*A course on:*

# Adaptive Control

## Methods, algorithms and applications

***From needs to applications***

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Grenoble, December 7-11, 2009

# Outline of the Course

1. Introduction
2. Parameter adaptation algorithms
3. Identification in open loop – a brief review
4. Iterative identification in closed loop and controller redesign
5. Direct and Indirect Adaptive Control
6. Robust control design for Adaptive Control
7. Parameter estimators for Adaptive Control
8. Adaptive regulation (versus adaptive control)
9. Adaptive control with multiple models

# Adaptive Control

Part 1: Introduction

# Adaptive Control

A set of techniques for automatic adjustment of the controllers in *real time*, in order to achieve or to maintain a desired *level of performance* of the control system when the parameters of the plant (disturbance) dynamic model are unknown and/or change in time

**Particular cases:**

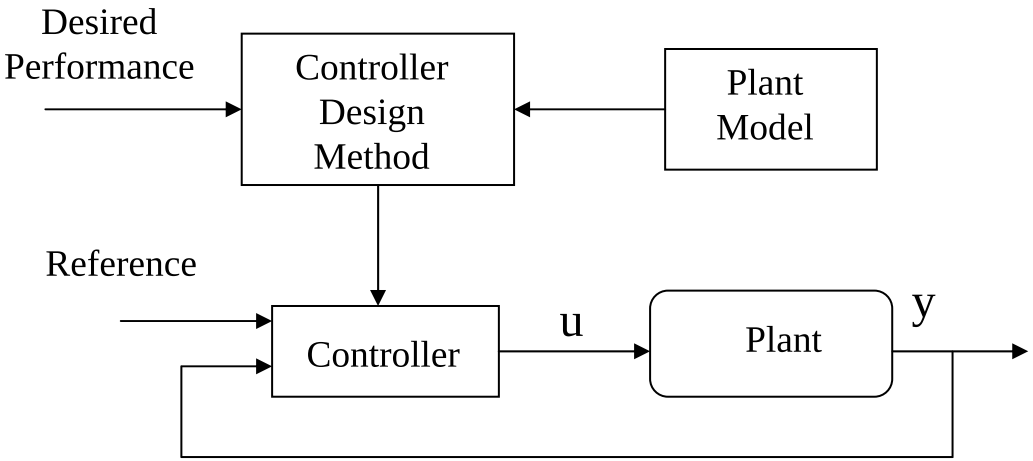
1 Automatic tuning of the controllers for unknown but constant plant parameters

2 Unpredictable change of the plant (disturbance) model in time

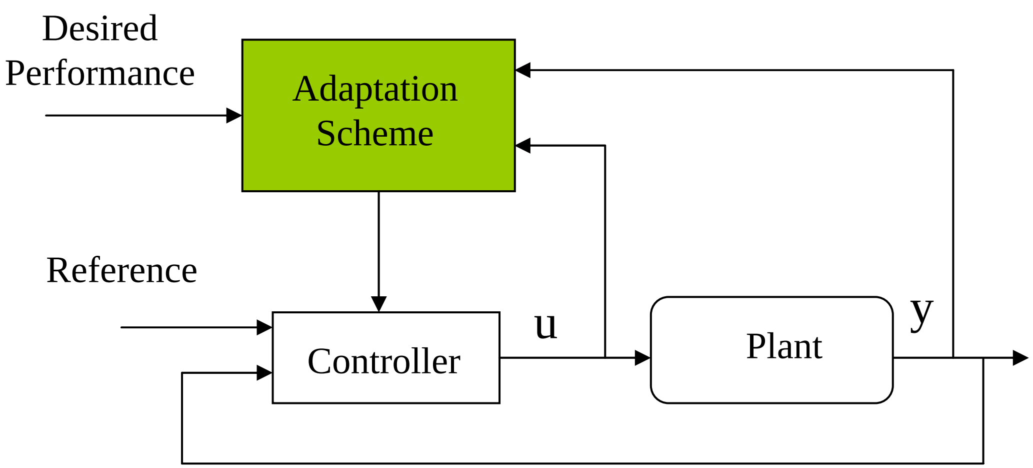
# Outline

- Concepts
- Basic schemes
- Adaptive control versus Robust control
- Adaptive control configurations
  (open loop adaptation, direct and indirect adaptive control)
- Parameter adaptation algorithms
- RST digital controller
- Adaptive control: regimes of operation
- Identification in closed and controller redesign
- Adaptive regulation
- Use of *a priori* available information
- Adaptive control with multiple models
- Example of applications

# Conceptual Structures




Principle of model based control design




An adaptive control structure

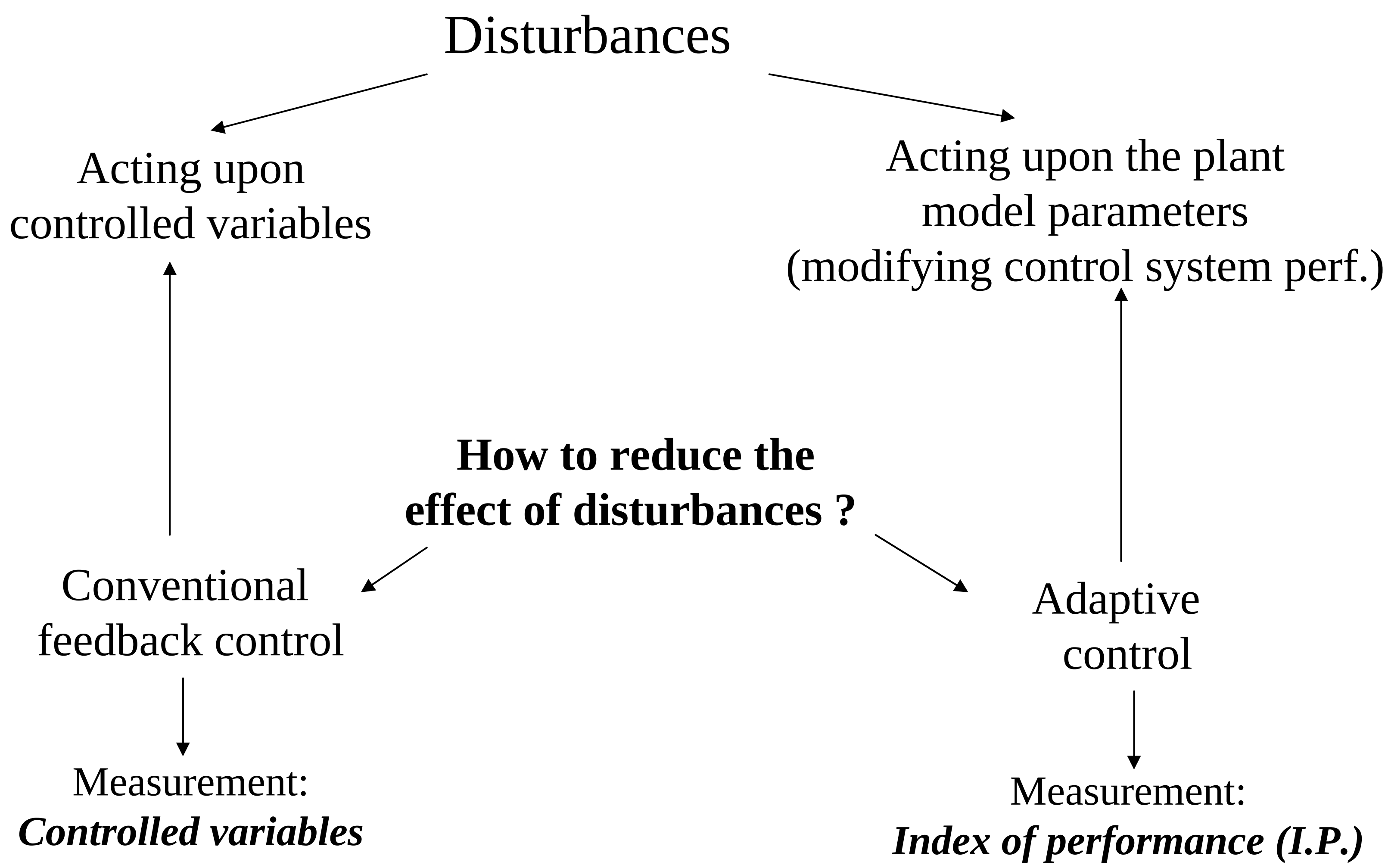
Remark:

An adaptive control system is *nonlinear* since controller parameters will depend upon *u* and *y*

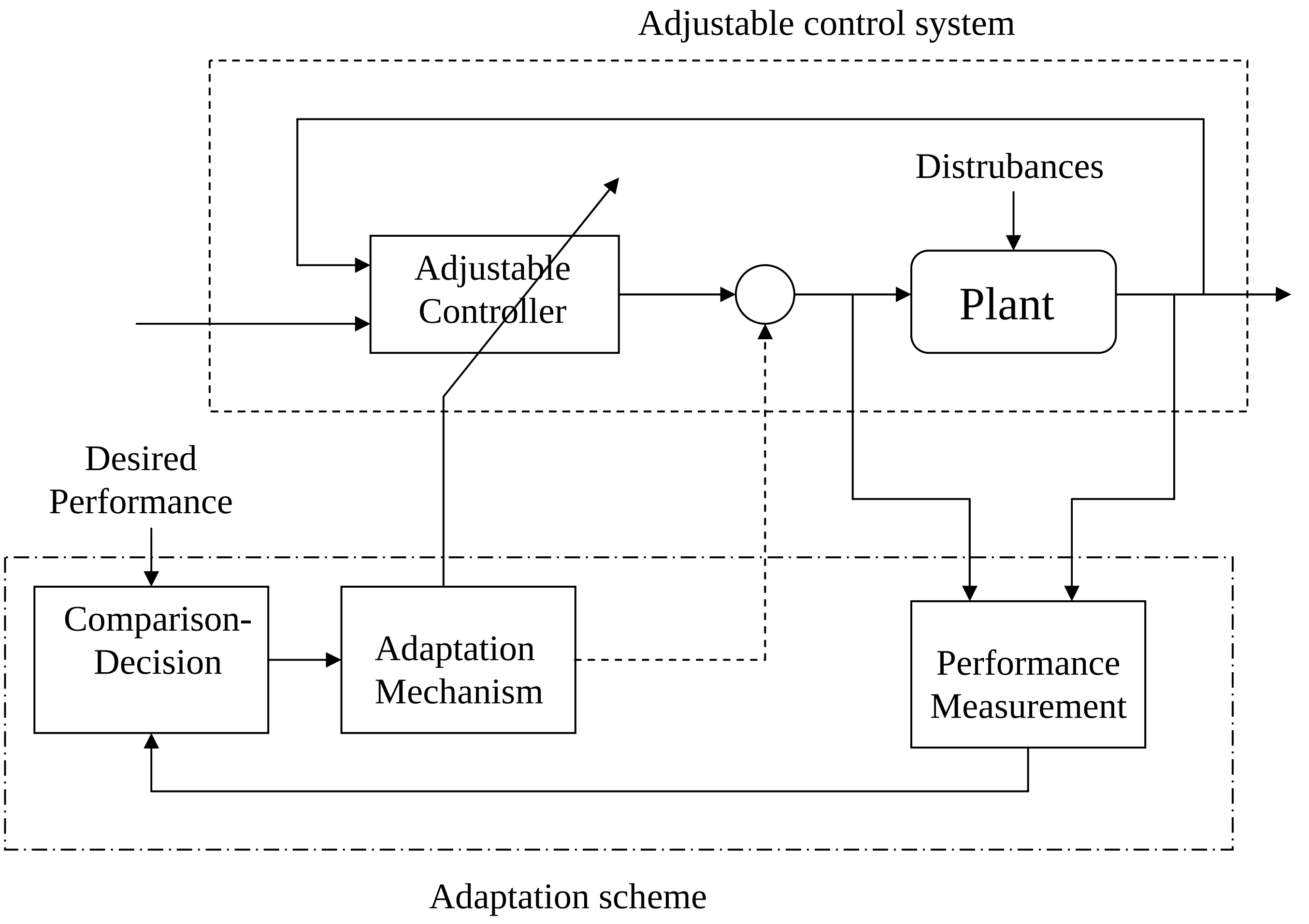
# Adaptive control- Why ?

- High performance control systems may require precise tuning of the controller but plant (disturbance) model parameters may be unknown or time-varying
- "Adaptive Control" techniques provide a systematic approach for automatic on-line tuning of controller parameters
- "Adaptive Control" techniques can be viewed as approximations of some nonlinear stochastic control problems (not solvable in practice)
- Objective of "Adaptive Control" : *to achieve and to maintain acceptable level of performance when plant (disturbance) model parameters are unknown or vary*

# Adaptive Control versus Conventional Feedback Control



# Adaptive Control – Basic Configuration



# Adaptive Control versus Conventional Feedback Control

| Conventional Feedback Control System | Adaptive Control System |
| --- | --- |
| Obj.: Monitoring of the “controlled” variables according to a certain IP for the case of known parameters | Obj.: Monitoring of the performance (IP) of the control system for unknown and varying parameters |
| Meas.: Controlled variables | Meas.: Index of performance (IP) |
| Transducer | IP measurement |
| Reference input | Desired IP |
| Comparison block | Comparison decision block |
| Controller | Adaptation mechanism |

# Adaptive Control versus Conventional Feedback Control

A conventional feedback control system is mainly dedicated to the *elimination of the effect of disturbances* upon the controlled variables.

An adaptive control system is mainly dedicated to the *elimination of the effect of parameter disturbances (variations)* upon the performance of the control system.

**Adaptive control system = hierarchical system:**

- **Level 1 : Conventional feedback system**
- **Level 2 : Adaptation loop**

# Fundamental Hypothesis in Adaptive Control

For any possible values of plant (disturbance) model parameters there is a controller with a fixed structure and complexity such that the specified performances can be achieved with appropriate values of the controller parameters

*The task of the adaptation loop is solely to search for the "good" values of the controller parameters*

# Adaptive Control versus Robust Control

Adaptive control can further improve the performance of a robust control system by:

- expanding the range of uncertainty for which performance specification can be achieved
- better tuning of the nominal controller

*For building an adaptive control systems robustness issues for the underlying controller design can not be ignored.*

The objective is to add adaptation capabilities to a robust controller and not to use adaptive approach for tuning a non robust controller.

# Conventional Control – Adaptive Control - Robust Control

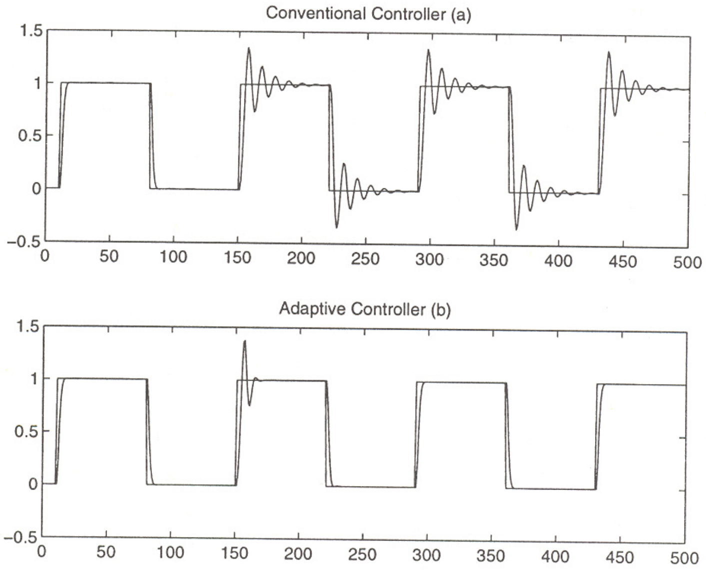



Figure 1.2.2: *Comparison of an adaptive controller with a conventional controller (fixed parameters), a) Fixed parameters controller, b) Adaptive controller*

Conventional versus Adaptive

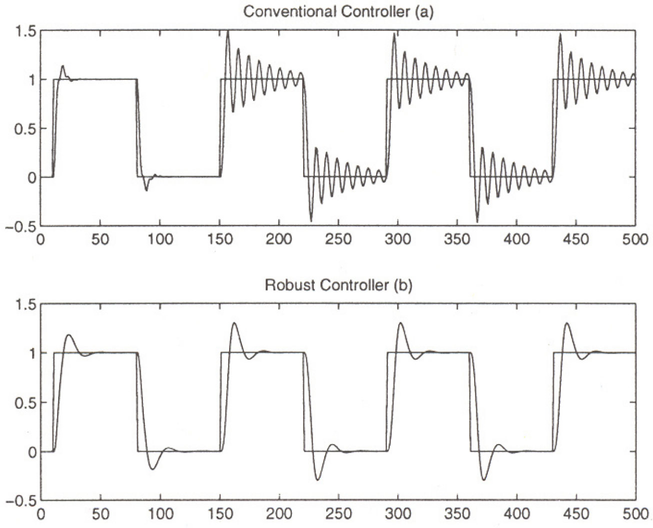



Figure 1.2.4: *Comparison of conventional feedback control and robust control, a) conventional design for the nominal model, b) robust control design*

Conventional versus Robust

# Conventional Control – Adaptive Control - Robust Control

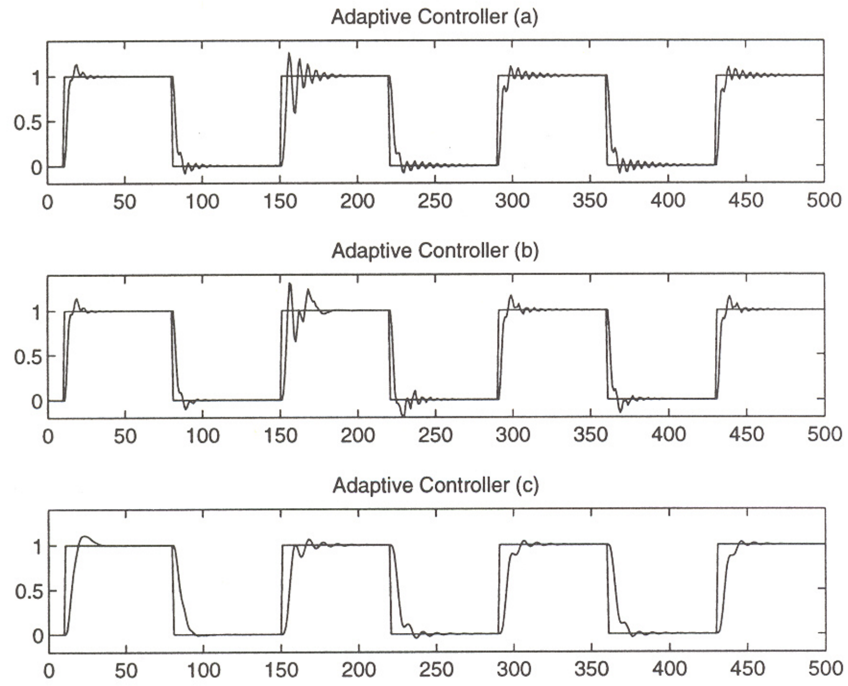


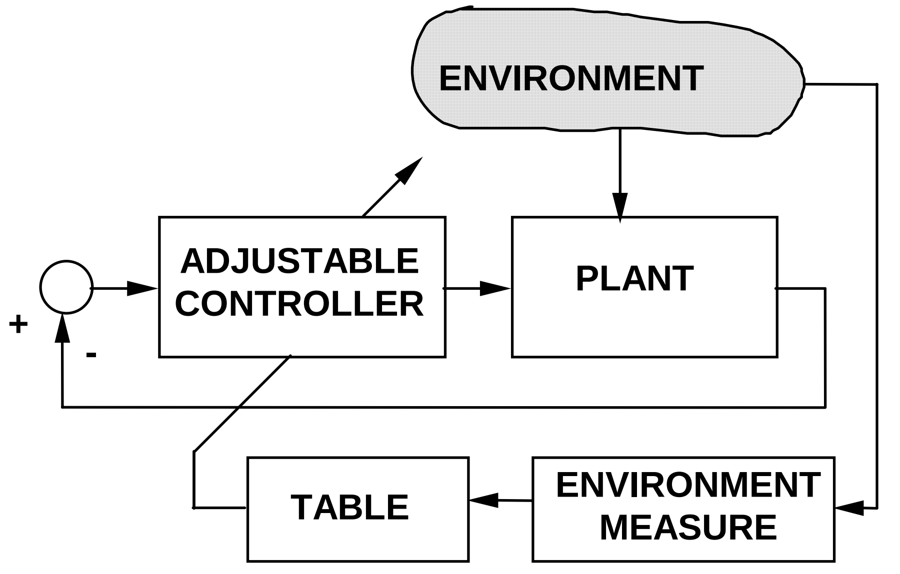

Figure 1.2.5: *Comparison of adaptive controller, a) adaptation added to the conventional controller (Fig. 1.2.4.a), b) robust adaptation added to the conventional controller (Fig. 1.2.4.a), c) adaptation added to the robust controller (Fig. 1.2.4.b)*

*Robust Adaptive Control* and *Adaptive Robust Control* are different.

***What we need*** : Robust Adaptation of a Robust Controller

# Basic Adaptive Control Configurations

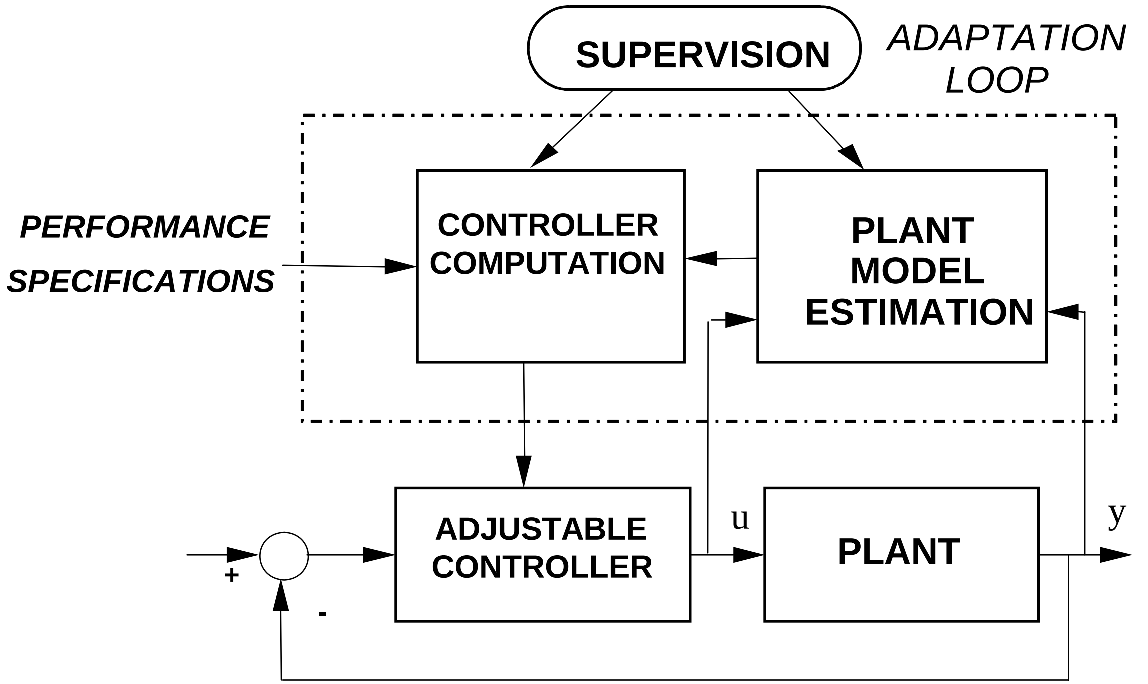
# Open Loop Adaptive Control



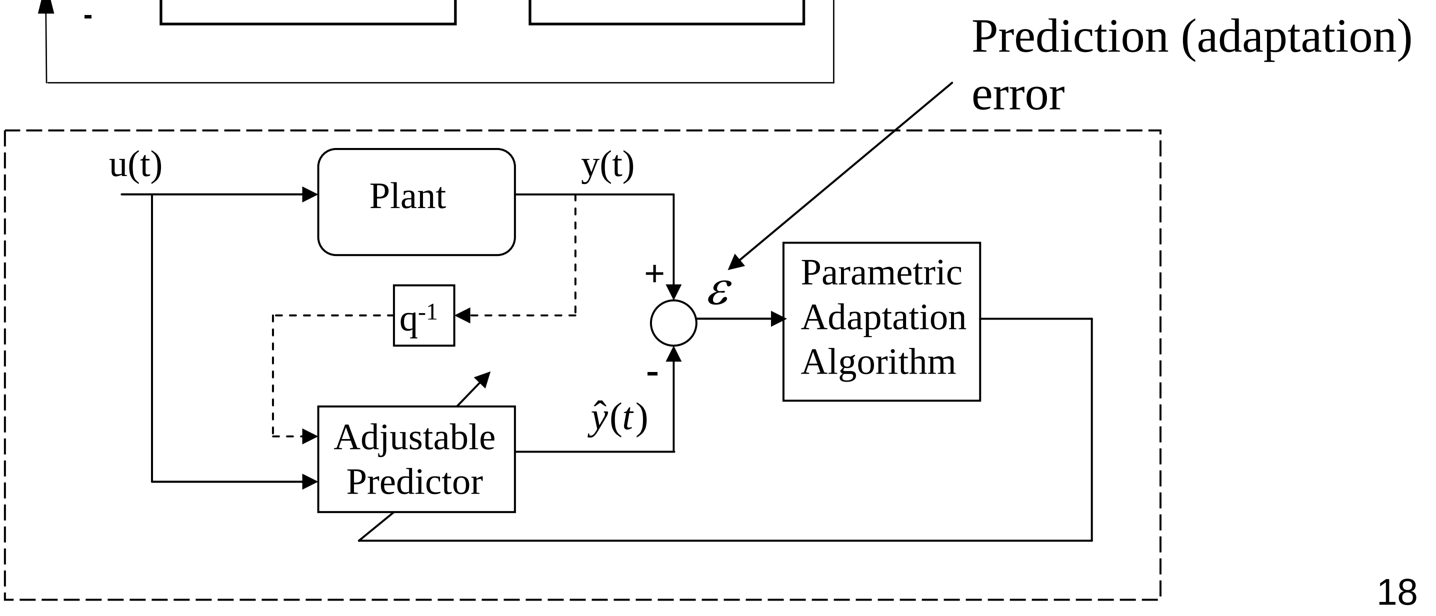

***Assumption*: known and rigid relationship between some measurable variables (characterizing the environment) and the plant model parameters**

Called also: gain-scheduling systems

# Indirect Adaptive Control

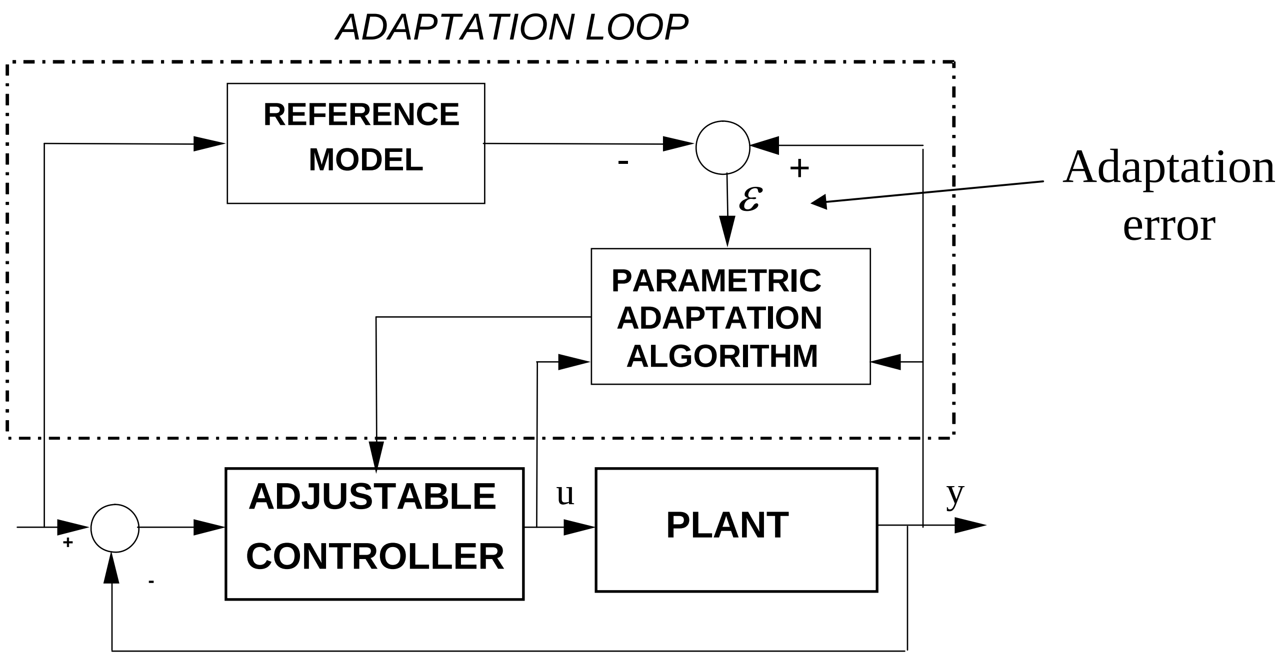



**Plant Model Basic Estimation Scheme**



# Direct Adaptive Control (model reference adaptive control)

The reference model gives the desired time trajectory of the plant output




Resemblance with plant parameter estimation scheme

Reference model ⟷ Plant

Adjustable feedback syst. ⟷ Adjustable predictor

# Parametric adaptation algorithm (PAA)

Parameter vector $\theta$ = contains all the parameters of the model (or of the controller)

$$\begin{bmatrix}\text{New parameters}\\ \text{estimation}\\ \text{(vector)}\end{bmatrix} = \begin{bmatrix}\text{Old parameters}\\ \text{estimation}\\ \text{(vector)}\end{bmatrix} +$$

$$\begin{bmatrix}\text{Adaptation}\\ \text{Gain}\\ \text{(matrix)}\end{bmatrix} \times \begin{bmatrix}\text{Measurement}\\ \text{function}\\ \text{(vector)}\end{bmatrix} \times \begin{bmatrix}\text{Error prediction}\\ \text{function}\\ \text{(scalar)}\end{bmatrix}$$

Regressor vector

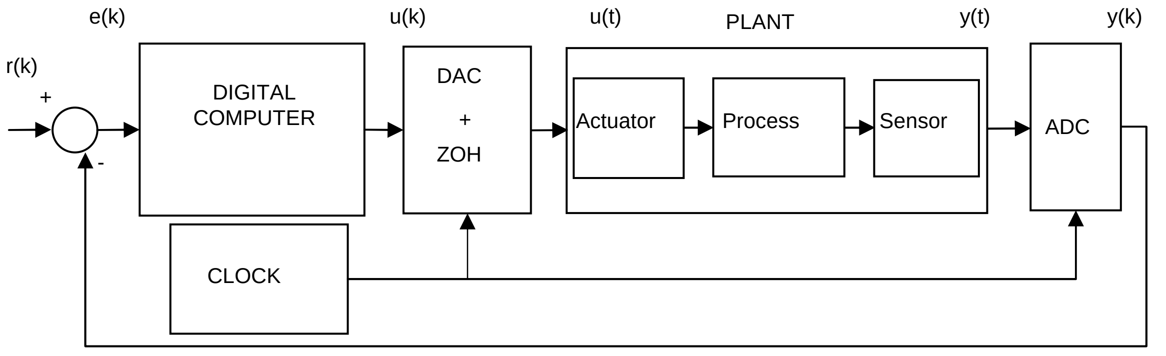
Estimated Parameter vector

$$\hat{\theta}(t+1) = \hat{\theta}(t) + F\Phi(t)\nu(t+1)$$

$(\nu = f(\varepsilon))$

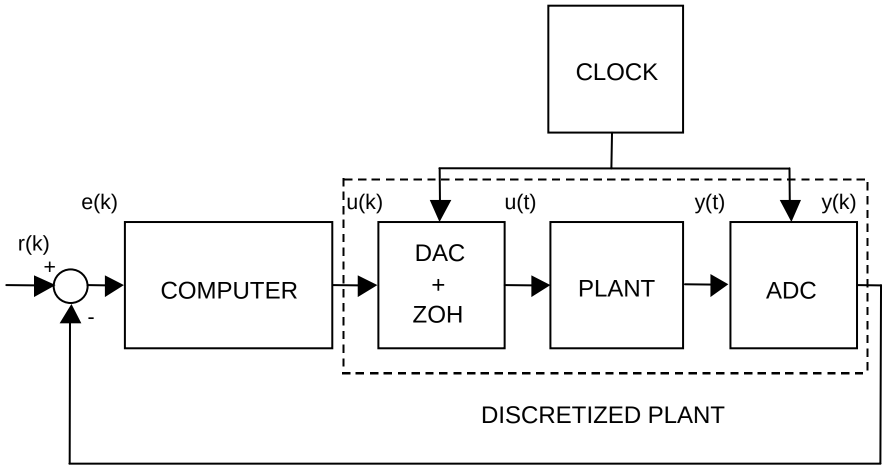
# Digital Control System

The *control law* is implemented on a digital computer



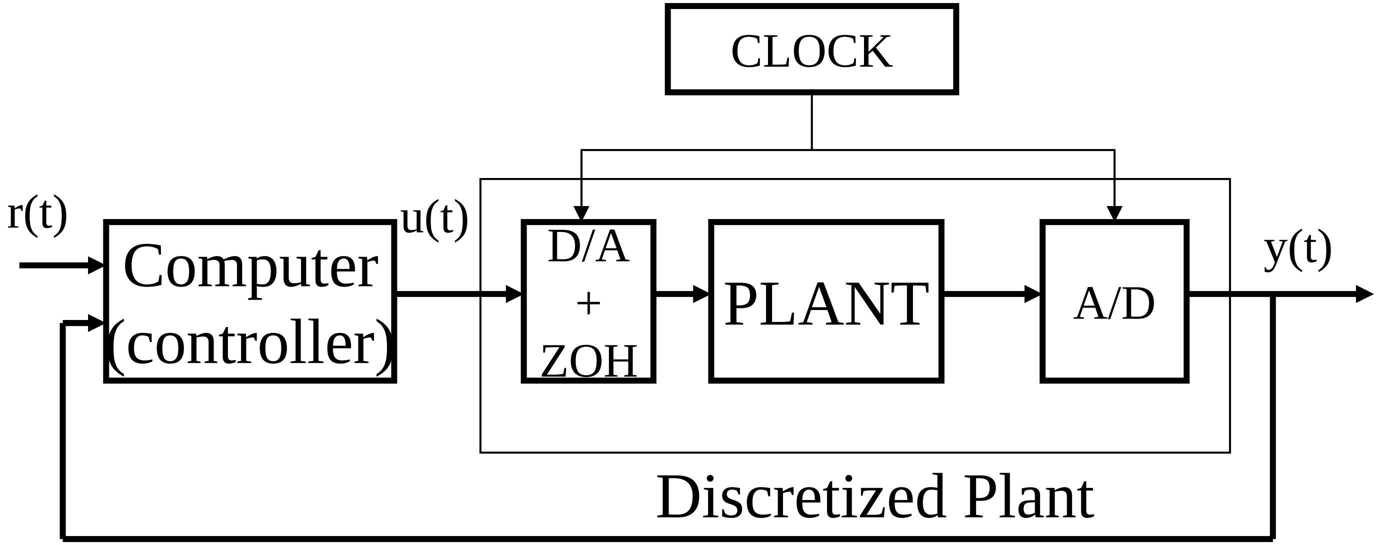

ADC: analog to digital converter
DAC: digital to analog converter
ZOH: zero order hold

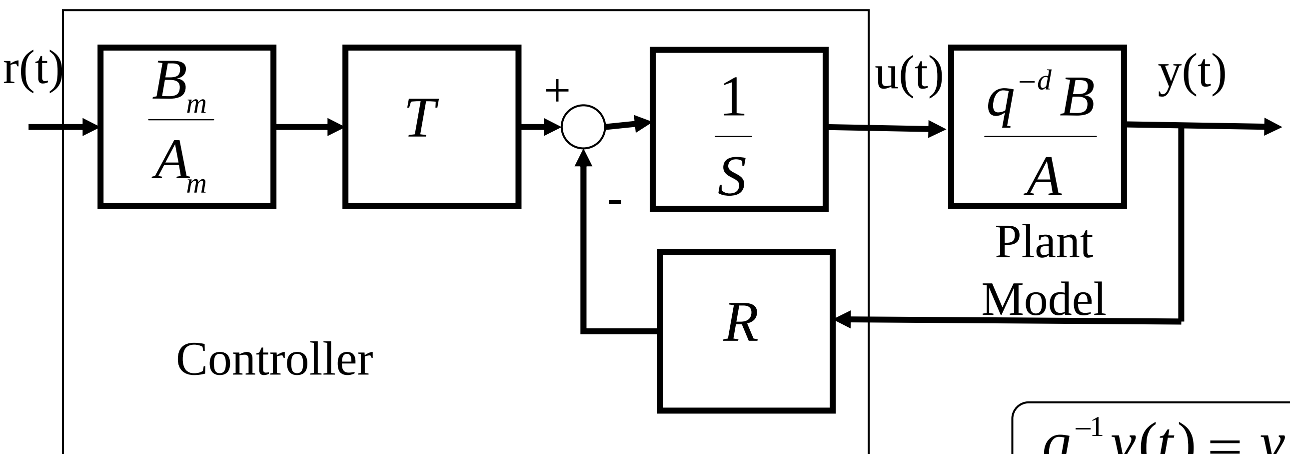
# Digital Control System




- Sampling time depends on the system bandwidth
- Efficient use of computer resources

# The R-S-T Digital Controller




$$q^{-1}y(t) = y(t-1)$$

# How to get a Direct Adaptive Control scheme ?

- Express the performance error in term of difference between the parameters of an unknown optimal controller and those of the adjustable controller

- Re-parametrize indirect adaptive control scheme (if possible) such that the adaptive predictor will provide directly the estimated parameters of the controller.

*See : Adaptive Control (Landau, Lozano, M'Saad) pg 19*

**The number of situation for which a direct adaptive control scheme can be developed is limited.**

# Adaptive Control Schemes. Regimes of operation

- **Adaptive regime**

1. Controller parameters are updated at every sampling time
2. Plant parameters are estimated at every sampling time but controller parameters are updated only every N samples (N small)
3 Adaptation works only when there is enough excitation

- **Self-tuning regime** (parameters are supposed unknown but constant)

1 Parameter adaptation algorithms with decreasing adaptation gain
2 Controller parameters are either updated at every sampling time or kept constant during parameter estimation
3 An external excitation is applied during tuning or plant identification

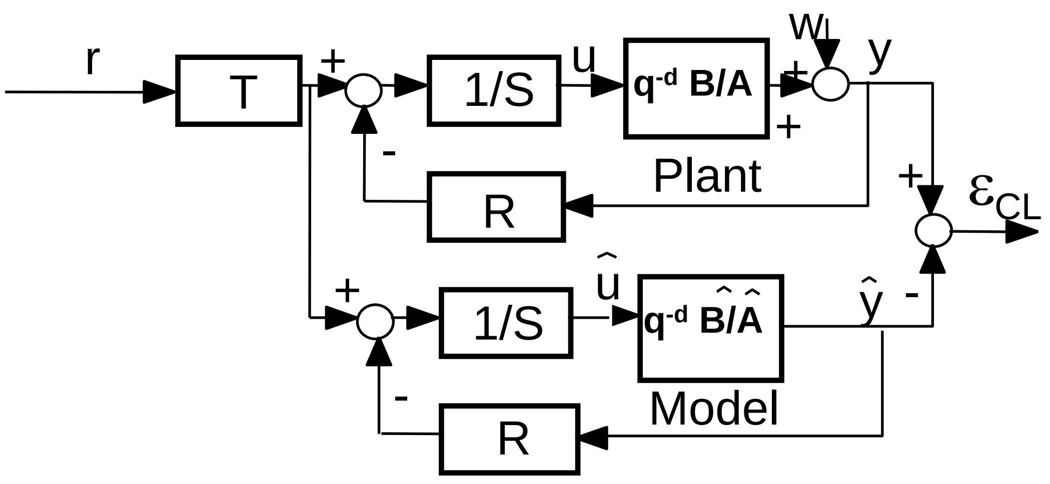
***Remark***

If controller parameters are kept constant during parameter estimation this is called "auto-tuning". For the indirect approach this corresponds to "plant identification in closed loop operation and controller redesign"

# Identification in Closed Loop and Adaptive Control

- Identification in closed loop operation using appropriate algorithms provides better models for design
- An iterative approach combining identification in closed loop followed by a re-design of the controller is a very powerful (auto-)tuning scheme

# Iterative Identification in Closed Loop and Controller Re-Design




*Step 1 : Identification in Closed Loop*

-Keep controller constant

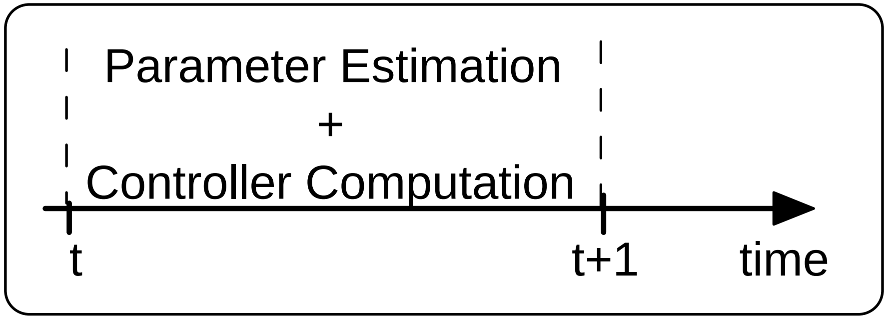
-Identify a new model such that $\varepsilon_{CL} \searrow$

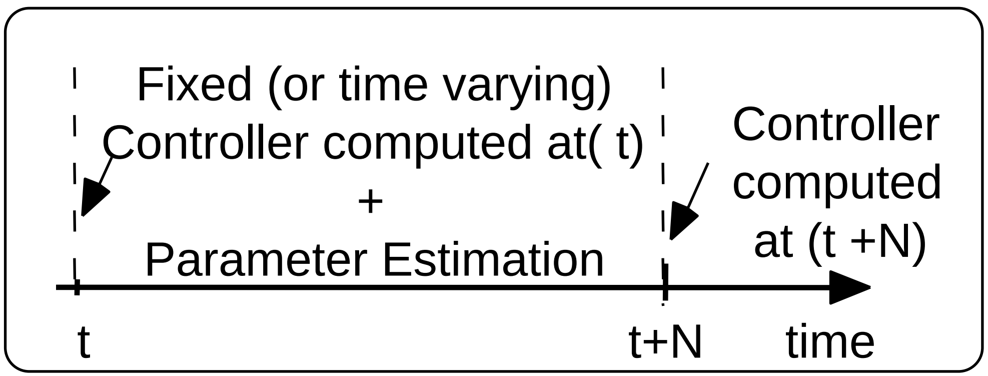
*Step 2 : Controller Re – Design*

- Compute a new controller such that $\varepsilon_{CL} \searrow$

Repeat 1, 2, 1, 2, 1, 2,…

# Iterative Identification and Controller Redesign versus (Indirect) Adaptive Control




*N = 1* : Adaptive Control

*N = Small*
Adaptive Control
*N = Large*
Iterative Identification in C.L.
And Controller Re-design

$N \Rightarrow \infty$
Plant Identification in C.L. +
Controller Re-design

The *iterative procedure* introduces a time scale separation between identification / control design

# Adaptive Control and Adaptive Regulation

**Adaptive Control**

*Plant model is unknown and time varying*

*The disturbance model is known and constant*

**Adaptive Regulation**

*Plant model is known and constant*

*The disturbance model is unknown and time varying*

**Adaptive control and regulation**

*Very difficult problem since is extremely hard to distinguish in the performance (prediction) error what comes from plant model error and what comes from disturbance model error*

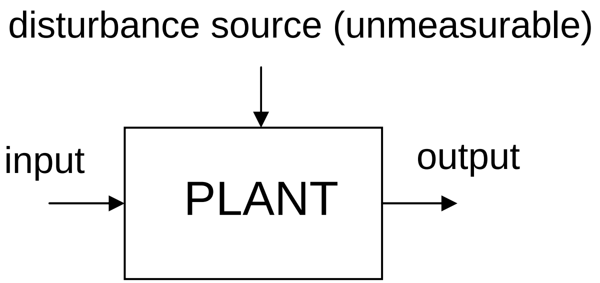
**Rem:**

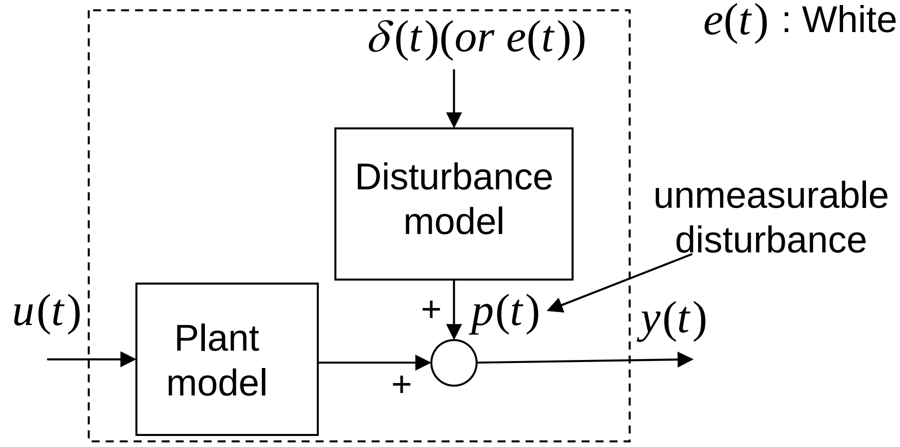
*The "internal model principle" has to be used in all the cases*

# Adaptive Control

$\delta(t)$: Dirac

$e(t)$ : White noise



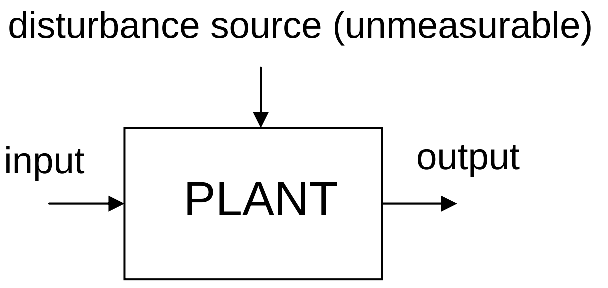



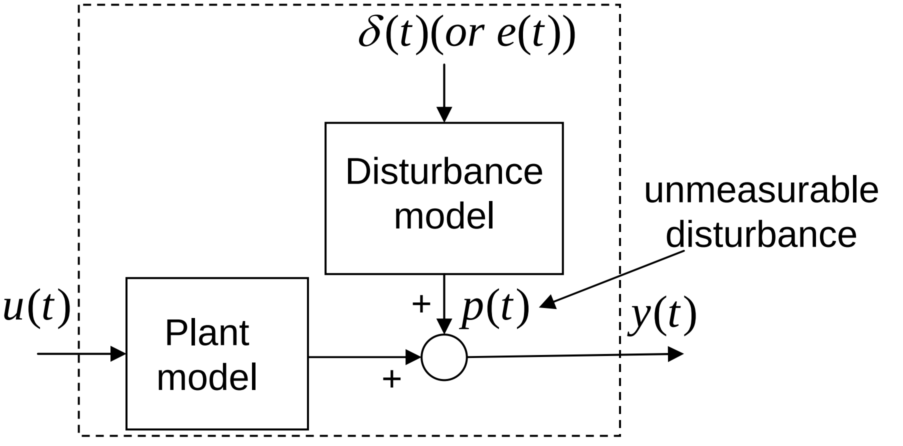

**Objective** : tracking/disturbance attenuation performance

- **Focus on adaptation with respect to plant model parameters variations**
- The model of the disturbance is assumed to be known and constant
- Only a level of attenuation in a frequency band is required*
- **No effort is made to simultaneously estimate the model of the disturbance**

***)** Except for known DC disturbances (use of integrators)

# Adaptive Regulation




**Objective** : Suppressing the effect of the (unknown) disturbance*

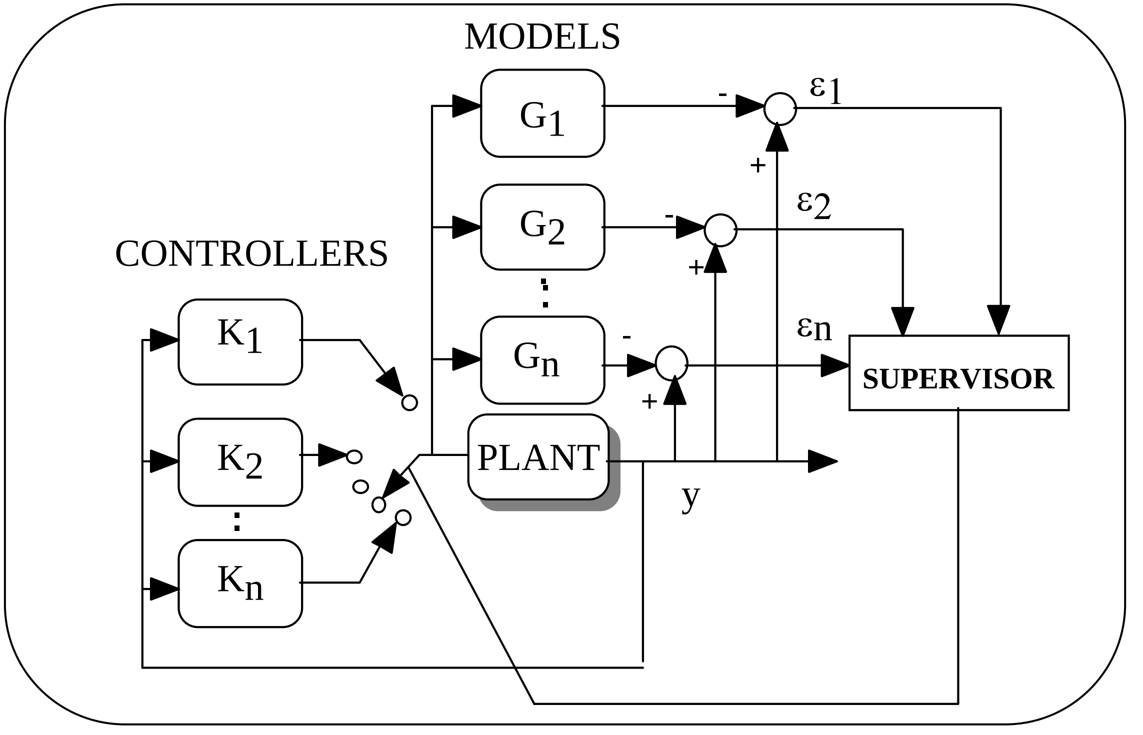
- **Focus on adaptation with respect to disturbance model parameters variations**
- Plant model is assumed to be known ( a priori system identification) and almost constant
- Small plant parameters variations handled by a robust control design
- **No effort is made to simultaneously estimate the plant model**

*) Assumed to be characterized by a rational power spectrum if stationary

# Use of a priori information for improving adaptation transients

- Before using an adaptive control scheme, an analysis of the system is done and this is followed by plant identification in various regimes of operation
- The availability of models for various regimes of operation allows to design robust controllers which can assure satisfactory performance in a region of the parameter space around each of the identified models.
- Provided that we can detect in what region the system is, the appropriate controller can be used
- "Indirect adaptive control" can not detect enough fast the region of operation but can make a "fine" tuning over a certain time.
- In case of rapid parameter changes the adaptation transients in indirect adaptive control may be unacceptable.
- There is a need to improve these transients by taking in account the available information

# Supervisory Control

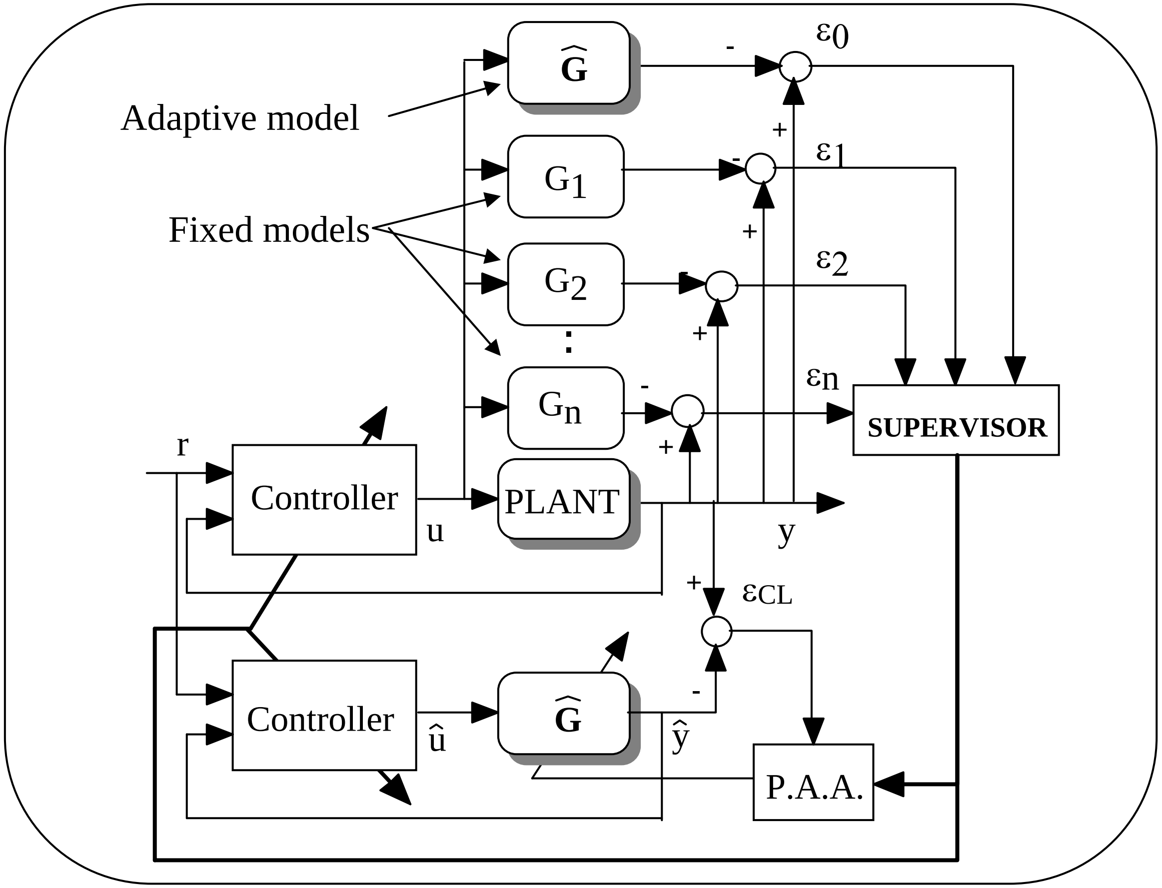



The "supervisor":

- will check what "plant-model" error is minimum
- will switch to the controller associated with the selected model

*Can provide a very fast decision (if there are not too many models) but not a fine tuning*

# Adaptive Control with Multiple Models

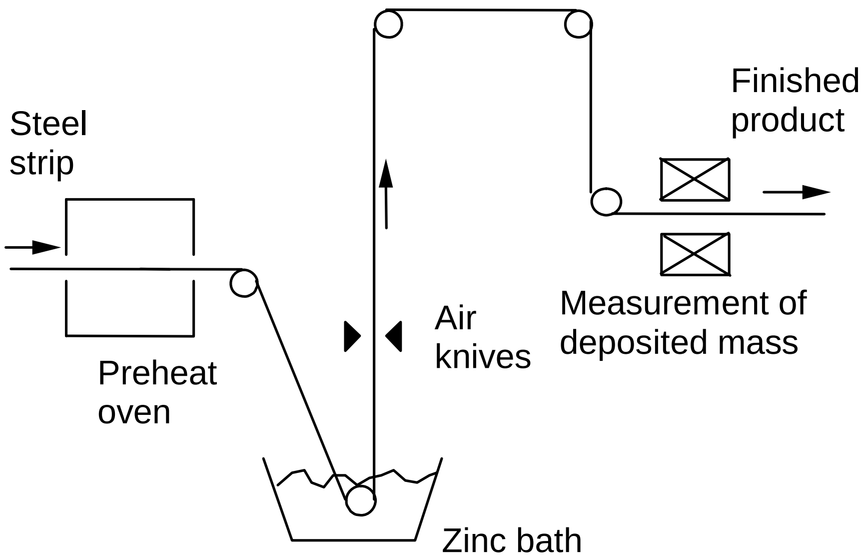


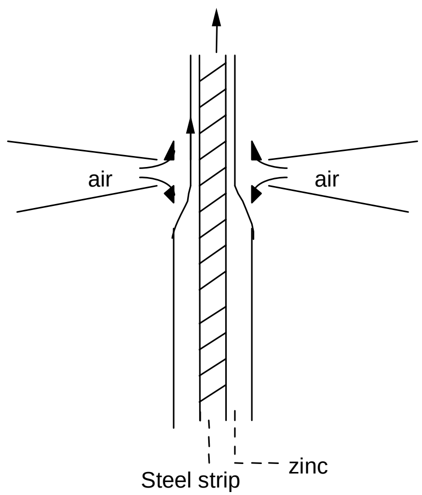

The supervisor select the best fixed model and then the adaptive model will be selected

Multiple fixed models : *improvement of the adaptation transients*
Adaptive plant model estimator (CLOE Estimator) : *performance improvement*

# Some Applications of Adaptive Control

# Open Loop Adaptive Control of Deposited Zinc in Hot-Dip Galvanizing



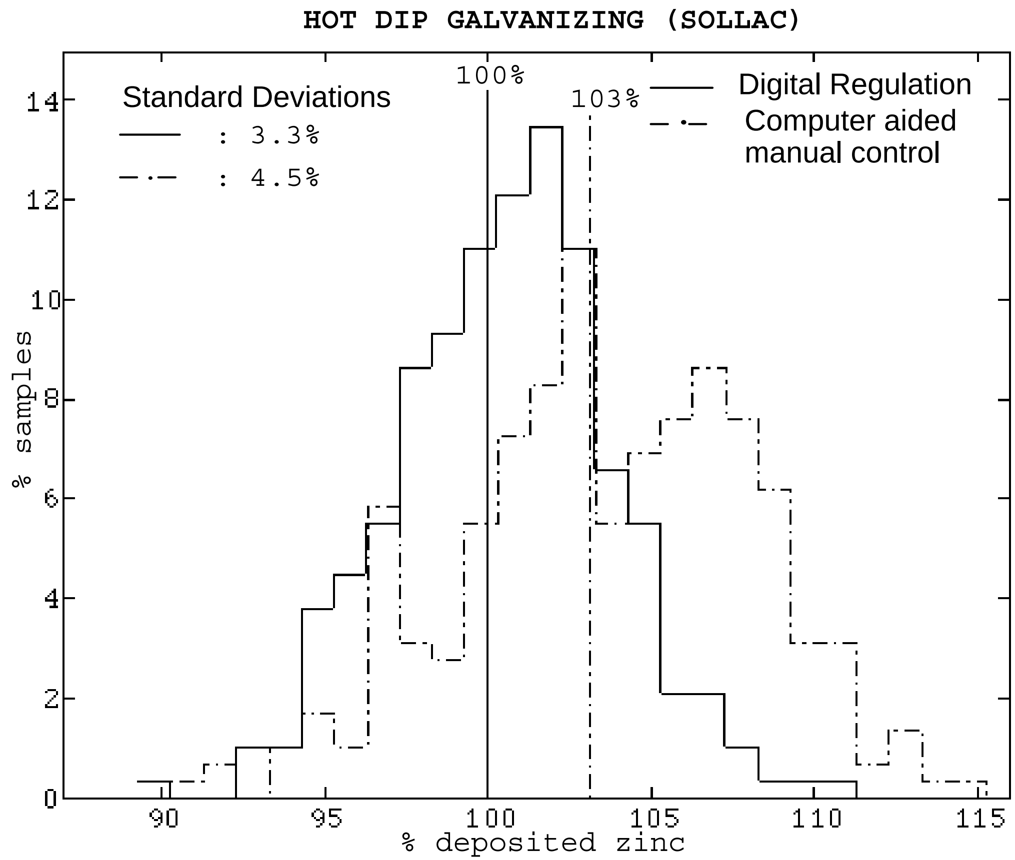

input: air knives pressure
output: measured deposited mass

$$H(s) = \frac{Ge^{-s\tau}}{1+sT} \quad ; \quad \tau = \frac{L}{V}$$

*L*- *distance knives –measure*
*V*- *strip speed*

- delay varies with the speed
- G and T depend upon strip speed and distance between knives and steel strip

# Open Loop Adaptive Control of Deposited Zinc in Hot-Dip Galvanizing




Adaptation done with respect to:

- Steel strip speed
- Distance between air knives and steel strip

*9 operation regions*

*The sampling period is tied to the strip speed to have constant discrete time delay*

# Direct Adaptive Control of a Phosphate Dryer Furnace

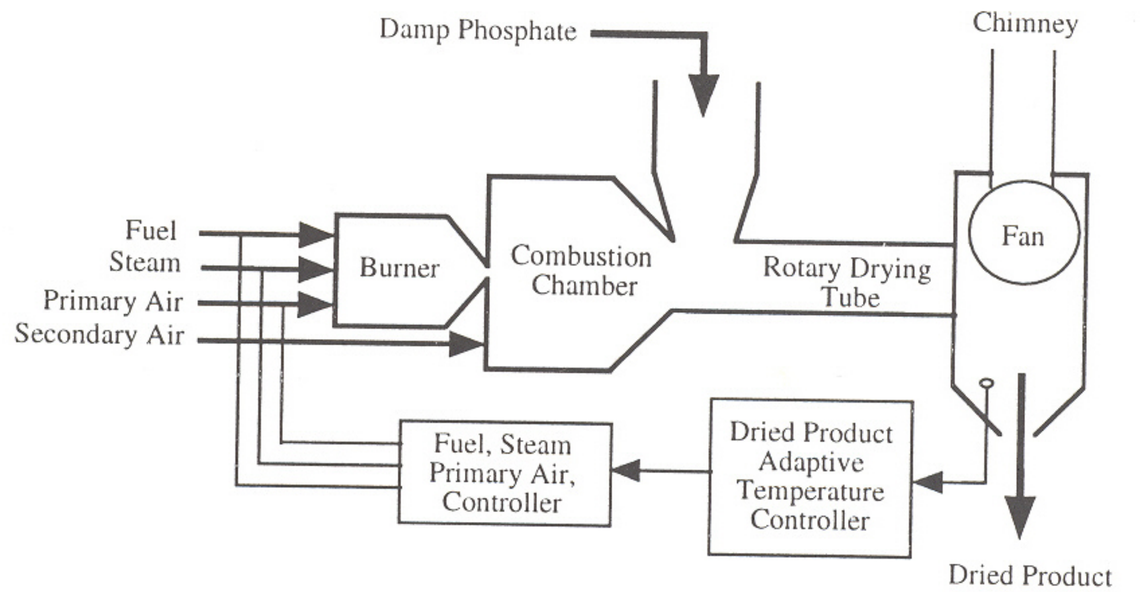


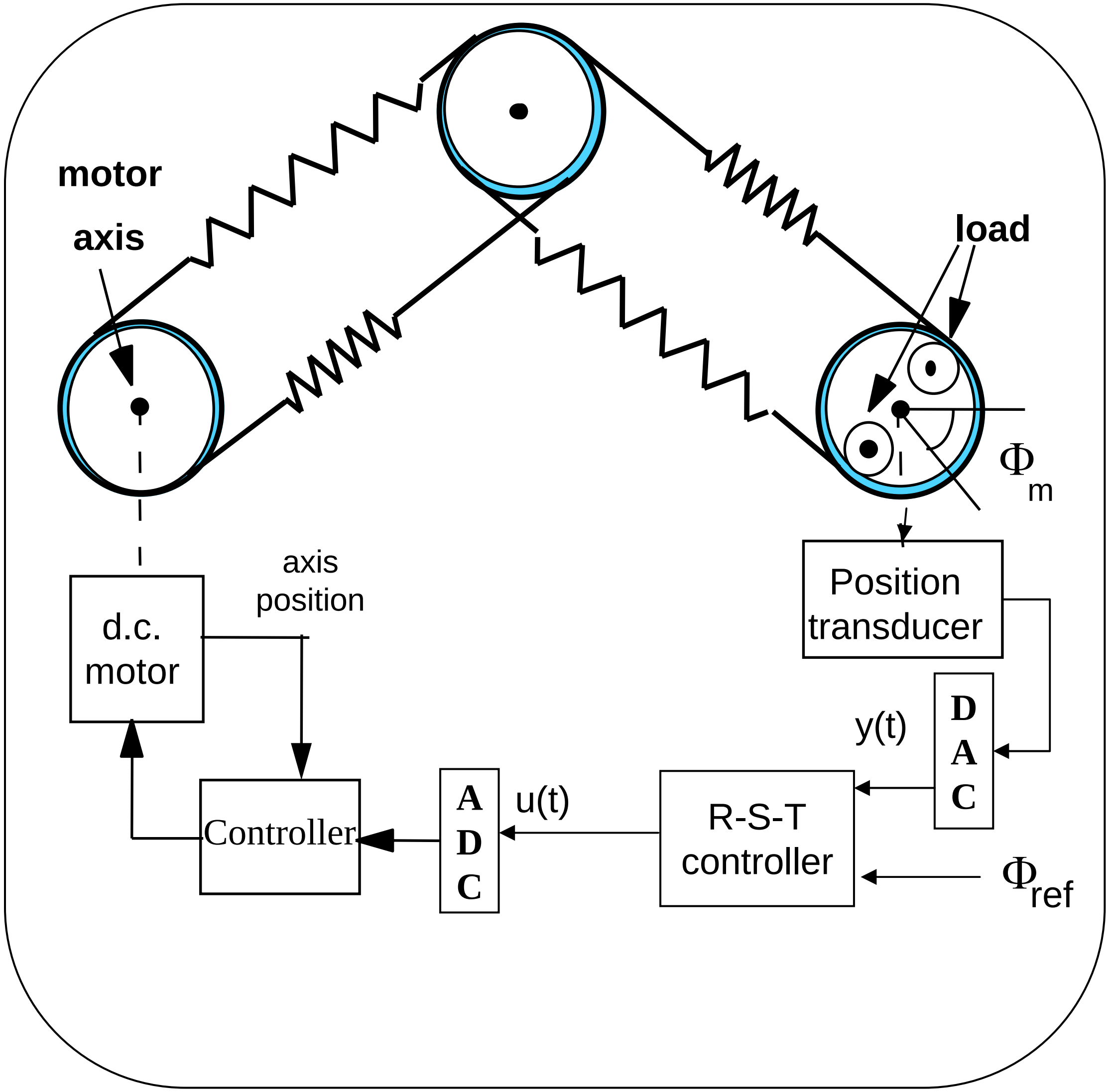

Figure 1.4.2: *Phosphate drying furnace*

Large delay : 90 s

Better quality( reduction of the humidity standard deviation)
Reduction of fuel comsumption and of the thermal stress.

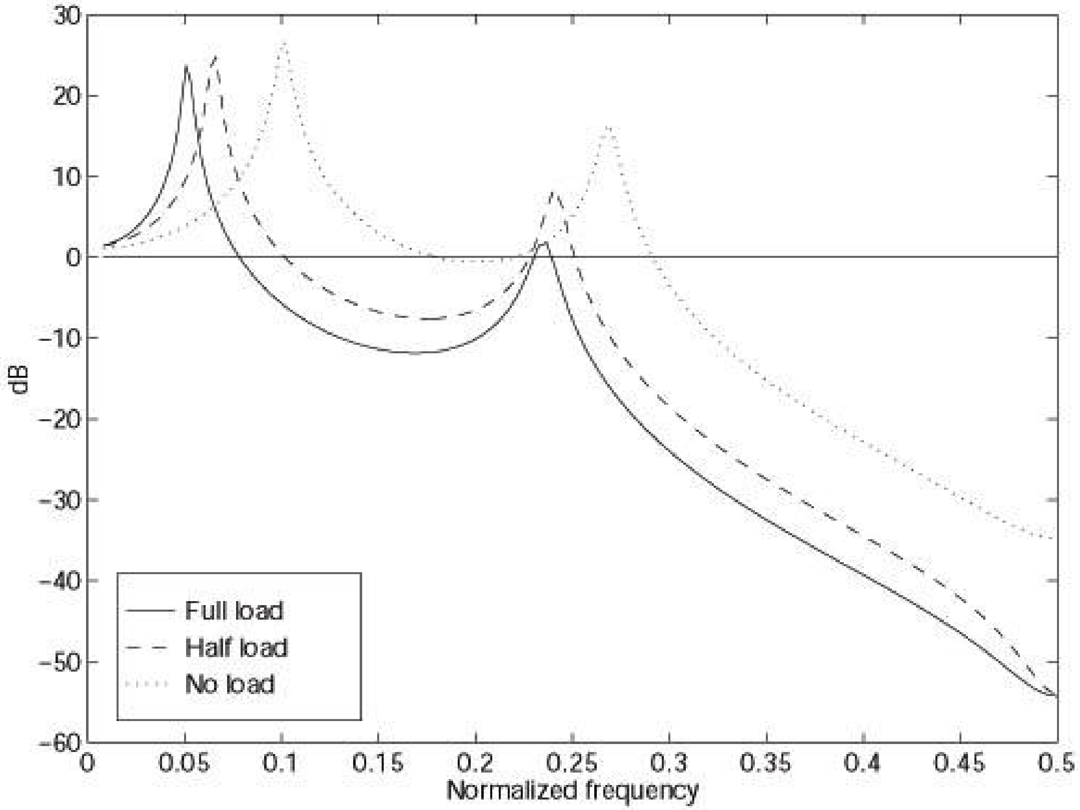
# Adaptive Control of a Flexible Transmission

The flexible transmission



# Adaptive Control of a Flexible Transmission

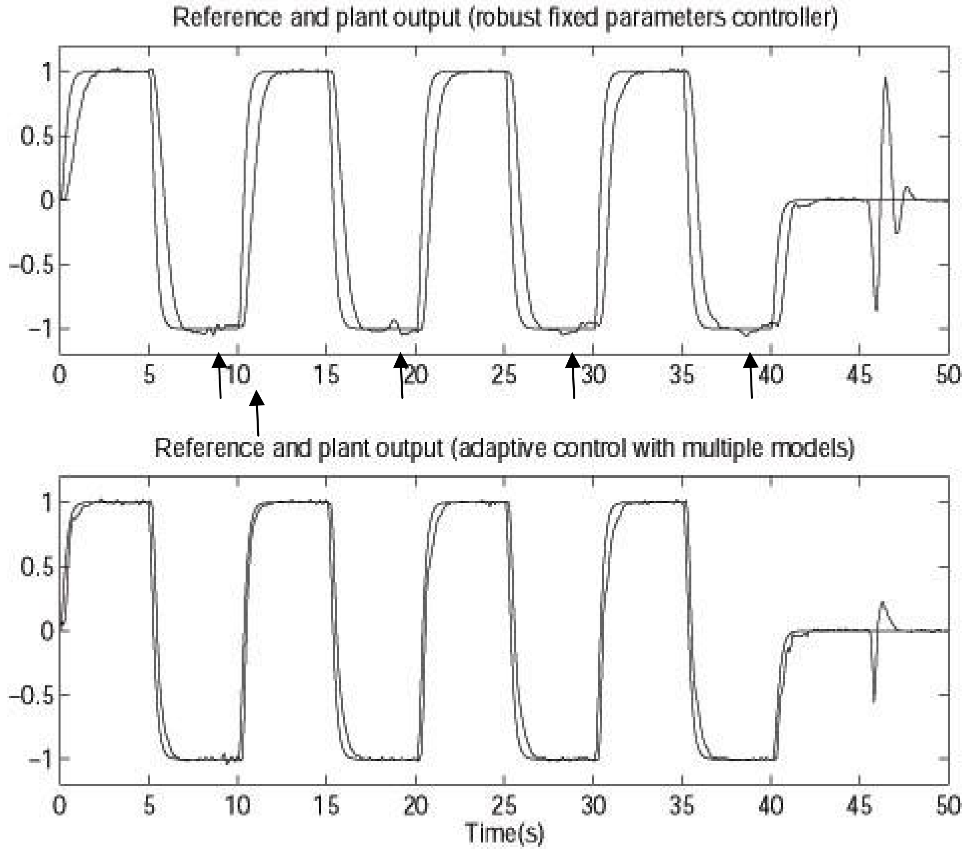
Frequency characteristics for various load




*Rem.:* the main vibration mode varies by 100%

**Solution :** Adaptive control with multiple models

# Adaptive Control versus Robust Control

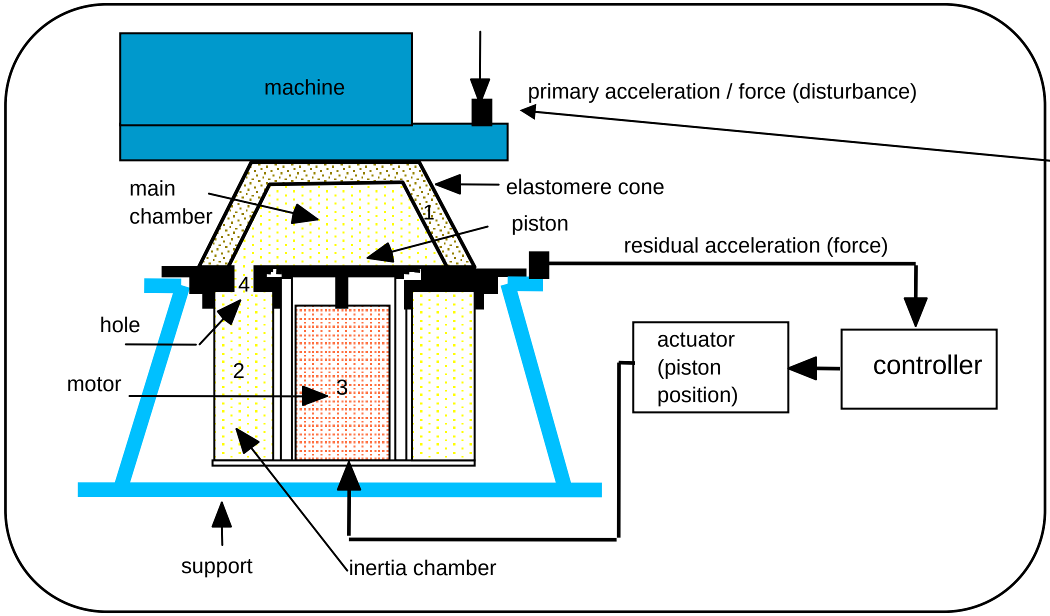



Load variations : 0% → 100% (in 4 steps, 25% each)

*Rem* : The robust controller used is the winner of an international benchmark test for robust control of the flexible transmission (EJC, no.2., 1995)

# Rejection of unknown narrow band disturbances in active vibration control

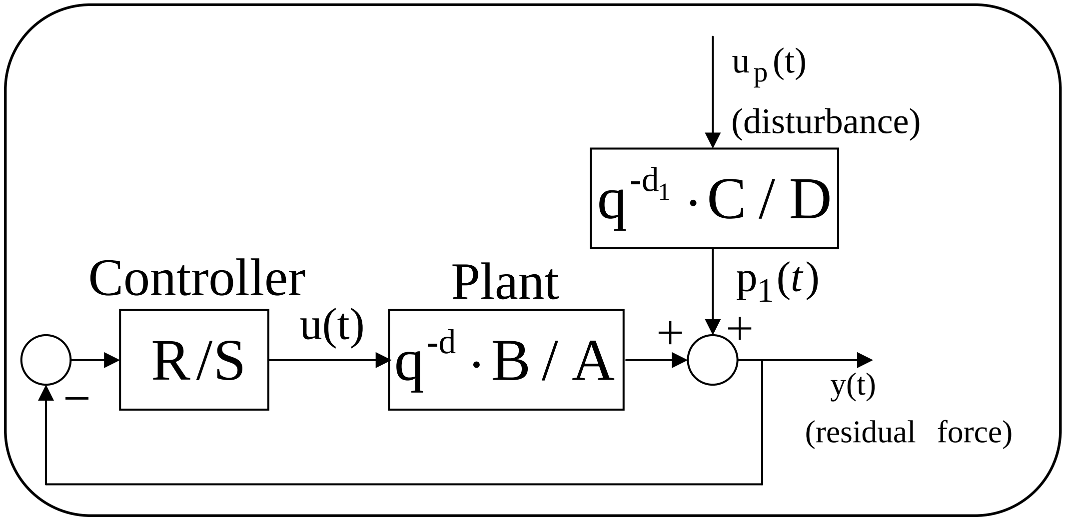
# The Active Suspension System




***Objective:***

**•Reject the effect of unknown and variable narrow band disturbances**

**•Do not use an aditional measurement**

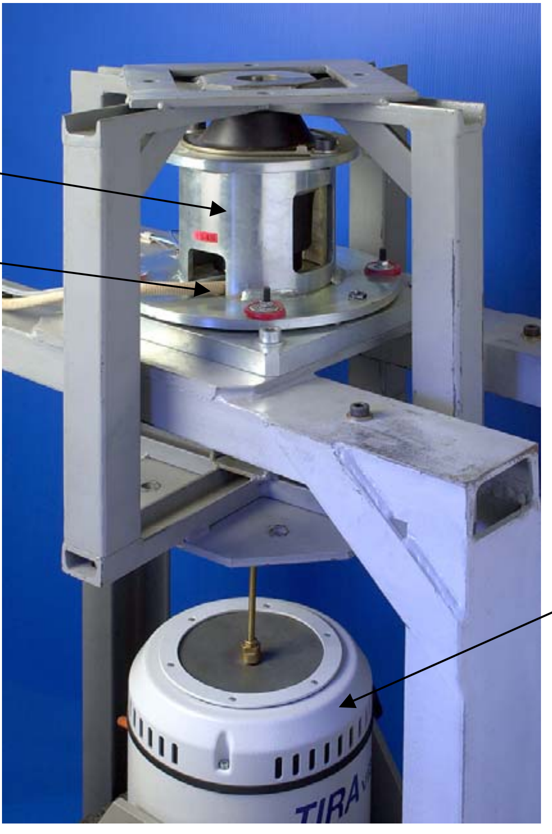



Two paths :

- •Primary
- •Secondary (double differentiator)

$$T_s = 1.25\,\text{ms}$$

# The Active Suspension

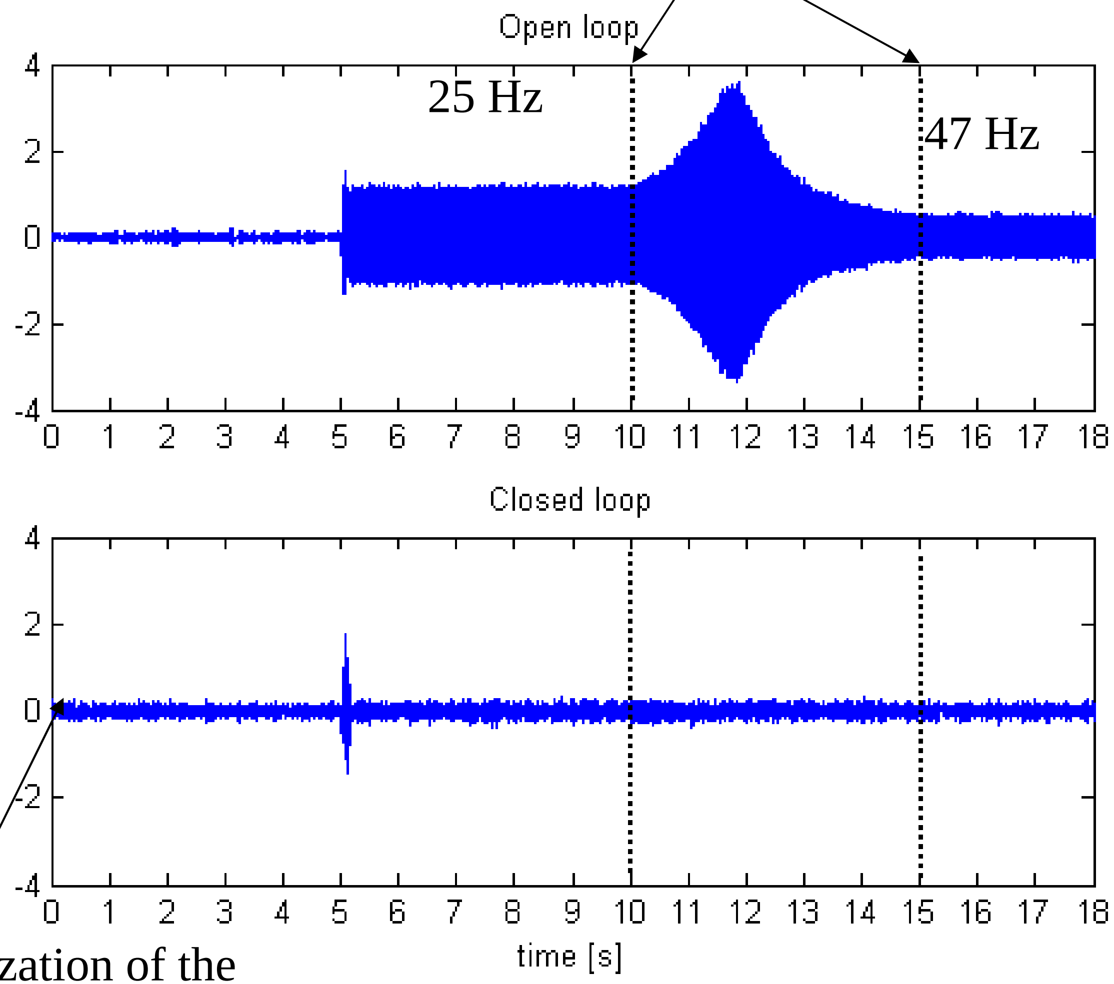



# Direct Adaptive Regulation : disturbance rejection

Disturbance : Chirp

Open loop

Closed loop

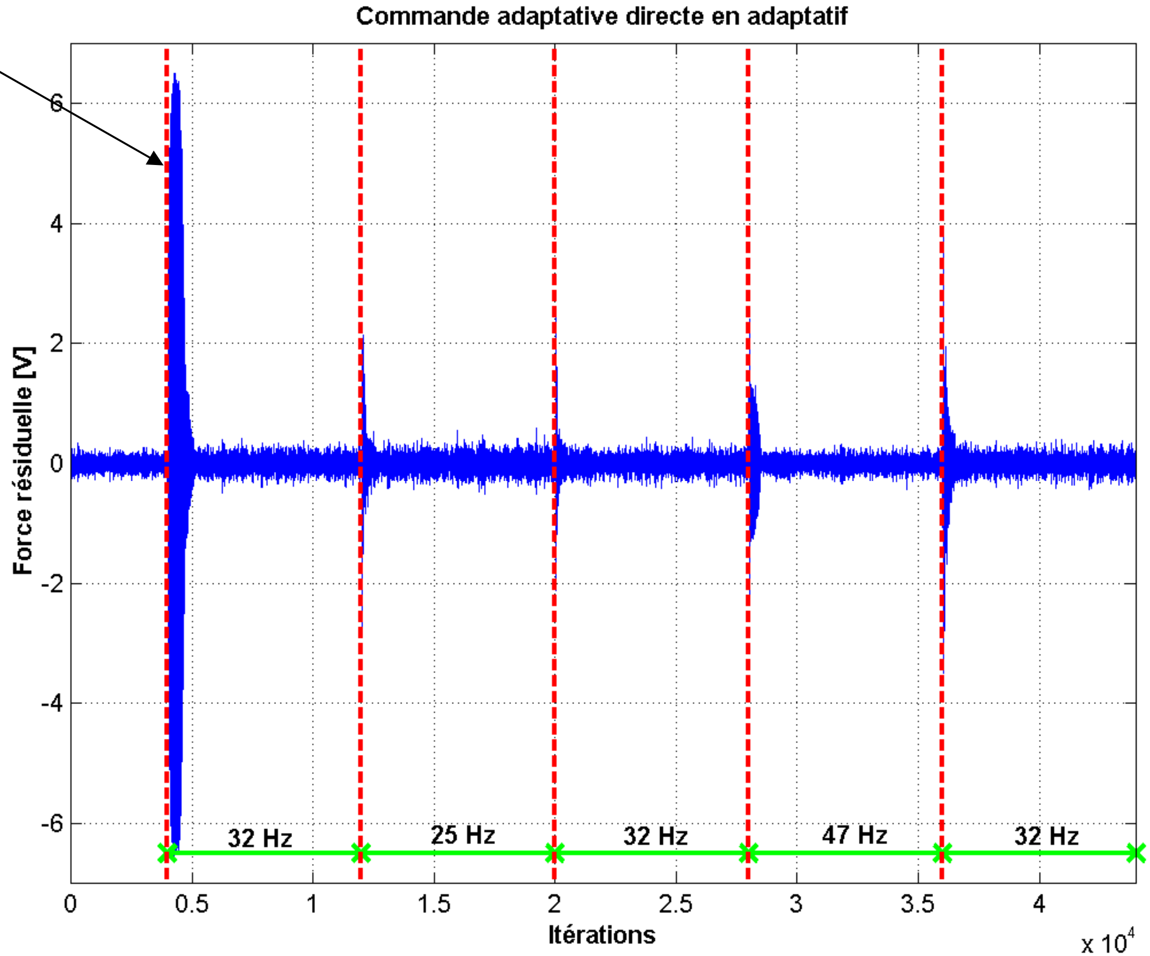



Initialization of the adaptive controller

# Direct Adaptive Regulation : rejection of sinusoidal disturbances

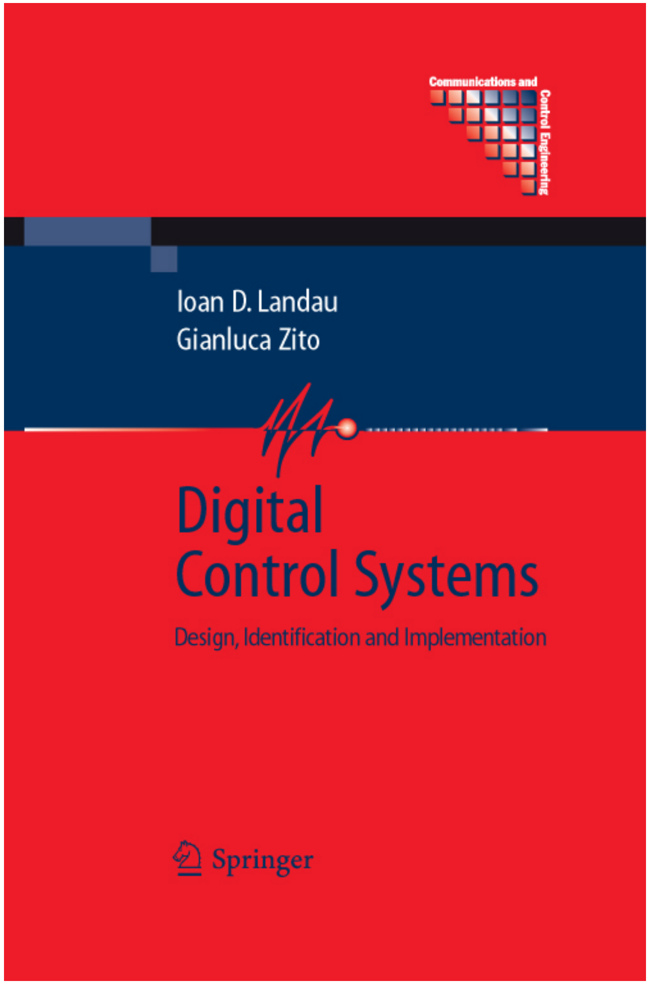
Simultaneous controller initialization and disturbance application

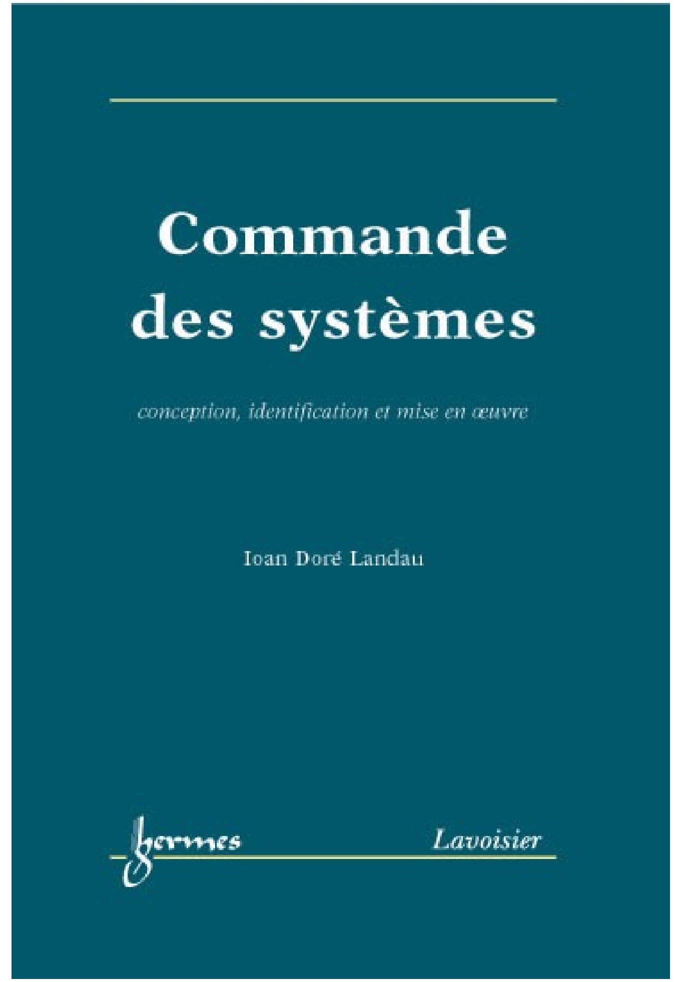
## Direct adaptive control




Step changes in the frequency of the disturbance

# References




ISBN: 1-84628-055-9

ISBN: 2-7462-0478-9

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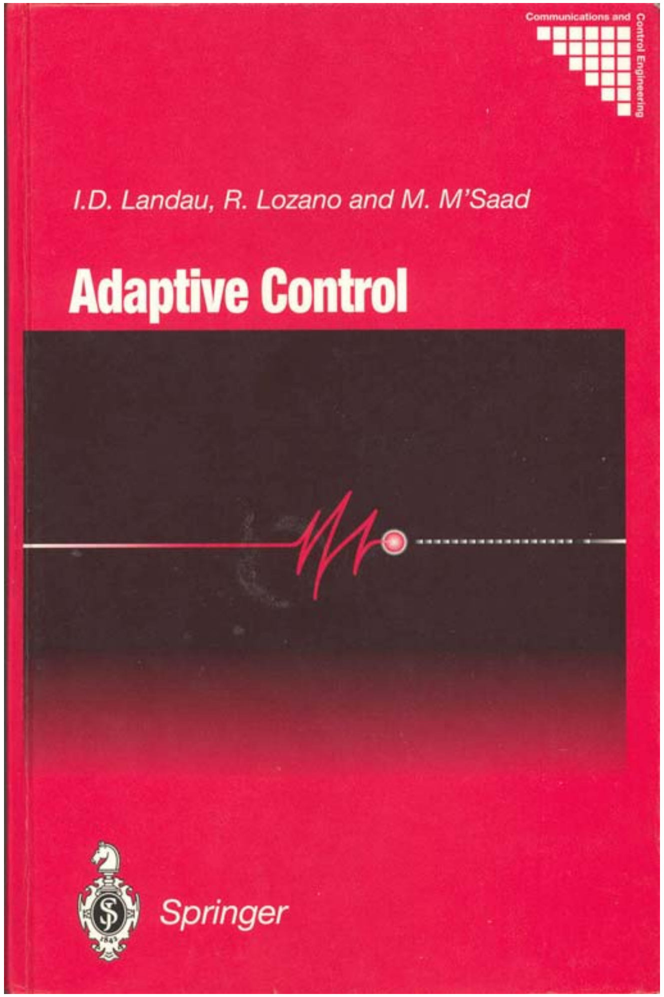


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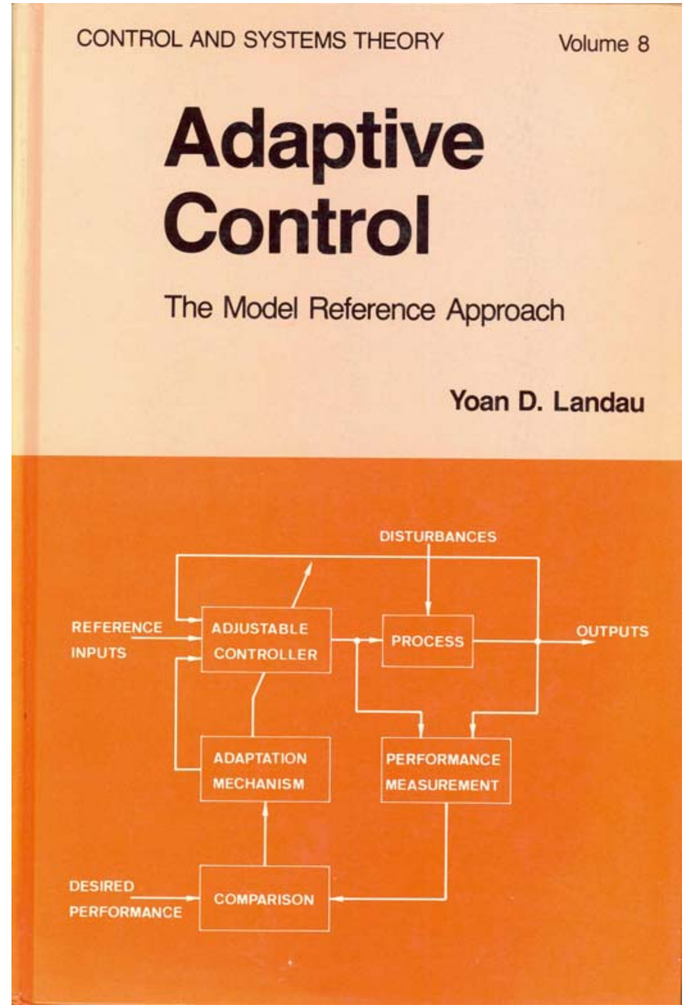


ISBN: 0-8247-6548-6

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The course is mainly based on the following books:


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- I.D. Landau ""Identification in closed loop- A powerful design tool (better models, simpler controllers), Control Eng. Practice, no. 1, Jan. 2001
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# Adaptive Control

Part 2: Parameter adaptation algorithms

# Parametric adaptation algorithms (PAA)

- Indirect adaptive control uses PAA in the plant model estimator
- Direct adaptive control uses PAA for controllers' parameter estimation
- Adaptive control with multiple models needs also PAA
- Self-tuning control using identification in closed loop needs also PAA

# Parametric adaptation algorithm (PAA)

Parameter vector = contains all the parameters of the model

$$\begin{bmatrix} \text{New parameters} \\ \text{estimation} \\ \text{(vector)} \end{bmatrix} = \begin{bmatrix} \text{Old parameters} \\ \text{estimation} \\ \text{(vector)} \end{bmatrix} +$$

$$\begin{bmatrix} \text{Adaptation} \\ \text{Gain} \\ \text{(matrix)} \end{bmatrix} \times \begin{bmatrix} \text{Measurement} \\ \text{function} \\ \text{(vector)} \end{bmatrix} \times \begin{bmatrix} \text{Error prediction} \\ \text{function} \\ \text{(scalar)} \end{bmatrix}$$

Regressor vector

*We will develop the PAA in the context of plant model estimation*

# Plant Model

$$G(q^{-1}) = \frac{q^{-d}B(q^{-1})}{A(q^{-1})} = \frac{q^{-d-1}B^*(q^{-1})}{A(q^{-1})}$$


$$A(q^{-1}) = 1 + a_1 q^{-1} + \ldots + a_{n_A} q^{-n_A} = 1 + q^{-1}A^*(q^{-1})$$

$$B(q^{-1}) = b_1 q^{-1} + \ldots + b_{n_B} q^{-n_B} = q^{-1}B^*(q^{-1})$$

$$y(t+1) = -A^*(q^{-1})y(t) + B^*(q^{-1})u(t-d) = \theta^T \phi(t)$$

$$\theta^T = \left[a_1, \ldots a_{n_A}, b_1, \ldots, b_{n_B}\right]$$ ← Parameter vector

$$\phi(t)^T = \left[-y(t) \ldots -y(t-n_A+1), u(t-d) \ldots u(t-d-n_B+1)\right]$$

↖ Measurement vector

# Algorithms for parameter estimation

Discrete time plant model (unknown parameters))

$$y(t+1) = -a_1 y(t) + b_1 u(t) = \theta^T \phi(t)$$

Measurement vector

$\theta^T = [a_1, b_1]$ ⟵ Parameter vector ; $\phi(t)^T = [-y(t), u(t)]$

Adjustable prediction model (à priori)

$$\hat{y}^o(t+1) = \hat{y}(t+1|\hat{\theta}(t)) = -\hat{a}_1(t)y(t) + \hat{b}_1(t)u(t) = \hat{\theta}(t)^T \phi(t)$$

$\theta(t)^T = [\hat{a}_1(t), \hat{b}_1(t)]$ ⟵ Vector of adjustable parameters

Prediction error (à priori)

$$\varepsilon^o(t+1) = y(t+1) - \hat{y}^o(t+1) = \varepsilon^o(t+1, \hat{\theta}(t))$$

Criterion to be minimized (objective):

$$J(t+1) = \left[\varepsilon^o(t+1)\right]^2 = \left[\varepsilon^o(t+1, \hat{\theta}(t))\right]^2$$

**?**

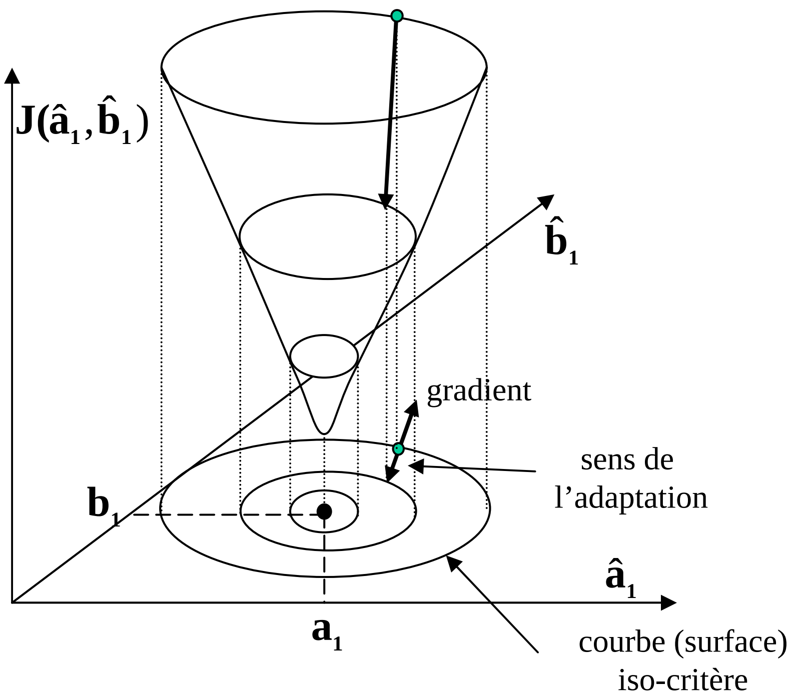
Parameter adaptation algorithm

$$\hat{\theta}(t+1) = \hat{\theta}(t) + \Delta\hat{\theta}(t+1) = \hat{\theta}(t) + f\left(\hat{\theta}(t), \phi(t), \varepsilon^o(t+1)\right)$$

# PAA – Gradient algorithm

Criterion to be minimized (objective):

$$\min_{\hat{\theta}(t)} J(t+1) = \left[\varepsilon^{o}(t+1)\right]^{2}$$




**gradient of the criterion**

$$\hat{\theta}(t+1) = \hat{\theta}(t) - F\frac{\partial J(t+1)}{\partial \hat{\theta}(t)}$$

$$F = \alpha I \quad (\alpha > 0)$$ **(*I* = unit matrix)**

$$\frac{1}{2}\frac{\partial J(t+1)}{\partial \hat{\theta}(t)} = \frac{\partial \varepsilon^{o}(t+1)}{\partial \hat{\theta}(t)}\varepsilon^{o}(t+1)$$

$$\varepsilon^{o}(t+1) = y(t+1) - \hat{y}^{o}(t+1) = y(t+1) - \hat{\theta}(t)^{T}\phi(t) \;\Rightarrow\; \frac{\partial \varepsilon^{o}(t+1)}{\partial \hat{\theta}(t)} = -\phi(t)$$

$$\hat{\theta}(t+1) = \hat{\theta}(t) + F\phi(t)\varepsilon^{o}(t+1)$$

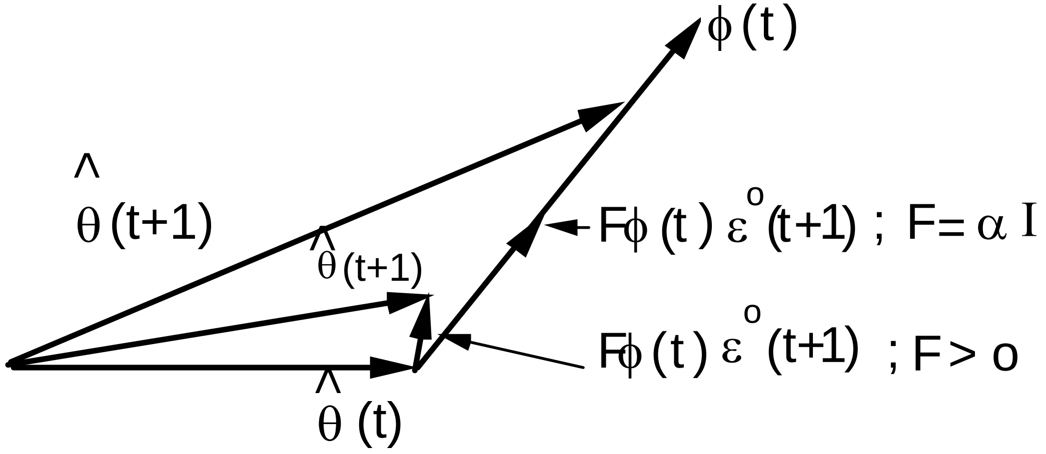
# PAA – Gradient algorithm

$$\hat{\theta}(t+1) = \hat{\theta}(t) + F\phi(t)\varepsilon^{o}(t+1)$$

Adaptation gain

$$\begin{cases} F = \alpha I & (\alpha > 0) \\ F > 0 & \text{Positive definite matrix} \end{cases}$$

Geometrical interpretation →




*Attention:* Instability risk if $F\ (\alpha)$ is large !!

(see book Landau-Zito, pg. 213 – 214 for details)

# PAA – Improved gradient algorithm

*a posteriori* output of the adjustable predictor

$$\hat{y}(t+1) = \hat{y}(t+1|\hat{\theta}(t+1)) = -\hat{a}_1(t+1)y(t) + \hat{b}_1(t+1)u(t) = \hat{\theta}(t+1)^T \phi(t)$$

Prediction error (a posteriori): $\varepsilon(t+1) = y(t+1) - \hat{y}(t+1)$

Criterion to be minimized (objective): $\min_{\hat{\theta}(t+1)} J(t+1) = [\varepsilon(t+1)]^2$

Gradient technique: $\hat{\theta}(t+1) = \hat{\theta}(t) - F \dfrac{\partial J(t+1)}{\partial \hat{\theta}(t)}$

$$\frac{1}{2}\frac{\partial J(t+1)}{\partial \hat{\theta}(t+1)} = \frac{\partial \varepsilon(t+1)}{\partial \hat{\theta}(t+1)} \varepsilon(t+1)$$

$$\varepsilon(t+1) = y(t+1) - \hat{y}(t+1) = y(t+1) - \hat{\theta}(t+1)^T \phi(t) \quad \Rightarrow \quad \frac{\partial \varepsilon(t+1)}{\partial \hat{\theta}(t+1)} = -\phi(t)$$

$$\hat{\theta}(t+1) = \hat{\theta}(t) + F\phi(t)\varepsilon(t+1)$$

**For implementation one should express**: $\varepsilon(t+1) = f(\hat{\theta}(t), \phi(t), \varepsilon^0(t+1))$

# PAA – Improved gradient algorithm

$$\varepsilon(t+1) = \underbrace{y(t+1) - \hat{\theta}(t)^T \phi(t)}_{\varepsilon^0(t+1)} - \left[\hat{\theta}(t+1) - \hat{\theta}(t)\right]^T \phi(t)$$

$$\hat{\theta}(t+1) = \hat{\theta}(t) + F\phi(t)\varepsilon(t+1) \longrightarrow \hat{\theta}(t+1) - \hat{\theta}(t) = F\phi(t)\varepsilon(t+1)$$

$$\varepsilon(t+1) = \varepsilon^0(t+1) - \phi(t)^T F\phi(t)\varepsilon(t+1) \longrightarrow \varepsilon(t+1) = \frac{\varepsilon^o(t+1)}{1+\phi(t)^T F\phi(t)}$$

$$\hat{\theta}(t+1) = \hat{\theta}(t) + \frac{F\phi(t)\varepsilon^o(t+1)}{1+\phi(t)^T F\phi(t)}$$

**Stable for any *F > 0***

***Implementation:***

1. Before *t+1* one has: $u(t), u(t-1), ..y(t), y(t-1), \phi(t), \hat{\theta}(t), F$
2. Before *t+1* one computes : $F\phi(t)/(1+\phi(t)^T F\phi(t)),\ \hat{y}^0(t+1) = \hat{\theta}^T\phi(t)$
3. At *t+1* acquisition of *y(t+1)* and *u(t+1) is sent*
4. Running of the AAP
   (computation of : $\varepsilon^0(t+1), \hat{\theta}(t+1)$)

# General form of the parameter adaptation algorithms

$$\hat{\theta}(t+1) = \hat{\theta}(t) + F(t+1)\Phi(t)\varepsilon^{o}(t+1)$$

$$F^{-1}(t+1) = \lambda_1(t)F^{-1}(t) + \lambda_2(t)\Phi(t)\Phi(t)^T$$

$$0 < \lambda_1(t) \leq 1 \;;\; 0 \leq \lambda_2(t) < 2 \;;\; F(0) > 0$$

Matrix inversion lemma

$$\left(F^{-1} + \phi\phi^T\right)^{-1} = F - \frac{F\phi\phi^T F}{1+\phi^T F\phi}$$

$$F(t+1) = \frac{1}{\lambda_1(t)}\left[F(t) - \frac{F(t)\phi(t)\phi(t)^T F(t)}{\frac{\lambda_1(t)}{\lambda_2(t)} + \phi(t)^T F(t)\phi(t)}\right]$$

$$\varepsilon^{o}(t+1) = y(t+1) - \hat{\theta}(t)^T \phi(t)$$

$\Phi(t)$ – *regressor vector*

$\varepsilon^{o}(t+1) =$ "*a priori*" *adaptation error*

*F(t) is a time varying adaptation gain (positive definite matrix).*

In "self-tuning" regime *F(t)* ⟹ *0* as t increases

In "adaptive regime" *F(t)* should remain > *0*

# Sequence of operations in the PAA (details)

Between $t$ and $t+1$ $(\lambda_1(t) = \lambda_2(t) = 1)$

$$F(t+1) = F(t) - \frac{F(t)\Phi(t)\Phi(t)^T F(t)}{1+\Phi(t)^T F(t)\Phi(t)}$$

$$F(t+1)\Phi(t) = \frac{F(t)\Phi(t)}{1+\Phi(t)^T F(t)\Phi(t)}$$

$$y^\circ(t+1) = \hat{\theta}^T \phi(t)$$

**At *t+1*: acquisition of *y(t+1)***

After $t+1$

$$\varepsilon^o(t+1) = y(t+1) - \hat{\theta}(t)^T \phi(t)$$

$$\hat{\theta}(t+1) = \hat{\theta}(t) + F(t+1)\Phi(t)\varepsilon^o(t+1)$$

# Choice of the adaptation gain $F(t)$

- *To optimize the performances of the PAA it is useful to tune the time profile of the adaptation gain matrix*
- *To understand the influence of the tuning parameters, it is more convenient to use the equation of the "inverse" of the adaptation gain (°°)*
- *To tune the time profile of the adaptation gain in (°°), 2 parameters are introduced* $(\lambda_1(t), \lambda_2(t))$

General form:

$$F(t+1)^{-1} = \lambda_1(t)F(t)^{-1} + \lambda_2(t)\phi(t)\phi(t)^T$$
$$0 < \lambda_1(t) \leq 1 \;\; ; \;\; 0 \leq \lambda_2(t) < 2 \;\; ; \;\; F(0) > 0 \qquad (°°)$$

*Which gives (using matrix inversion lemma):*

$$F(t+1) = \frac{1}{\lambda_1(t)}\left[F(t) - \frac{F(t)\phi(t)\phi(t)^T F(t)}{\frac{\lambda_1(t)}{\lambda_2(t)} + \phi(t)^T F(t)\phi(t)}\right]$$

# Choice of the adaptation gain *F(t)*

Forme générale:

$$F(t+1)^{-1} = \lambda_1(t)F(t)^{-1} + \lambda_2(t)\phi(t)\phi(t)^T$$
$$0 < \lambda_1(t) \le 1 \ ; \ 0 \le \lambda_2(t) < 2 \ ; \ F(0) > 0$$

**A.1** Decreasing gain (RLS): $\lambda_1(t) = \lambda_1 = 1 \ ; \ \lambda_2(t) = 1$ — $t \nearrow F(t)^{-1} \nearrow F(t) \searrow$

*Parameter estimation (constant parameters). To be used in self-tuning regime*

**Forgetting factor**

**A.2** Fixed forgetting factor: $\lambda_1(t) = \lambda_1 \ ; \ 0 < \lambda_1 < 1 \ ; \ \lambda_2(t) = \lambda_2 = 1$

Typical values for $\lambda_1$: $\lambda_1 = 0.95,...,0.99$

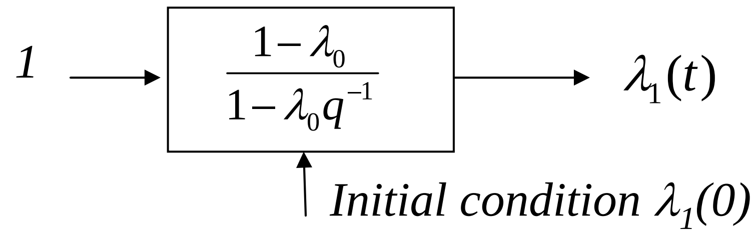
Minimized criterion: $$J(t) = \sum_{i=1}^{t} \lambda_1^{(t-i)} \left[ y(i) - \hat{\theta}(t)^T \phi(i-1) \right]^2$$

*One gives more weight in the criterion the last prediction error (weigtht=1).*
*The weight on the first prediction error is very small (weight* $= \lambda^{(t-1)} << 1$ *; $\lambda$<1 and t>100)*

*Parameter estimation for slowly time variiyng plants*
*Dangerous algorithm if there is no excitation (adaptation gain goes to infinity)!*

# Choice of the adaptation gain *F(t)*

**A.3** Variable forgetting factor: $\lambda_1(t) = \lambda_0 \lambda_1(t-1) + (1-\lambda_0)1 \;\; ; \;\; 0 < \lambda_0 < 1$




$t \to \infty ; \lambda_1 \to 1$

*Asymptotically tends toward a decreasing adaptation gain*

$$\lambda_2(t) = \lambda_2 = 1$$

Typical values: $\lambda_1(0) = 0.95,...,0.99 \;\; ; \;\; \lambda_0 = 0.95,...,0.99$

Minimized criterion:

$$J(t) = \sum_{i=1}^{t} \left[ \prod_{j=1}^{t-1} \lambda_1(j-i) \right] \left[ y(i) - \hat{\theta}(t)^T \phi(i-1) \right]^2$$

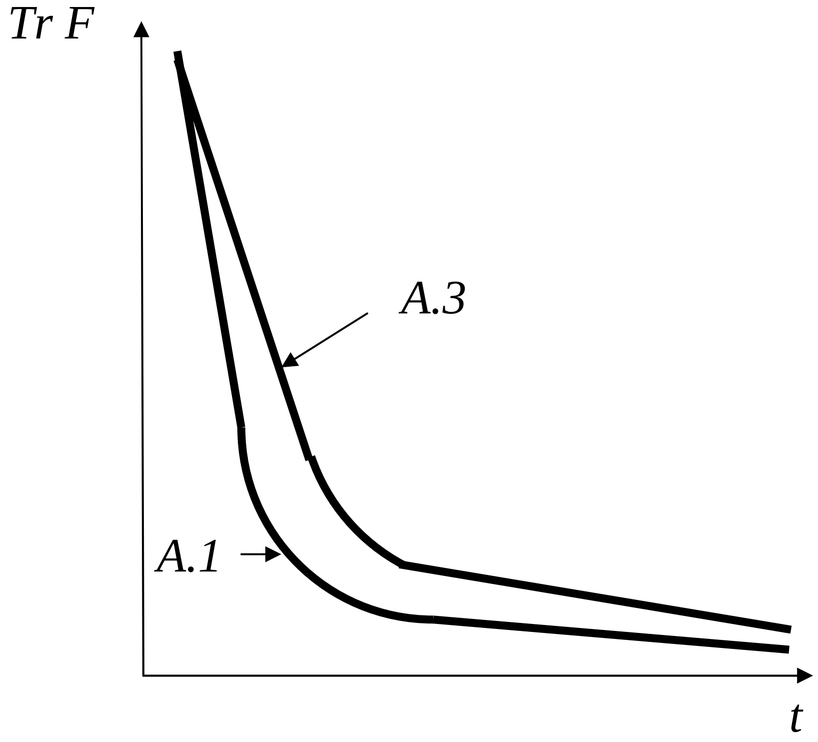
Since $\lambda_1(t)$ goes towards *1* for large *i*, one forgets intial valus

- *Parameter estimation for plants with constant parameters*
- *Use in « self-tuning regime ».*
- *Maintain a larger gain then A.1 at the beginning*
- *Gives in general better performances than A.1*

# Comparison A.1/A.3

*Tr F=Trace of matrix F (sum of the diagonal terms of gain matrix F)*

*The trace is a measure of the"gain" for the case of matrix gains*



# Choice of the adaption gain

**A.4** Constant trace: $trF(t+1) = trF(t) = trF(0) = nGI$

$$F(0) = \begin{bmatrix} GI & & & 0 \\ & \cdot & & \\ & & \cdot & \\ 0 & & & GI \end{bmatrix}$$

$n$ = number of parameters

*GI = (0.01)0.1 to 4*

$$trF(t+1) = \frac{1}{\lambda_1(t)} tr\left[ F(t) - \frac{F(t)\phi(t)\phi(t)^T F(t)}{\alpha(t) + \phi(t)^T F(t)\phi(t)} \right] = trF(t)$$

One computes: $\lambda_1(t)$, *for* $\alpha(t) = \lambda_1(t)/\lambda_2(t)$ *fixed*

*Parameter estimation for time varying systems (adaptive control regime)*

**A.5** Decreasing gain + constant trace

One switches from A.1 to A.4 when: $trF(t) \leq nG$ ; $G = (0.01)0.1$ to 4

*Parameter estimation of time varying systems in the absence of initial information upon the parameters (adaptive control regime)*

# Choice of the adaptation gain

**A.6** Variable forgetting factor + constant trace

One switches from A.3 to A.4 when: $trF(t) \leq nG \;\; ; \;\; G = (0.01)0.1 \text{ to } 1$

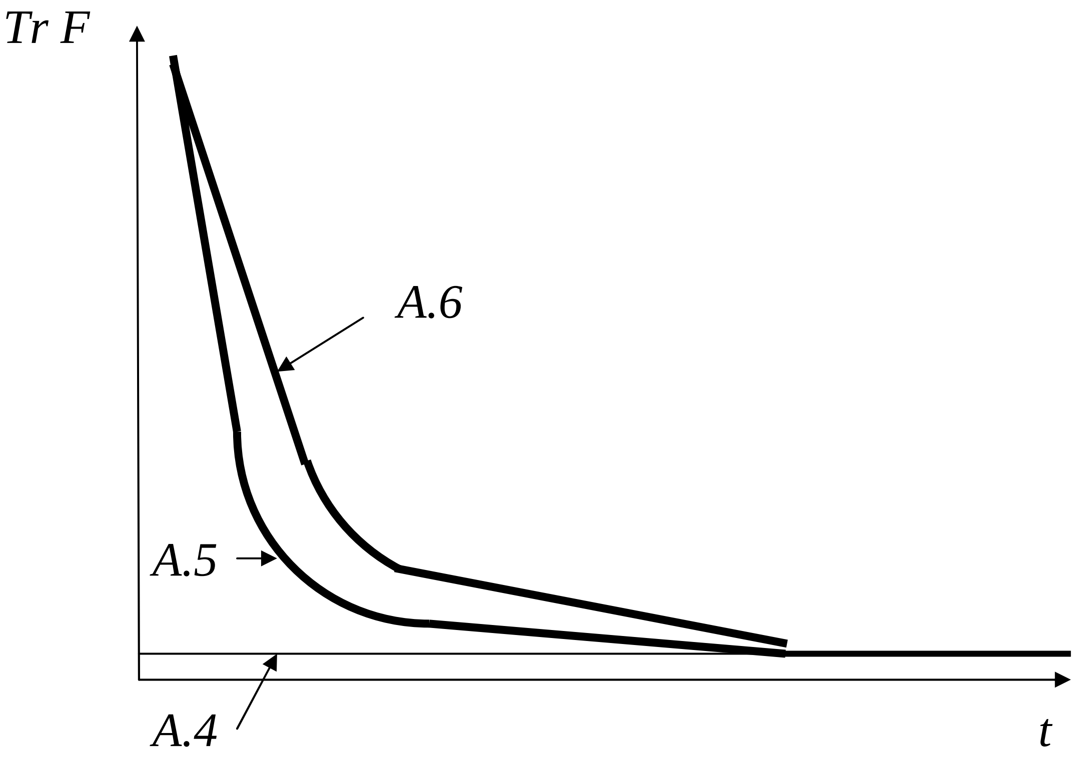
*Parameter estimation of time varying systems in the absence of initial information upon the parameters (adaptive control regime)*

**A.7** Constant gain (improved gradient algorithm)

$$\lambda_1(t) = \lambda_1 = 1 \;\; ; \;\; \lambda_2(t) = \lambda_2 = 0 \quad \longrightarrow \quad F(t+1) = F(t) = F(0)$$

*Estimation of systems with few parameters* $(\leq 3)$ *and low noise level.*
*Simple implementation but performance inferior to A.1, A.2, A.3 and A.4 (adaptive control regime)*

# Comparison A.4/A.5/A.6



# Choice of the initial adaptation gain *F(0)*

$$F(0) = \frac{1}{\delta} I = (GI) I$$

The adaption gain can be interpreted as a measure of the Parametric error (precision of the estimation).

Without initial information upon the parameters: $GI = 1000\,;\ \hat{\theta}(0) = 0$

Initial parameter estimation is available: $GI = << 1\,;\ \hat{\theta}(0) = \hat{\theta}_0$

*The trace of the gain matrix is a measure of the « value » of the adaptation gain*

Remark:

*If the trace of F(t) does not decrease significantly, in general the parameter estimation is bad.*

(can happens when the excitation signals are not appropriate)

# Alternative form for the PAA

$$\hat{\theta}(t+1) = \hat{\theta}(t) + F(t)\Phi(t)\varepsilon(t+1)$$

$$F(t+1) = F(t) - \frac{F(t)\Phi(t)\Phi(t)^T F(t)}{1+\Phi(t)^T F(t)\Phi(t)}$$

$$\varepsilon(t+1) = \frac{\varepsilon^0(t+1)}{1+\Phi(t)^T F(t)\Phi(t)}$$

$\varepsilon(t+1) = \varepsilon(t+1/\hat{\theta}(t+1)$ *"a posteriori" adaptation error*

$\varepsilon^\circ(t+1) = \varepsilon(t+1/\hat{\theta}(t))$ *"a priori" adaptation error*

*Used for synthesis and analysis of adaptive systems*

# How to generate a parameter adaptation algorithm ?

Define $\theta^*$ as the "unknown" optimal value of the parameter vector
Define $\hat{\theta}(t)$ as the "adjustable" parameter vector to be tuned

*One can generate a PAA if one can express a signal error* $\varepsilon(t+1)$

$$\varepsilon(t+1) = H(q^{-1})\left[\theta^* - \hat{\theta}(t+1)\right]^T \Phi(t) \qquad \textbf{(x)}$$

The associated PAA is:

$$\hat{\theta}(t+1) = \hat{\theta}(t) + F(t)\Phi(t)\varepsilon(t+1)$$
$$F(t+1)^{-1} = \lambda_1(t)F(t)^{-1} + \lambda_2(t)\phi(t)\phi(t)^T \qquad \textbf{(xx)}$$
$$0 < \lambda_1(t) \le 1 \ \ ; \ \ 0 \le \lambda_2(t) < 2 \ \ ; \ \ F(0) > 0$$
$$\varepsilon(t+1) = \frac{\varepsilon^0(t+1)}{1 + \Phi(t)^T F(t)\Phi(t)}$$

$\varepsilon(t+1) = \varepsilon(t+1/\hat{\theta}(t+1)$ "*a posteriori*" *adaptation error*
$\varepsilon^\circ(t+1) = \varepsilon(t+1/\hat{\theta}(t))$ "*a priori*" *adaptation error*

## Does it always work ?

Define:

$$H'(z^{-1}) = H(z^{-1}) - \frac{\lambda}{2} \quad with \;\; 2 > \lambda \geq \max_t \lambda_2(t)$$

It works provided that:

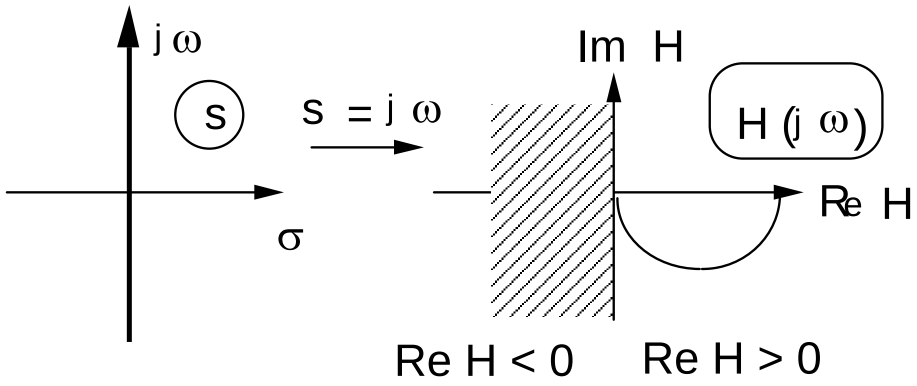
- $H'(z^{-1})$ is a *strictly positive real* discrete time transfer function
- $\Phi(t)$ is bounded

This will assure that $\varepsilon(t+1)$ and $\varepsilon^0(t+1)$ go to zero as $t \rightarrow \infty$

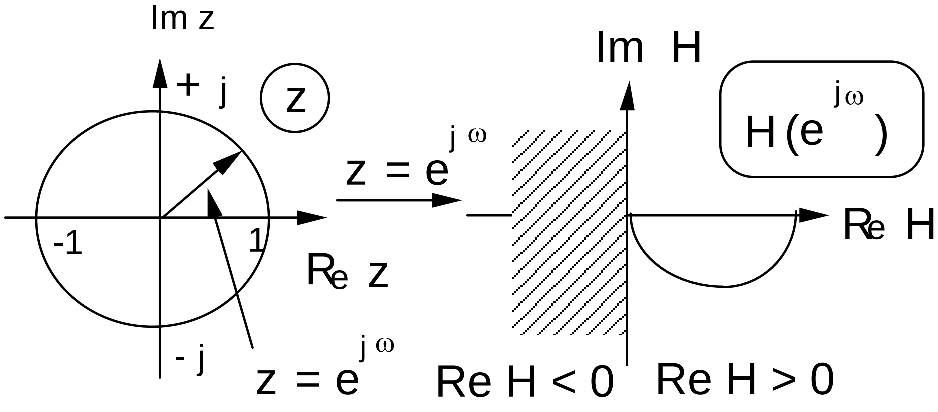
**Convergence of the parameters** requires in addition that *Φ(t)* is a "persistenly exciting" signal (to be discussed)

# Strictly positive real transfer function (SPR)

**Continuous - time**




**Discrete - time**




- asymptotically stable
- $\operatorname{Re} H(e^{j\omega}) > 0 \;\; \textit{for all} \; |e^{j\omega}| = 1, \; (0 < \omega < \pi)$ *(discrete-time case)*

# Generation of a PAA – example

Discrete time plant model (unknown parameters))

$$y(t+1) = -a_1 y(t) + b_1(t)u(t) = \theta^T \phi(t)$$

Measurement vector

$\theta^T = [a_1, b_1]$ ← Parameter vector ; $\phi(t)^T = [-y(t), u(t)]$

Adjustable prediction model

$$\hat{y}^o(t+1) = \hat{y}(t+1|\hat{\theta}(t)) = -\hat{a}_1(t)y(t) + \hat{b}_1(t)u(t) = \hat{\theta}(t)^T \phi(t)$$ a priori

$$\hat{y}(t+1) = \hat{y}(t+1|\hat{\theta}(t+1)) = -\hat{a}_1(t+1)y(t) + \hat{b}_1(t+1)u(t) = \hat{\theta}(t+1)^T \phi(t)$$ a posteriori

$\theta(t)^T = [\hat{a}_1(t), \hat{b}_1(t)]$ ← Vector of adjustable parameters

Prediction error

$$\varepsilon^o(t+1) = y(t+1) - \hat{y}^o(t+1) = \varepsilon^o(t+1/\hat{\theta}(t))$$

$$\varepsilon(t+1) = y(t+1) - \hat{y}(t+1) = \varepsilon(t+1/\hat{\theta}(t+1))$$

**But:**

$$\varepsilon(t+1) = \left[\theta - \hat{\theta}(t+1)\right]^T \phi(t)$$

Is of the form (x) and we can use the PAA (xx) with $\Phi(t) = \phi(t)$

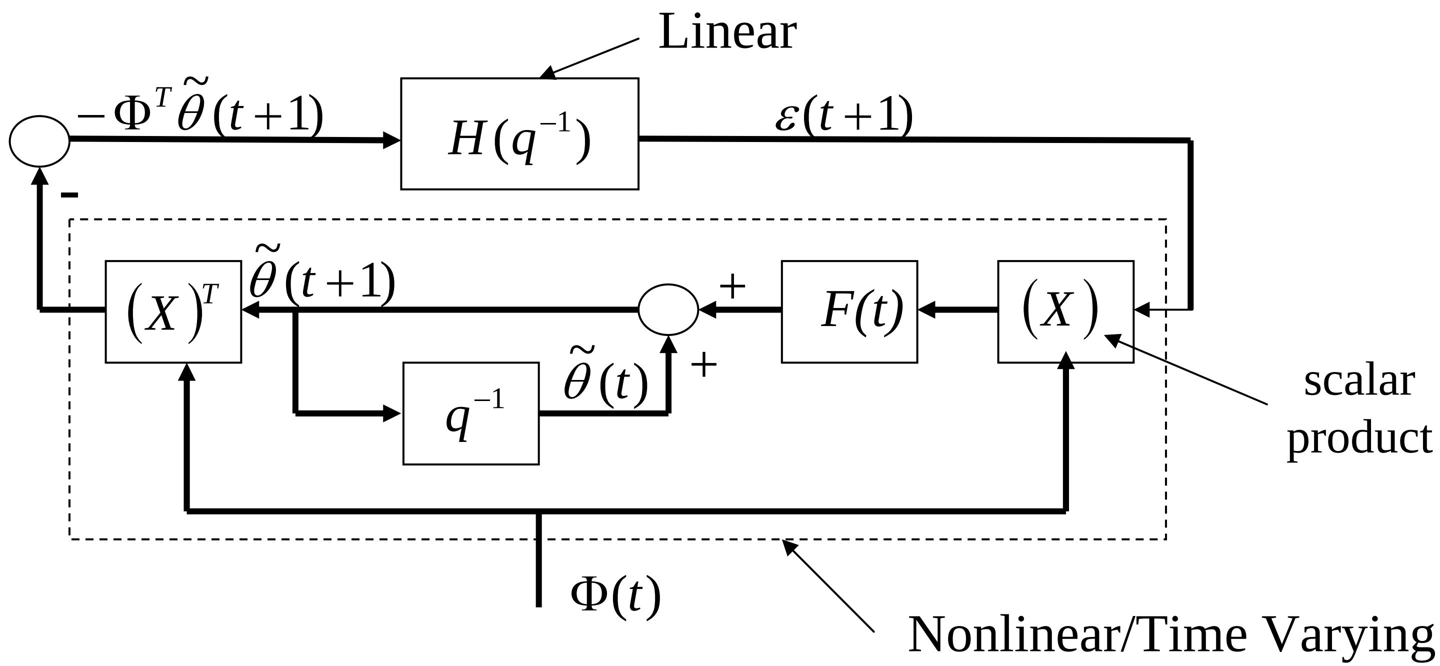
# An equivalent feedback representation

Eqs (**x**) and (**xx**) allow to define an equivalent feedback system

Define the *parameter error :* $\tilde{\theta}(t) = \hat{\theta}(t) - \theta$

$$\varepsilon(t+1) = -H(q^{-1})\tilde{\theta}^T(t+1)\Phi(t) \qquad \textbf{(x)}$$

From (**xx**) one gets: $$\tilde{\theta}(t+1) = \tilde{\theta}(t) + F(t)\Phi(t)\varepsilon(t+1) \qquad \textbf{(xxx)}$$




*The stability of the adaptive systems is studied using this equivalent representation*

# Convergence of estimated parameters

Convergence towards 0 (zero) of the *adaptation error* does not necessarily implies the convergence of the estimated parameter vector $\hat{\theta}$ towards the optimal value $\theta$ *!!!!*

A supplementary condition upon the input is required (*persistence of excitation*)

# Persistenly exciting regressor

$\Phi$(t) is such that:

$$\left[\theta^* - \hat{\theta}\right]^T \Phi(t) = 0 \; only \; if \; \left[\theta^* - \hat{\theta}\right] = 0$$

This imply that:

- $\Phi(t)$ is different from zero
- It does not exist:

$$\Phi(t) \neq 0 \; such \; that \; it \; exists \; a \; vector \; \hat{\theta} \neq \theta^* \;\; for \; which \left[\theta^* - \hat{\theta}\right]^T \Phi(t) = 0$$

**Rem.:** the condition above can be converted in many cases in terms of a condition upon the excitation only.

# Convergence of the parameters

« null prediction error » does not implies in all the cases « estimation of the true parameters »!!

Plant model: $y(t+1) = -a_1 y(t) + b_1 u(t)$

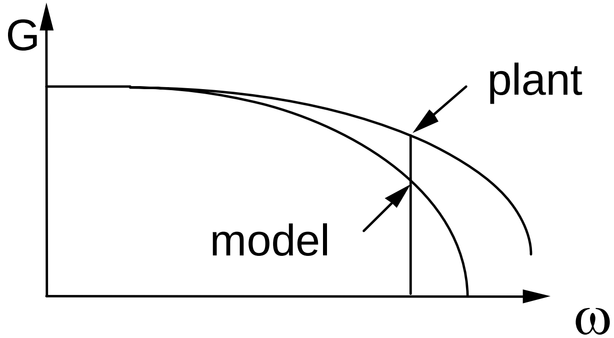
Estimated model: $\hat{y}(t+1) = -\hat{a}_1 y(t) + \hat{b}_1 u(t)$

$u(t) = const.$ $\quad \dfrac{b_1}{1+a_1} = \dfrac{\hat{b}_1}{1+\hat{a}_1}$

**The two models have the same static gain but** $\hat{a}_1 \neq a_1 ; \hat{b}_1 \neq b_1$

$y(t+1) = y(t) = \dfrac{b_1}{1+a_1} u$ et $\hat{y}(t+1) = \hat{y}(t) = \dfrac{\hat{b}_1}{1+\hat{a}_1} u$

$$\varepsilon(t+1) = y(t+1) - \hat{y}(t+1) = 0 \quad \text{for } u(t) = const.; \hat{a}_1 \neq a_1 ; \hat{b}_1 \neq b_1$$




***If we would like to distinguish the two models one should apply:*** $u(t) = \sin(\omega t) ; \omega \neq 0$

# Analysis

$$\varepsilon(t+1) = y(t+1) - \hat{y}(t+1) = -[a_1 - \hat{a}_1]y(t) + [b_1 - \hat{b}_1]u(t) = 0$$

$$y(t) = \frac{b_1 q^{-1}}{1 + a_1 q^{-1}} u(t)$$

$$\varepsilon(t+1) = \left[ (\hat{a}_1 - a_1) b_1 q^{-1} + (b_1 - \hat{b}_1)(1 + a_1 q^{-1}) \right] u(t) = 0$$

$$\varepsilon(t+1) = \left[ (b_1 - \hat{b}_1) + q^{-1}(b_1 \hat{a}_1 - a_1 \hat{b}_1) \right] u(t) = (\alpha_0 + \alpha_1 q^{-1}) u(t) = 0 \qquad \textbf{(*)}$$

Solution of the recursive equation: $u(t) = z^t = e^{sT_e t}$

$$(\alpha_0 + z^{-1}\alpha_1) z^t = 0 \Longrightarrow z = -\frac{\alpha_1}{\alpha_0} = e^{\sigma T_e} \; ; \sigma = réel$$

$$\boxed{u(t) = e^{\sigma T_e t}}$$

$$u(t) = const \Rightarrow \sigma = 0 \Rightarrow z = 1 \Rightarrow -\alpha_1 = \alpha_0 \Rightarrow b_1 - \hat{b}_1 = a_1 \hat{b}_1 - b_1 \hat{a}_1 \Rightarrow \frac{b_1}{1 + a_1} = \frac{\hat{b}_1}{1 + \hat{a}_1}$$

**Problem : find *u(t)* such that:** $\varepsilon = 0 \Rightarrow \hat{a}_1 = a_1 ; \hat{b}_1 = b_1$

**Answer: *u(t)* should not be a solution of (*).**

**An example :** $u(t) = e^{j\omega T_e t} \quad or \quad e^{-j\omega T_e t} \; or \sin \omega T_e t, \; \omega \neq 0$

# General case – choice of the excitation signal

Structure of the model to be indentified:

$$y(t) = -\sum_{i=1}^{n_A} a_i y(t-i) + \sum_{i=1}^{n_B} b_i u(t-d-i)$$

Number of the parameters to be identified: $= n_A + n_B$

Excitation signal: $$u(t) = -\sum_{i=1}^{p} \sin \omega_i T_e t$$

**One should choose *p* such that *u(t)* can not be a solution of the recursive equation for $\varepsilon$ whch features the parametr errors.**

$$\left.\begin{array}{ll} n_A + n_B = even & p \geq \dfrac{n_A + n_B}{2} \\ n_A + n_B = odd & p \geq \dfrac{n_A + n_B + 1}{2} \end{array}\right\}$$

*In order to identify correctly one should use an input « rich » in frequencies.*

**Standard solution**: *Pseudo Random Binary Sequence (PRBS)*

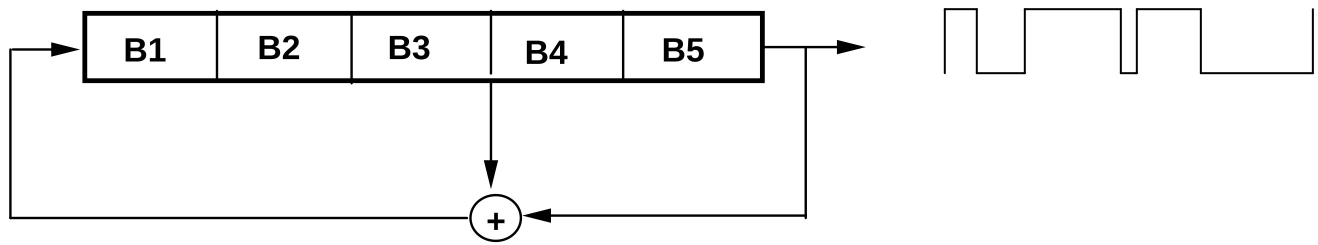
# Pseudo random binary sequence (PRBS)

Sequence of rectangular pulses, modulated in width

(rich in frequencies – almost uniform spectral density from 0 to $0.5f_s$)

Generation: *shift registers connected in feedbck*

Example : generation of a PRBS of length $31=2^5-1$




( addition modulo 2 )

***Length of the sequence*** **: gives its periodicity.**

**Random variation of the pulse width within a sequence**

*Characteristic parameters:*

- number of cells (N)
- Maximum pulse duration ( $t_{im}=Nt_e$ )
- length of the sequence ( $L=2^N-1$ )

C++ code and .m file for generation of PRBS: see the book website

# PAA – Concluding remarks

- PAA are a fundamental component of any closed loop adaptive control scheme either “direct” or “indirect”
- PAA are also used in sytem identification (recursive algorithms)
- There are several choices for the profile of the adaptation gain
- For a secure implementation the implementation of the updating algorithm for the adaption gain should use the U-D factorization (see L-Z book appendix F)
- Convergence towards good performance does not necessarily implies that the optimal parameters have been correctly identified
- Convergence towards good performance simply implies that one has the good parameters for the specific excitation (reference signals) acting on the system
- If convergence to the optimal parameters is required, special external excitation should be applied (testing signals satisfying a persistence of excitation condition)