance for RLS and CLOE

CLOE gives to switching control a better performance and switching control assure the stability of adaptive control with CLOE

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