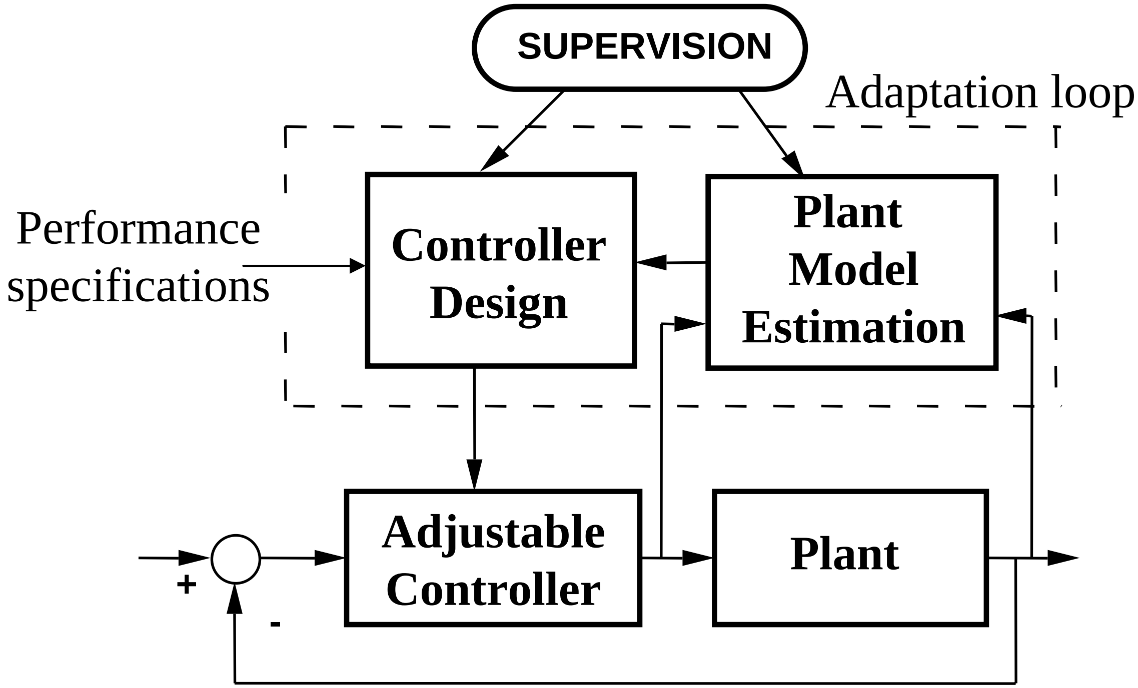
Adaptive Control

Part 5: Direct and Indirect Adaptive Control

Adaptive Control – A Basic Scheme



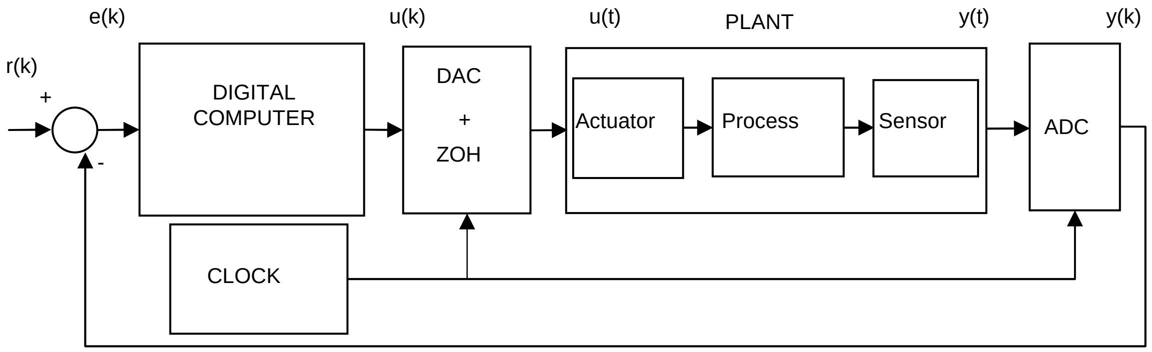
- Indirect adaptive control
- Direct adaptive control *(the controller is directly estimated)*

Outline

- Digital control systems
- Tracking and regulation with independent objectives (known parameters)
- Adaptive tracking and regulation with independent objectives (direct adaptive control)
- Pole placement (known parameters)
- Adaptive pole placement (indirect adaptive control)

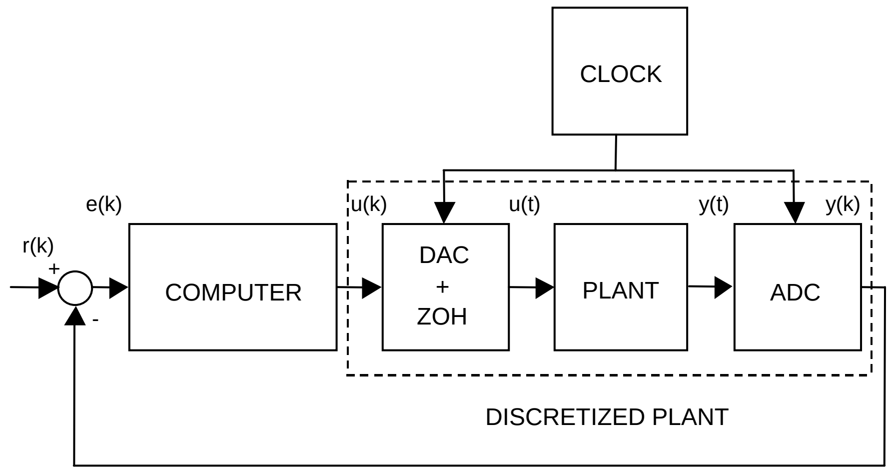
Digital Control System

The *control law* is implemented on a digital computer



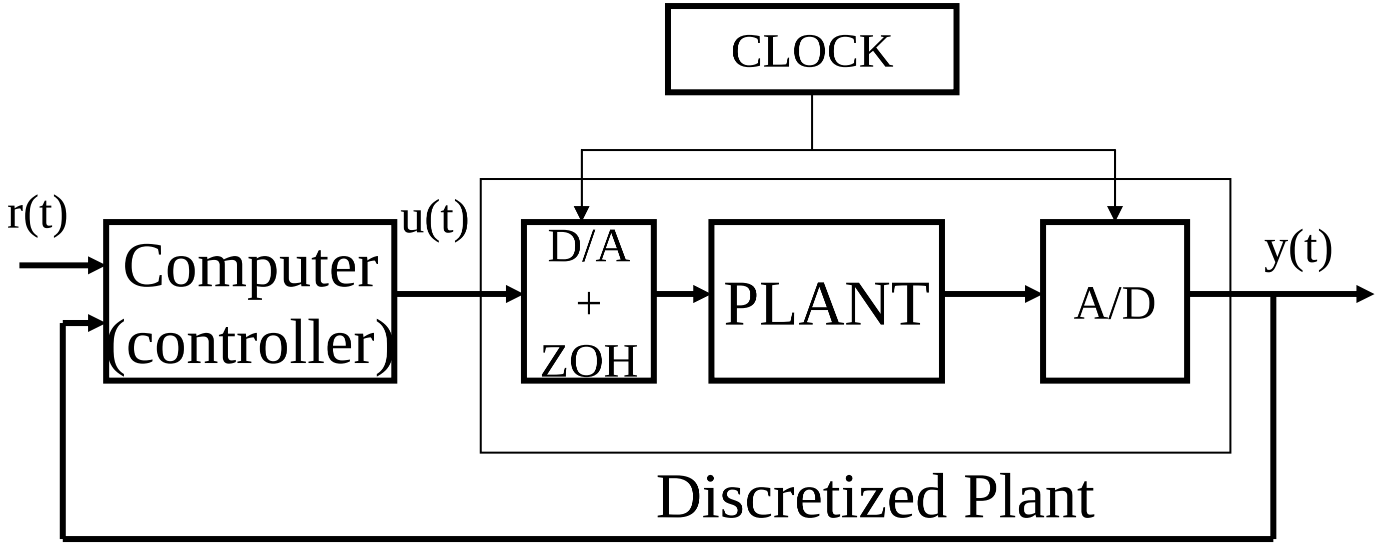
ADC: analog to digital converter
DAC: digital to analog converter
ZOH: zero order hold

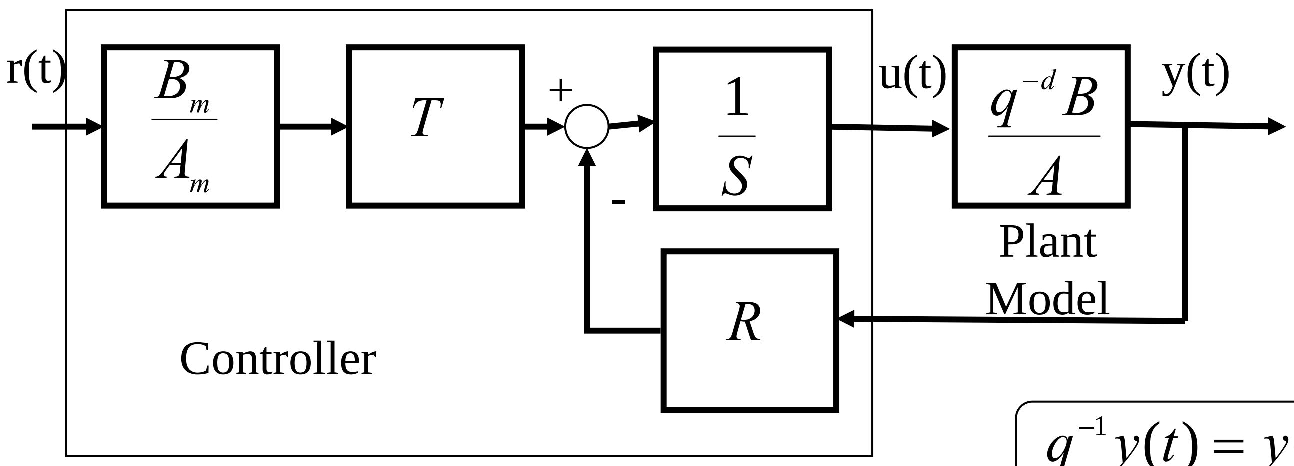
Digital Control System



- Sampling time depends on the system bandwidth
- Efficient use of computer resources

The R-S-T Digital Controller





$$q^{-1}y(t) = y(t-1)$$

Discrete time model – *General form*

$$(*)\quad y(t) = -\sum_{i=1}^{n_A} a_i y(t-i) + \sum_{i=1}^{n_B} b_i u(t-d-i)$$

d –delay (integer multiple of the sampling period)

$$1+\sum_{i=1}^{n_A} a_i q^{-i} = A(q^{-1}) = 1 + q^{-1}A^*(q^{-1}) \;;\quad A^*(q^{-1}) = a_1 + a_2 q^{-1} + \ldots + a_{n_A} q^{-n_A+1}$$

$$\sum_{i=1}^{n_B} b_i q^{-i} = B(q^{-1}) = q^{-1}B^*(q^{-1}) \qquad ; \quad B^*(q^{-1}) = b_1 + b_2 q^{-1} + \ldots + b_{n_B} q^{-n_B+1}$$

$$(*)\; A(q^{-1})y(t) = q^{-d}B(q^{-1})u(t)$$

$$(*)\; A(q^{-1})y(t+d) = B(q^{-1})u(t)$$ (Predictive form)

$$(*)\; y(t) = H(q^{-1})u(t)\;; \quad H(q^{-1}) = \frac{q^{-d}B(q^{-1})}{A(q^{-1})}$$ - pulse transfer operator

$$q^{-1} \to z^{-1} \qquad H(z^{-1}) = \frac{q^{-z}B(z^{-1})}{A(z^{-1})}$$ - transfer function

First order systems with delay

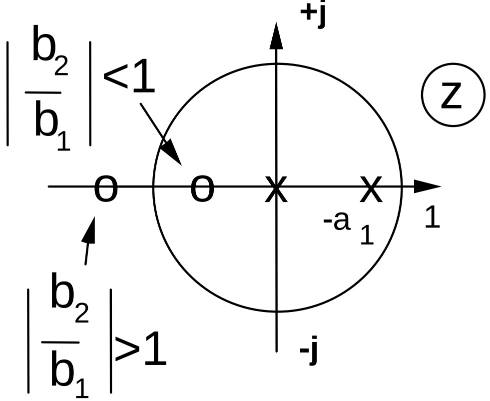
Fractional delay

Continuous time model $H(s) = \dfrac{G\,e^{-s\tau}}{1+T_s}$ $\qquad \tau = d.T_s + L\,;\ \ 0 < L < T_s$

Discrete time model $H(z^{-1}) = \dfrac{z^{-d}(b_1 z^{-1} + b_2 z^{-2})}{1+a_1 z^{-1}} = \dfrac{z^{-d-1}(b_1 + b_2 z^{-1})}{1+a_1 z^{-1}}$

$a_1 = -e^{-\frac{T_s}{T}}$ $\qquad b_1 = G(1 - e^{\frac{L-T_s}{T}})$ $\qquad b_2 = Ge^{-\frac{T_s}{T}}(e^{\frac{L}{T}} - 1)$

Remark: For $L > 0.5T_s \Rightarrow b_2 > b_1 \Rightarrow unstable\ \ zero\ (\left|-\dfrac{b_2}{b_1}\right| > 1)$



o - zero
x - pole

Tracking and regulation with independent objectives

It is a particular case of pole placement
(the closed loop poles contain the plant zeros))

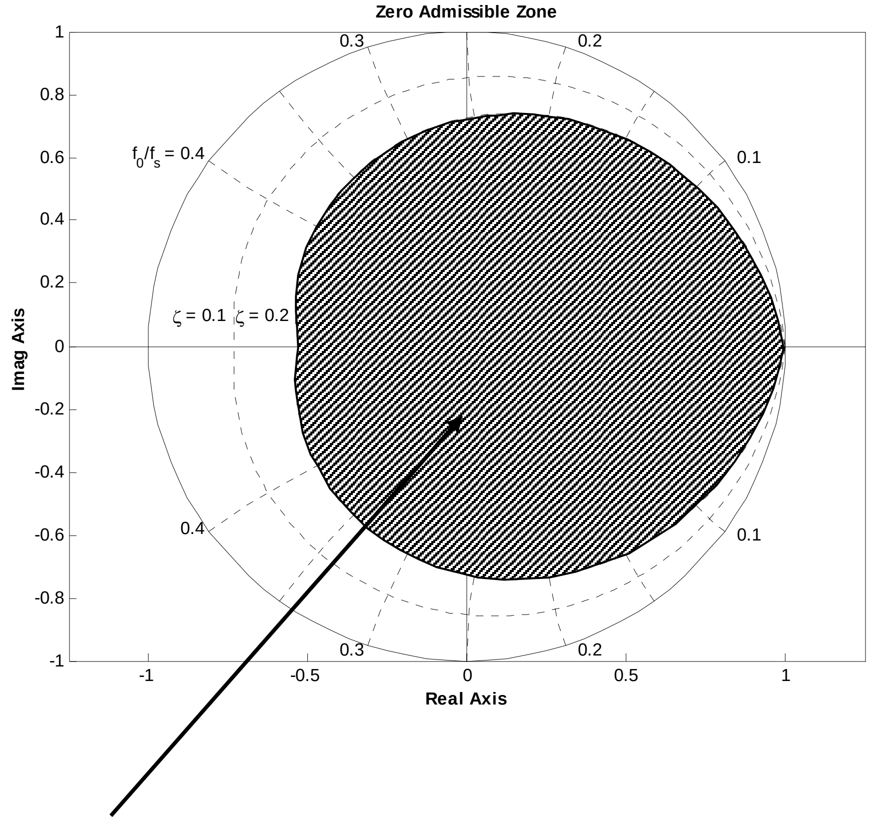
It is a method which simplifies the plant zeros
Allows exact achievement of imposed performances

Allows to design a RST controller for:
- stable or unstable systems
- without restrictions upon the degrees of the polynomials A et B
- without restriction upon the integer delay d of the plant model
- discrete-time plant models with *stable zeros!!!*

Remarks:
- *Does not tolerate fractional delay > 0.5 T_S (unstable zero)*
- High sampling frequency generates unstable discrete time zeros !

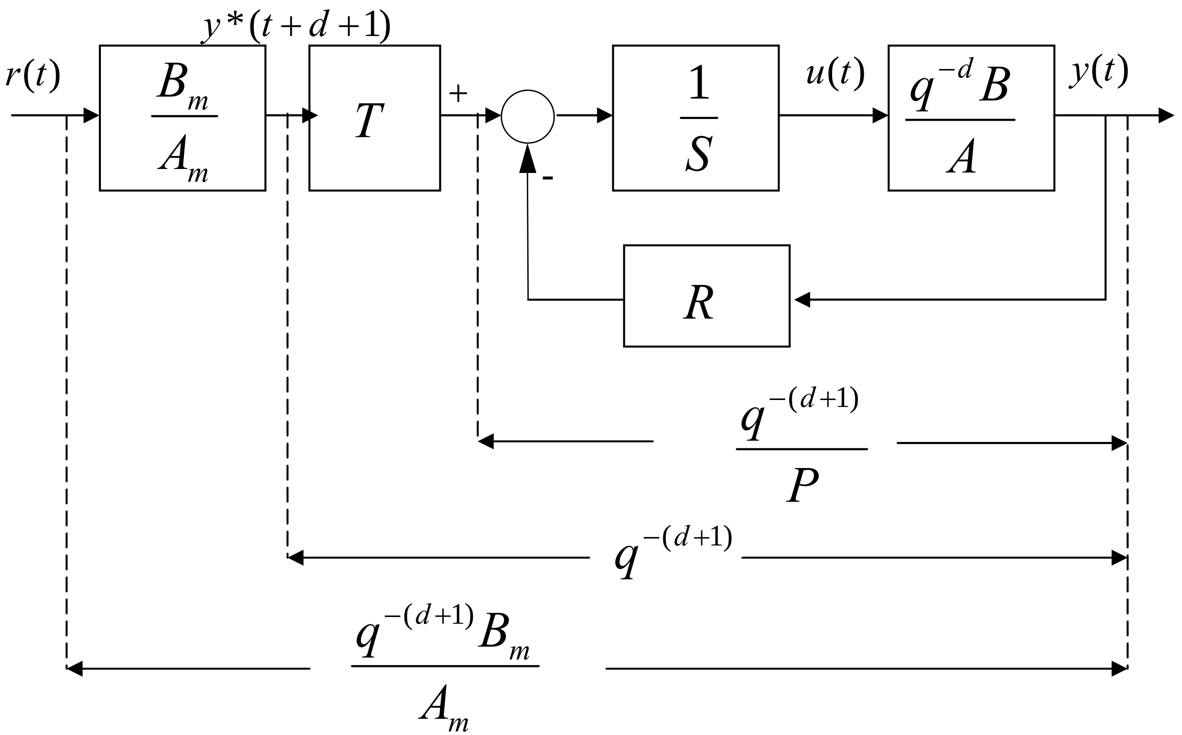
Tracking and regulation with independent objectives

The model zeros should be stable and enough damped



Admissibility domain for the zeros of the discrete time model

Tracking and regulation with independent objectives



$$P(q^{-1}) = P_D(q^{-1})P_F(q^{-1})$$

Reference signal (tracking): $y^*(t+d+1) = \dfrac{B_m(q^{-1})}{A_m(q^{-1})} r(t)$

Controller: $S(q^{-1})u(t) = T(q^{-1})y^*(t+d+1) - R(q^{-1})y(t)$

Regulation (computation of $R(q^{-1})$ and $S(q^{-1})$)

T.F. of the closed loop without T:

$$H_{CL}(q^{-1}) = \frac{q^{-d+1}B^*(q^{-1})}{A(q^{-1})S(q^{-1}) + q^{-d+1}B^*(q^{-1})R(q^{-1})} = \frac{q^{-d+1}}{P(q^{-1})} = \frac{q^{-d+1}B^*(q^{-1})}{B^*(q^{-1})P(q^{-1})}$$

The following equation has to be solved :

$$A(q^{-1})S(q^{-1}) + q^{-d+1}B^*(q^{-1})R(q^{-1}) = B^*(q^{-1})P(q^{-1}) \quad (*)$$

S should be in the form: $S(q^{-1}) = s_0 + s_1 q^{-1} + \ldots + s_{n_S} q^{-n_S} = B^*(q^{-1})S'(q^{-1})$

After simplification by B^*,(*) becomes:

$$\boxed{A(q^{-1})S'(q^{-1}) + q^{-d+1}R(q^{-1}) = P(q^{-1})} \qquad (**)$$

Unique solution if: $n_P = deg\, P(q^{-1}) = n_A + d$; $\boxed{deg\, S'(q^{-1}) = d}$; $deg\, R(q^{-1}) = n_A - 1$

$$R(q^{-1}) = r_0 + r_1 q^{-1} + \ldots r_{n_A-1} q^{-n_A-1} \qquad S'(q^{-1}) = 1 + s'_1 q^{-1} + \ldots s'_d\, q^{-d}$$

Regulation (computation of $R(q^{-1})$ and $S(q^{-1})$)

(**) is written as: $Mx = p \longrightarrow x = M^{-1}p$

$$
\begin{array}{cc}
\overbrace{\hphantom{xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx}}^{n_A + d + 1} & \\
\left[\begin{array}{cccccccc}
1 & 0 & & & & & & 0 \\
a_1 & 1 & & & & & & . \\
a_2 & a_1 & & 0 & & & & . \\
: & : & & 1 & & & & . \\
a_d & a_{d-1} & \ldots & a_1 & 1 & & & . \\
a_{d+1} & a_d & & & a_1 & 1 & & . \\
a_{d+2} & a_{d+1} & & & a_2 & 0 & & . \\
 & & & & . & & & . \\
 & & & & . & & & 0 \\
0 & 0 & \ldots & 0 & a_{n_A} & 0 & 0 & 1
\end{array}\right] & \left.\vphantom{\begin{array}{c}1\\1\\1\\1\\1\\1\\1\\1\\1\\1\end{array}}\right\} n_A + d + 1 \\
\underbrace{\hphantom{xxxxxxxxxxxxxxxxxxxxxxxx}}_{d+1}\;\underbrace{\hphantom{xxxxxxxxxx}}_{n_A} &
\end{array}
$$

$$x^T = [1, s'_1, \ldots, s'_d, r_0, r_1, \ldots, r_{n-1}] \qquad p^T = [1, p_1, p_2, \ldots, p_{n_A}, p_{n_A+1}, \ldots, p_{n_A+d}]$$

Use of WinReg or *predisol.sci(.m)* for solving (**)

Insertion of pre specified parts in R and S is possible

Tracking (computation of $T(q^{-1})$)

Closed loop T.F.: $r \longrightarrow y$

$$H_{BF}(q^{-1}) = \frac{q^{-(d+1)}B_m(q^{-1})}{A_m(q^{-1})} = \frac{B_m(q^{-1})T(q^{-1})q^{-(d+1)}}{A_m(q^{-1})P(q^{-1})}$$

Desired T.F.

It results : $\boxed{T(q^{-1}) = P(q^{-1})}$

Controller equation:

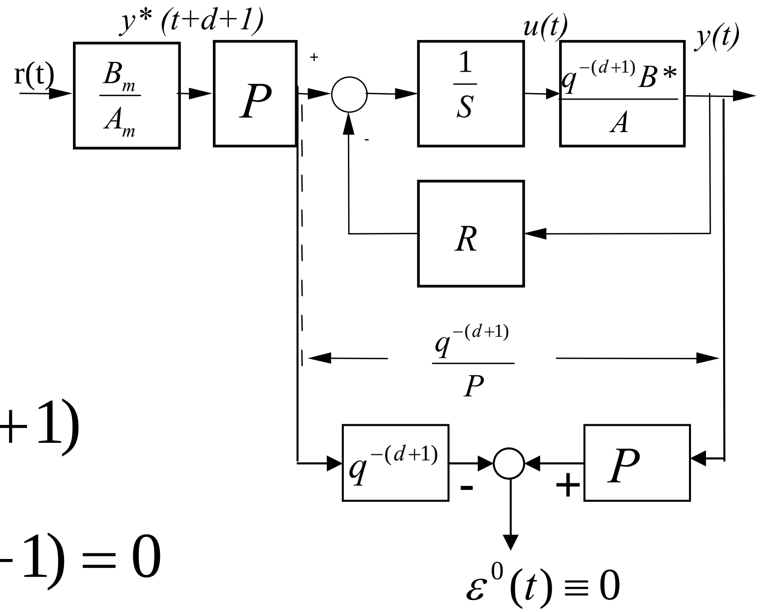
$$S(q^{-1})u(t) = P(q^{-1})y^*(t+d+1) - R(q^{-1})y(t)$$

$$u(t) = \frac{P(q^{-1})y^*(t+d+1) - R(q^{-1})y(t)}{S(q^{-1})}$$

$$u(t) = \frac{1}{b_1}\left[P(q^{-1})y^*(t+d+1) - S^*(q^{-1})u(t-1) - R(q^{-1})y(t)\right] \quad (s_0 = b_1)$$

Reference signal (tracking) : $y^*(t+d+1) = \frac{B_m(q^{-1})}{A_m(q^{-1})}r(t)$

Tracking and regulation with independent objectives
A time domain interpretation



$$y(t)=\frac{q^{-(d+1)}}{P(q^{-1})}y^*(t+d+1)$$

$$P(q^{-1})y(t)=q^{-(d+1)}y^*(t+d+1)$$

$$P(q^{-1})y(t)-q^{-(d+1)}y^*(t+d+1)=0$$

(in case of correct tuning)

Reformulation of the "design problem":

Find a controller which generate *u(t)* such that:

$$\varepsilon^0(t+d+1)=P\left[y(t+d+1)-y^*(t+d+1)\right]=0$$

Tracking and regulation with independent objectives
Synthesis in the time domain – an example

For *d=0 (S'=1)*

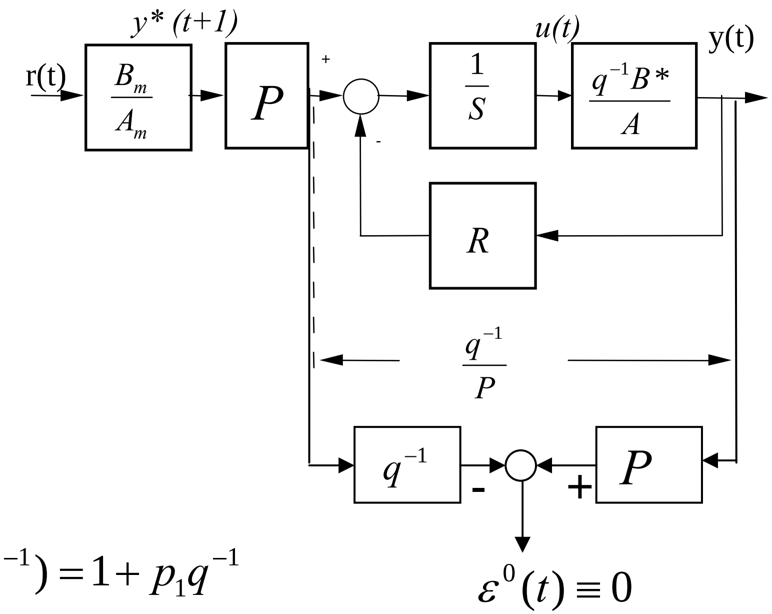
$$S(q^{-1}) = B^*(q^{-1})$$

$$A(q^{-1}) + q^{-1}R(q^{-1}) = P(q^{-1})$$

$$R(q^{-1}) = P^*(q^{-1}) - A^*(q^{-1})$$

$$P(q^{-1}) = 1 + q^{-1}P^*(q^{-1})$$

$$A(q^{-1}) = 1 + q^{-1}A^*(q^{-1})$$



Example:

$$y(t+1) = -a_1 y(t) + b_1 u(t) + b_2 u(t-1); \quad P(q^{-1}) = 1 + p_1 q^{-1}$$

$$\varepsilon^\circ(t+1) = P\left[y(t+1) - y^*(t+1)\right] = y(t+1) + p_1 y(t) - Py^*(t+1) =$$

$$= \left[-a_1 y(t) + b_1 u(t) + b_2 u(t-1) + p_1 y(t) - Py^*(t+1)\right] = 0$$ ← Solve for u(t)

$$u(t) = \frac{Py^*(t+1) - b_2 u(t-1) - r_0 y(t)}{b_1}; \qquad r_0 = p_1 - a_1$$

Controller satisfies:

$$\boxed{\begin{aligned} &P(q^{-1})y^*(t+1) = b_1 u(t) + b_2 u(t-1) + r_0 y(t) = \theta^T \phi(t) \\ &\theta^T = \left[b_1, b_2, r_0\right] \ \phi^T(t) = \left[u(t), u(t-1), y(t)\right] \end{aligned}}$$

Adaptive tracking and regulation with independent objectives

Three techniques:

- Model reference adaptive control (direct)
- Plant model estimation + computation of the controller (indirect)
- Re-parametrized plant model estimation (direct)

Model Reference Adaptive Control

Objective: $$\lim_{t\to\infty} \varepsilon^0(t+1) = \lim_{t\to\infty} P(q^{-1})\left[y(t+1) - y^*(t+1)\right] = 0$$

Adjustable controller: $$u(t) = \frac{Py^*(t+1) - \hat{b}_2(t)u(t-1) - \hat{r}_0(t)y(t)}{\hat{b}_1(t)}$$

$$P(q^{-1})y^*(t+1) = \hat{\theta}^T(t)\phi(t)$$

$$\hat{\theta}^T(t) = \left[\hat{b}_1(t), \hat{b}_2(t), \hat{r}_0(t)\right];\ \phi^T(t) = \left[u(t), u(t-1), y(t)\right]$$

But for the correct values of controller parameters one has:

$$P(q^{-1})y(t+1) = P(q^{-1})y^*(t+1) = \theta^T\phi(t)$$

And therefore one has: $$\varepsilon^0(t+1) = \left[\theta - \hat{\theta}(t)\right]^T\phi(t)$$

Define the a posteriori adaptation error: $$\varepsilon(t+1) = \left[\theta - \hat{\theta}(t+1)\right]^T\phi(t)$$

Use P.A.A.

However one should show in addition that $\|\phi(t)\|$ is bounded (i.e. plant input and output are bounded)

Plant model estimation + computation of the controller (indirect)

Step 1 : Plant model estimation

Plant model (unknown): $y(t+1) = -a_1 y(t) + b_1 u(t) + b_2 u(t-1) = \theta_P^T \phi(t)$

Adjustable predictor:

$$\hat{y}^0(t+1) = -\hat{a}_1(t) y(t) + \hat{b}_1(t) u(t) + \hat{b}_2(t) u(t-1) = \hat{\theta}_P^T(t)\phi(t)$$

$$\hat{\theta}_P^T(t) = \left[\hat{b}_1(t), \hat{b}_2(t), -\hat{a}_1(t)\right]; \; \phi^T(t) = \left[u(t), u(t-1), y(t)\right]$$

a priori prediction error: $\varepsilon^0(t+1) = y(t+1) - \hat{y}^0(t+1) = \left[\theta_P - \hat{\theta}_P(t)\right]^T \phi(t)$

a posteriori prediction error: $\varepsilon(t+1) = y(t+1) - \hat{y}(t+1) = \left[\theta_P - \hat{\theta}_P(t+1)\right]^T \phi(t)$

Use PAA

Step 2 : Computation of the controller

Compute at each instant t : $\hat{r}_0(t) = p_1 - \hat{a}_1(t)$

Adjustable controller: $P(q^{-1}) y^*(t+1) = \hat{\theta}^T(t)\phi(t)$

From $\hat{\theta}_P(t)$

$$\phi^T(t) = \left[u(t), u(t-1), y(t)\right]; \; \hat{\theta}^T(t) = \left[\hat{b}_1(t), \hat{b}_2(t), \hat{r}_0(t)\right]$$

In the general case d > 0 one will have to solve equation (**)

Re-parametrized plant model estimation (direct)

Plant model (unknown):

$$y(t+1) = -a_1 y(t) + b_1 u(t) + b_2 u(t-1) \pm p_1 y(t)$$
$$= -p_1 y(t) + (\underbrace{p_1 - a_1}_{r_0}) y(t) + b_1 u(t) + b_2 u(t-1) = -p_1 y(t) + \theta^T \phi(t)$$

Re-parametrized adjustable predictor:

$$\hat{y}^0(t+1) = -p_1 y(t) + \hat{r}_0(t) y(t) + \hat{b}_1(t) u(t) + \hat{b}_2(t) u(t-1)$$
$$= -p_1 y(t) + \hat{\theta}^T(t)\phi(t)$$
$$\hat{\theta}^T(t) = \left[\hat{b}_1(t), \hat{b}_2(t), r_0(t)\right]; \ \phi^T(t) = \left[u(t), u(t-1), y(t)\right]$$

a priori prediction error:

$$\varepsilon^0(t+1) = y(t+1) - \hat{y}^0(t+1) = \left[\theta - \hat{\theta}(t)\right]^T \phi(t)$$

a posteriori prediction error:

$$\varepsilon(t+1) = y(t+1) - \hat{y}(t+1) = \left[\theta - \hat{\theta}(t+1)\right]^T \phi(t)$$

Use PAA

One estimates directly the parameters of the controller

One has:

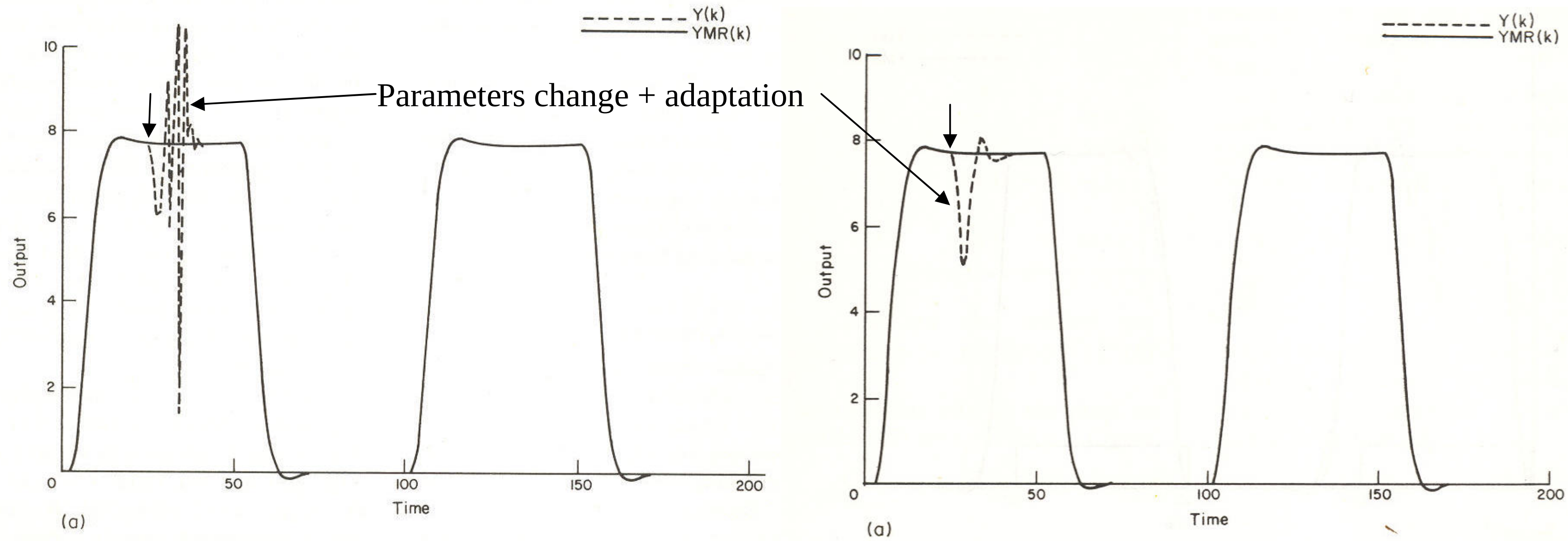
$$\lim_{t\to\infty} \varepsilon^0(t+1) = \lim_{t\to\infty} P(q^{-1})\left[y(t+1) - y^*(t+1)\right] = 0$$

Adaptive tracking and regulation with independent objectives

- Easy generalization for the case d > 0
- Elegant and simple solution for adaptation (direct)
- **Unfortunately restricted use in practice because it requires that the plant zeros remains always stable and well damped**

Direct Adaptive Control – Simulations results

Tracking

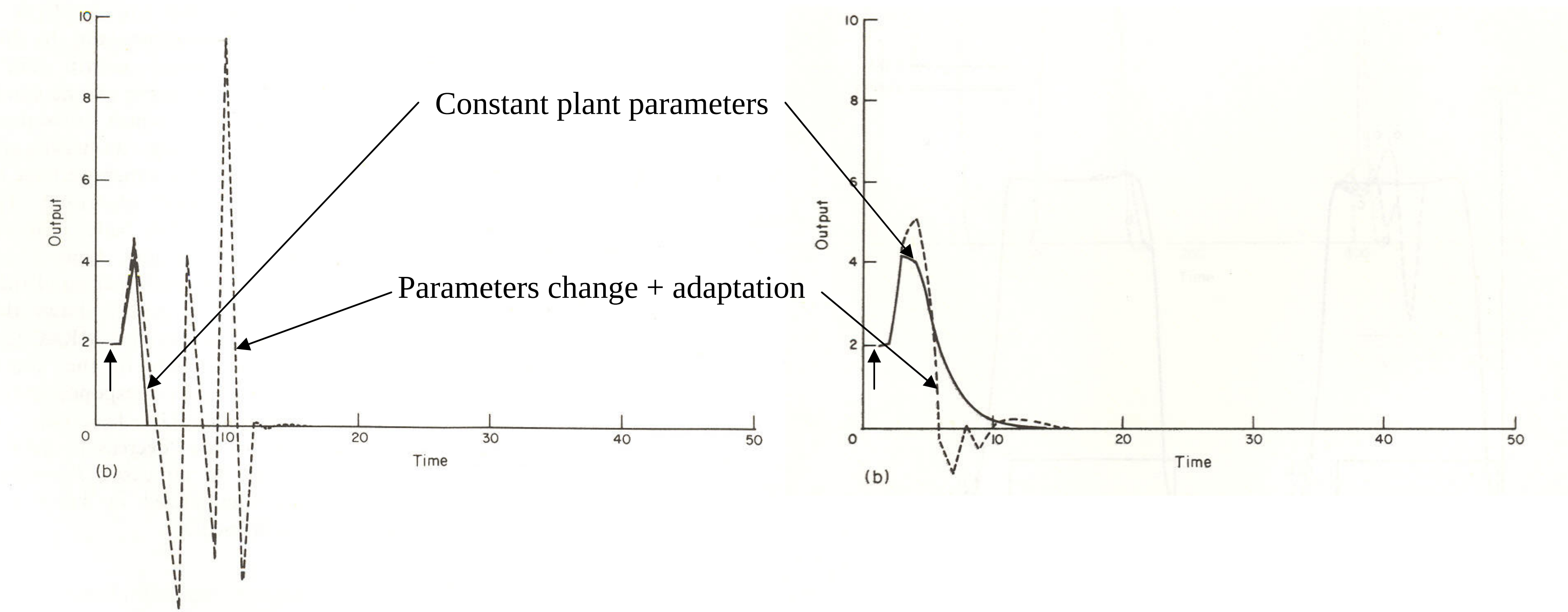


$$P(q^{-1}) = 1 \qquad\qquad P(q^{-1}) = (1 - 0.4q^{-1})$$

The choice of the poles for the closed loop (regulation) has a great influence upon adaptation transient behavior!

Direct Adaptive Control – Simulations results

Regulation



$$P(q^{-1}) = 1 \qquad\qquad P(q^{-1}) = (1 - 0.4q^{-1})$$

The choice of the poles for the closed loop (regulation) has a great influence upon adaption transient behavior!

Indirect Adaptive Control
of non-necessarily zeros- stable plants

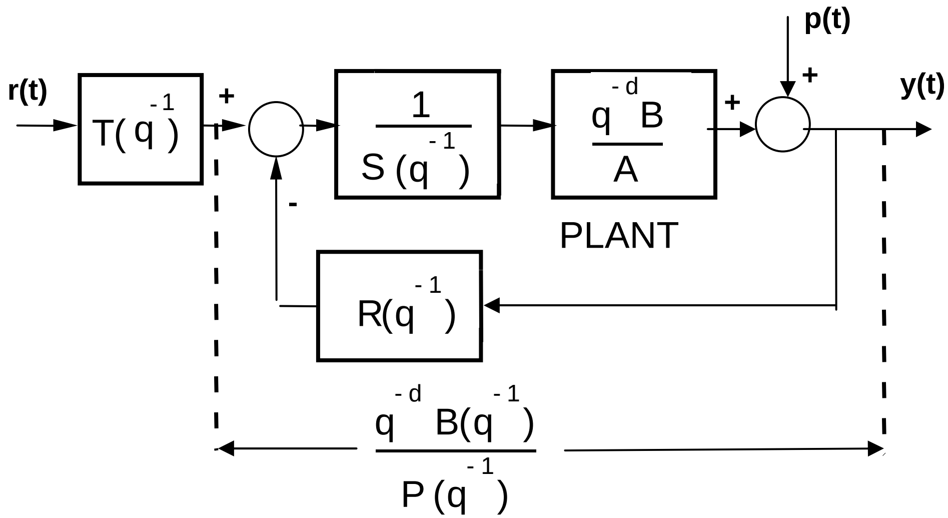
Pole placement

The pole placement allows to design a R-S-T controller for

- stable or unstable systems
- without restriction upon the degrees of A and B polynomials
- without restrictions upon the plant model zeros (stable or unstable)
- but A and B polynomials should not have common factors (controllable/observable model for design)

It is a method that does not simplify the plant model zeros

Structure



Plant:

$$H(q^{-1}) = \frac{q^{-d}B(q^{-1})}{A(q^{-1})}$$

$$A(q^{-1}) = 1 + a_1 q^{-1} + ... + a_{n_A} q^{-n_A} \qquad B(q^{-1}) = b_1 q^{-1} + b_2 q^{-2} + ... + b_{n_B} q^{-n_B} = q^{-1} B^*(q^{-1})$$

Pole placement

Closed loop T.F. (r $\rightarrow$ y) *(reference tracking)*

$$H_{BF}(q^{-1}) = \frac{q^{-d}T(q^{-1})B(q^{-1})}{A(q^{-1})S(q^{-1}) + q^{-d}B(q^{-1})R(q^{-1})} = \frac{q^{-d}T(q^{-1})B(q^{-1})}{P(q^{-1})}$$

$$P(q^{-1}) = A(q^{-1})S(q^{-1}) + q^{-d}B(q^{-1})R(q^{-1}) = 1 + p_1 q^{-1} + p_2 q^{-2} + \ldots.$$

Defines the (desired)closed loop poles

Closed loop T.F. (p $\rightarrow$ y) *(disturbance rejection)*

$$S_{yp}(q^{-1}) = \frac{A(q^{-1})S(q^{-1})}{A(q^{-1})S(q^{-1}) + q^{-d}B(q^{-1})R(q^{-1})} = \frac{A(q^{-1})S(q^{-1})}{P(q^{-1})}$$

Output sensitivity function

Choice of desired closed loop poles (polynomial *P*)

$$P(q^{-1}) = P_D(q^{-1})P_F(q^{-1})$$

Dominant poles **Auxiliary poles**

Choice of $P_D(q^{-1})$(dominant poles)

Specification in continuous time (t_M, M) $\longrightarrow$ 2^{nd} order (ω_0, ζ) $\xrightarrow[T_e]{\text{discretization}}$ $P_D(q^{-1})$

$$0.25 \le \omega_0 T_e \le 1.5$$
$$0.7 \le \zeta \le 1$$

Auxiliary poles

- *Auxiliary poles are introduced for robustness purposes*
- *They usually are selected to be faster than the dominant poles*

Regulation(computation of $R(q^{-1})$ and $S(q^{-1})$)

(Bezout) $$A(q^{-1})S(q^{-1}) + q^{-d}B(q^{-1})R(q^{-1}) = P(q^{-1}) \quad \textbf{(*)}$$

? **?**

A and B do not have common factors

$$n_A = \deg A(q^{-1}) \quad n_B = \deg B(q^{-1})$$

unique minimal solution for :

$$n_P = \deg P(q^{-1}) \le n_A + n_B + d - 1$$

$$n_S = \deg S(q^{-1}) = n_B + d - 1 \qquad n_R = \deg R(q^{-1}) = n_A - 1$$

$$S(q^{-1}) = 1 + s_1 q^{-1} + \dots s_{n_S} q^{-n_S} = 1 + q^{-1} S^*(q^{-1})$$

$$R(q^{-1}) = r_0 + r_1 q^{-1} + \dots r_{n_R} q^{-n_R}$$

Computation of *R(q-1)* and *S(q-1)*

Equation (*) is written as: $\boxed{Mx = p} \longrightarrow x = M^{-1}p$

$$x^T = [1, s_1, \ldots, s_{n_S}, r_0, \ldots, r_{n_R}] \qquad p^T = [1, p_1, \ldots, p_i, \ldots, p_{n_P}, 0, \ldots, 0]$$

$$\begin{array}{cc}
\begin{array}{c}\overbrace{\hspace{10em}}^{n_B + d}\end{array} & \begin{array}{c}\overbrace{\hspace{10em}}^{n_A}\end{array} \\
\end{array}$$

$$\underbrace{\left[\begin{array}{cccc|cccc}
1 & 0 & \ldots & 0 & 0 & \ldots & \ldots & 0 \\
a_1 & 1 & & . & b'_1 & & & \\
a_2 & & & 0 & b'_2 & & & b'_1 \\
 & & & 1 & . & & & b'_2 \\
 & & & a_1 & . & & & . \\
a_{n_A} & & & a_2 & b'_{n_B} & & & . \\
0 & & & . & 0 & . & . & . \\
0 & \ldots & 0 & a_{n_A} & 0 & 0 & 0 & b'_{n_B}
\end{array}\right]}_{n_A + n_B + d} \left.\vphantom{\begin{array}{c}1\\1\\1\\1\\1\\1\\1\\1\end{array}}\right\} n_A + n_B + d$$

$b'_i = 0$ pour $i = 0, 1 \ldots d$; $b'_i = b_{i-d}$ pour $i > d$

Use of WinReg or *bezoutd.sci(.m)* for solving (*)

Structure of *R(q^-1)* and *S(q^-1)*

R and S may include pre-specified fixed parts (ex: integrator)

$$R(q^{-1}) = R'(q^{-1})H_R(q^{-1}) \qquad\qquad S(q^{-1}) = S'(q^{-1})H_S(q^{-1})$$

H_R, H_S, - pre-specified polynomials

$$R'(q^{-1}) = r'_0 + r'_1 q^{-1} + \dots r'_{n_{R'}} q^{-n_{R'}} \quad S'(q^{-1}) = 1 + s'_1 q^{-1} + \dots s'_{n_{S'}} q^{-n_{S'}}$$

- The pre specified filters H_R and H_S will allow to impose certain properties of the closed loop.
- They can influence performance and/or robustness

$$A(q^{-1})H_S(q^{-1})S'(q^{-1}) + q^{-d}B(q^{-1})H_R(q^{-1})R'(q^{-1}) = P(q^{-1})$$

? ?

Fixed parts (H_R, H_S). Examples

Zero steady state error (S_{yp} should be null at certain frequencies)

$$S_{yp}(q^{-1}) = \frac{A(q^{-1})H_S(q^{-1})S'(q^{-1})}{P(q^{-1})}$$

Step disturbance : $H_S(q^{-1}) = 1 - q^{-1}$

Sinusoidal disturbance : $H_S = 1 + \alpha q^{-1} + q^{-2}$; $\alpha = -2\cos\omega T_s$

Signal blocking (S_{up} should be null at certain frequencies)

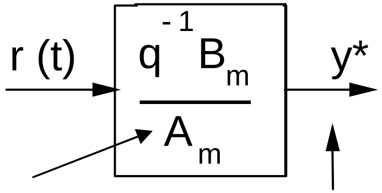
$$S_{up}(q^{-1}) = -\frac{A(q^{-1})H_R(q^{-1})R'(q^{-1})}{P(q^{-1})}$$

Sinusoidal signal: $H_R = 1 + \beta q^{-1} + q^{-2}$; $\beta = -2\cos\omega T_s$

Blocking at 0.5f_S: $H_R = (1 + q^{-1})^n$; $n = 1,2$

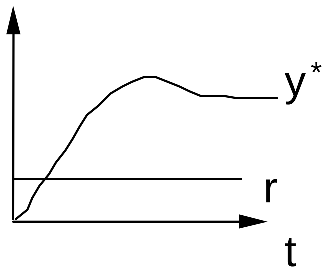
Tracking (computation of $T(q^{-1})$)

Ideal case



Tracking reference model (H_m)

desired trajectory for y (t)



$$H_m(q^{-1}) = \frac{B_m(q^{-1})}{A_m(q^{-1})}$$

$$B_m(q^{-1}) = b_{m0} + b_{m1}q^{-1} + \dots$$

$$A_m(q^{-1}) = 1 + a_{m1}q^{-1} + a_{m2}q^{-2} + \dots$$

Specification in continuous time (t_M, M) $\longrightarrow$ 2^{nd} order (ω_0, ζ) $\xrightarrow[T_s]{\text{discretization}}$ $H_m(q^{-1})$

$$0.25 \le \omega_0 T_s \le 1.5$$

$$0.7 \le \zeta \le 1$$

The ideal case can not be obtained (delay, plant zeros)

Objective : to approach $y^*(t)$

$$y^*(t) = \frac{q^{-(d+1)}B_m(q^{-1})}{A_m(q^{-1})} r(t)$$

Tracking (computation of $T(q^{-1})$)

Build: $$y^*(t+d+1) = \frac{B_m(q^{-1})}{A_m(q^{-1})} r(t)$$

Choice of $T(q^{-1})$:

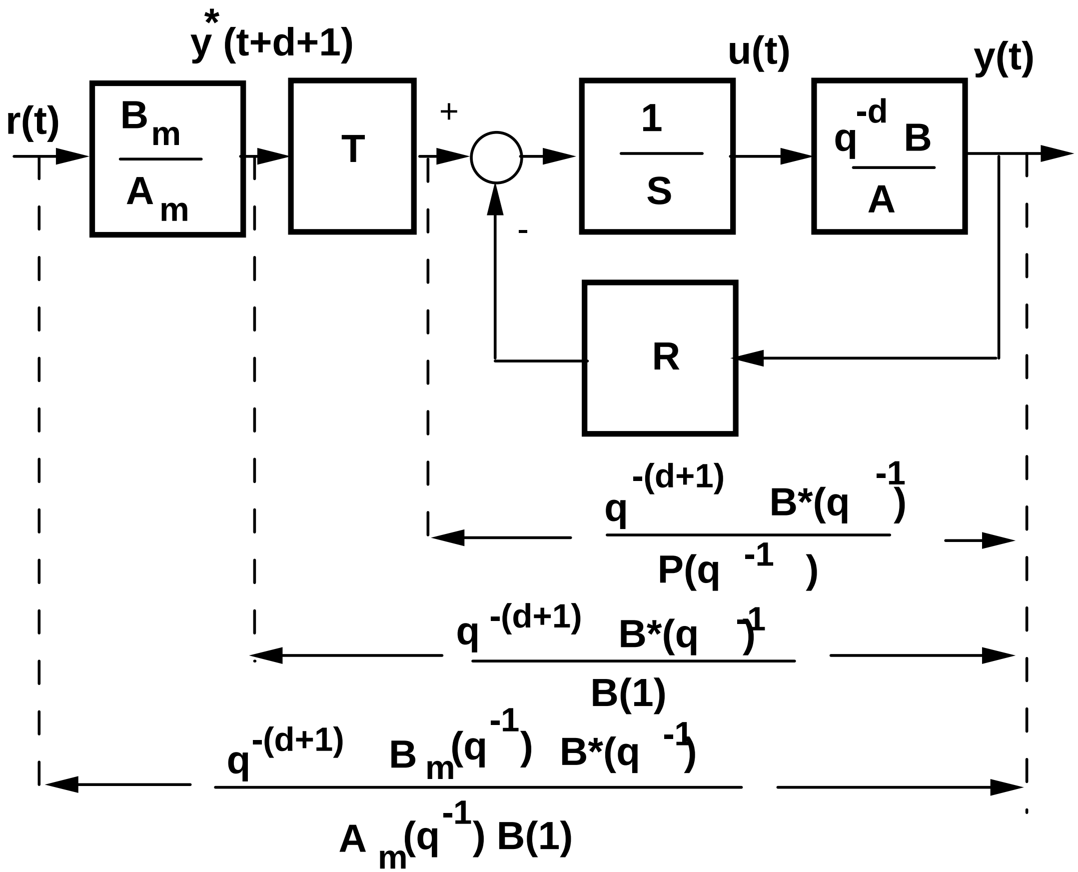
- Imposing unit static gain between y^* and y
- Compensation of regulation dynamics $P(q^{-1})$

$$T(q^{-1}) = GP(q^{-1}) \qquad G = \begin{cases} 1/B(1) & if \quad B(1) \neq 0 \\ 1 & if \quad B(1) = 0 \end{cases}$$

F.T. $r \longrightarrow y$: $$H_{BF}(q^{-1}) = \frac{q^{-(d+1)} B_m(q^{-1})}{A_m(q^{-1})} \cdot \frac{B^*(q^{-1})}{B(1)}$$

$$Particular\ case : P = A_m \qquad T(q^{-1}) = G = \begin{cases} \dfrac{P(1)}{B(1)} & if \quad B(1) \neq 0 \\ 1 & if \quad B(1) = 0 \end{cases}$$

Pole placement. Tracking and regulation



$$S(q^{-1})u(t) + R(q^{-1})y(t) = T(q^{-1})y^*(t+d+1)$$

Pole placement. Control law

$$u(t) = \frac{T(q^{-1})y^*(t+d+1) - R(q^{-1})y(t)}{S(q^{-1})}$$

$$S(q^{-1})u(t) + R(q^{-1})y(t) = GP(q^{-1})y^*(t+d+1) = T(q^{-1})y^*(t+d+1)$$

$$S(q^{-1}) = 1 + q^{-1}S^*(q^{-1})$$

$$\boxed{u(t) = P(q^{-1})Gy^*(t+d+1) - S^*(q^{-1})u(t-1) - R(q^{-1})y(t)}$$

$$y^*(t+d+1) = \frac{B_m(q^{-1})}{A_m(q^{-1})}r(t)$$

$$A_m(q^{-1}) = 1 + q^{-1}A_m^*(q^{-1})$$

$$\boxed{y^*(t+d+1) = -A_m^*(q^{-1})y(t+d) + B_m(q^{-1})r(t)}$$

$$B_m(q^{-1}) = b_{m0} + b_{m1}q^{-1} + \ldots \qquad A_m(q^{-1}) = 1 + a_{m1}q^{-1} + a_{m2}q^{-2} + \ldots$$

Indirect adaptive control

At each sampling instant:

Step I : Estimation of the plant model $(\hat{A}, \hat{B})$

ARX identification (Recursive Least Squares)

Step II: Computation of the controller

Solving Bezout equation (for S' and R')

$$\hat{A} H_S S' + q^{-d} \hat{B} H_R R' = P$$

Compute:

$$R(q^{-1}) = R'(q^{-1}) H_R(q^{-1}) \qquad S(q^{-1}) = S'(q^{-1}) H_S(q^{-1})$$

$$T = \hat{G} P = \begin{cases} \hat{G} = \dfrac{1}{\hat{B}(1)} & if \quad \hat{B}(1) \neq 0 \\ \hat{G} = 1 \quad if \quad \hat{B}(1) = 0 \end{cases}$$

Remark:

It is time consuming for large dimension of the plant model

Supervision

Estimation:

- Check if input is enough "persistently exciting"
  (if not, do not take in account the estimations)
- Check if $\hat{A}$ and $\hat{B}$ are numerically "sound" (condition number)
  (no close poles/zeros)
- If necessary, add external excitation (testing signal)

Control:

- Check if desired dominant closed loop poles are compatible with estimated plant poles
- Check robustness margins

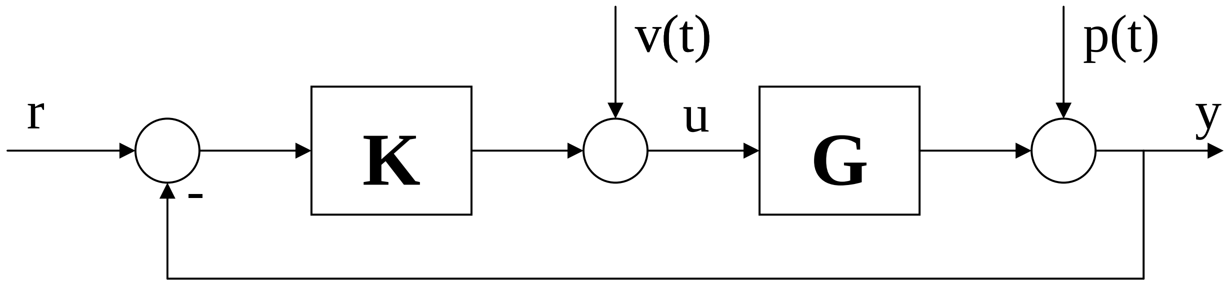
Additional problem:

- How to deal with neglected dynamics ?
  (filtering of the data, robustification of PAA)

Adaptive Control

Part 6: Robust Control Design for Adaptive Control

Notations



$$G(q^{-1}) = \frac{q^{-d}B(q^{-1})}{A(q^{-1})} \qquad K(q^{-1}) = \frac{R(q^{-1})}{S(q^{-1})}$$

Sensitivity functions :

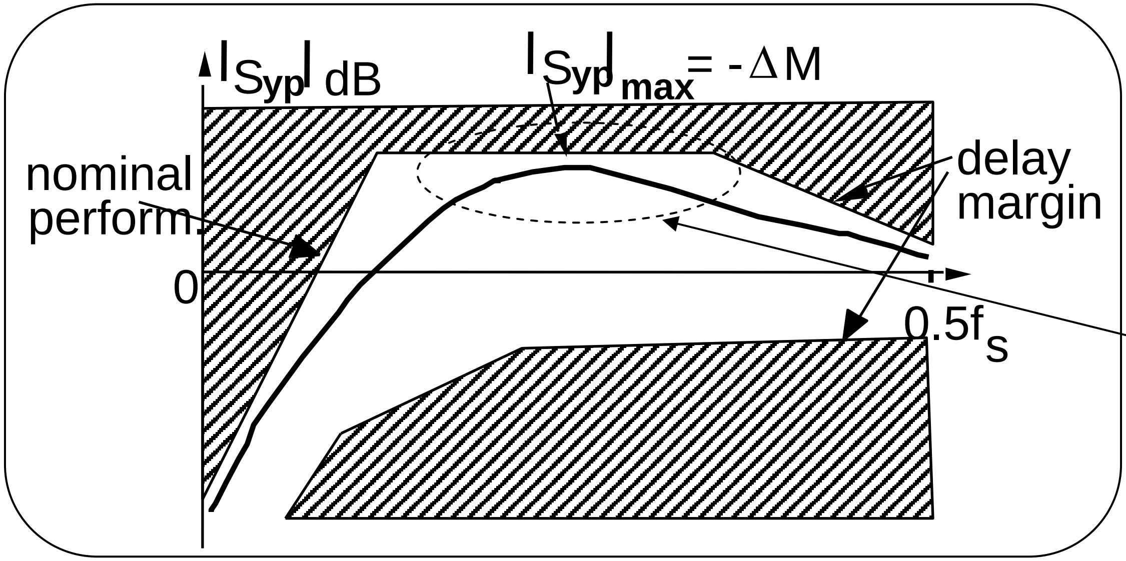
$$S_{yp}(z^{-1}) = \frac{1}{1+KG} \;;\; S_{up}(z^{-1}) = -\frac{K}{1+KG} \;;\; S_{yv}(z^{-1}) = \frac{G}{1+KG} \;;\; S_{yr}(z^{-1}) = \frac{KG}{1+KG}$$

Closed loop poles : $P(z^{-1}) = A(z^{-1})S(z^{-1}) + z^{-d}B(z^{-1})R(z^{-1})$

True closed loop system :(K,G), P, S_{xy}

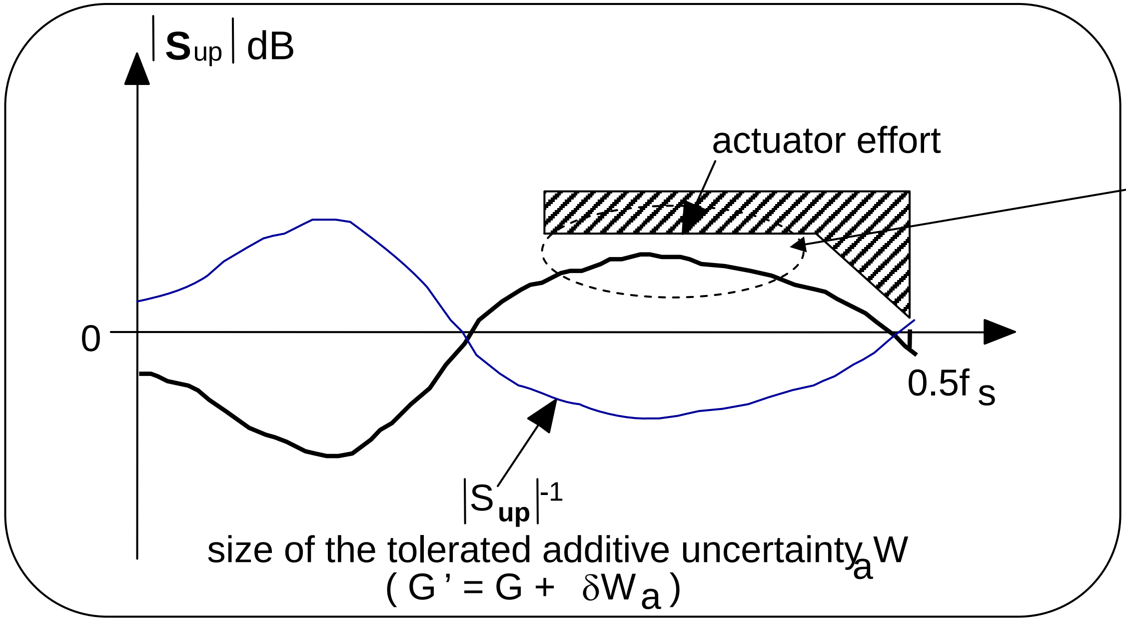
Nominal simulated(estimated) closed loop :$(K, \hat{G}), \hat{P}, \hat{S}_{xy}$

Templates for the Sensitivity Functions



Output Sensitivity Function

Critical frequency region for control



Input Sensitivity Function

Robust Control Design for Adaptive Control

parameter variations (low frequency) → **Adaptation**

unstructured uncertainities (high frequency) → **Robust Design**

Basic rule : The *input sensitivity function* (S_{up}) should be small in medium and high frequencies

How to achieve this ?

Pole Placement :

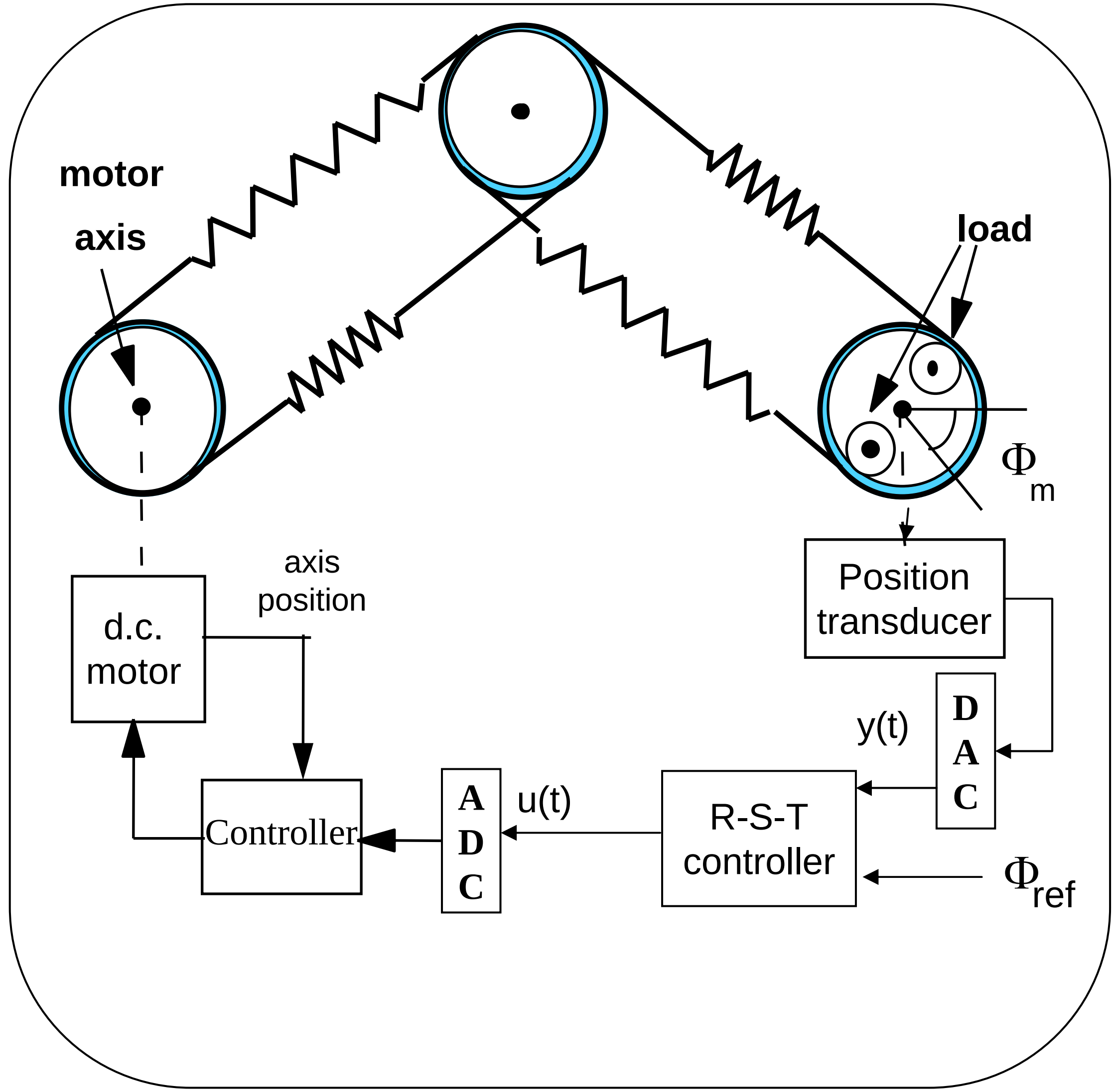
- Opening the loop in high frequencies (at $0.5f_s$)
- Placing auxiliary closed loop poles near the high frequency poles of the plant model

Generalized Predictive Control :

- Appropriate weighting filter on the control term in the criterion

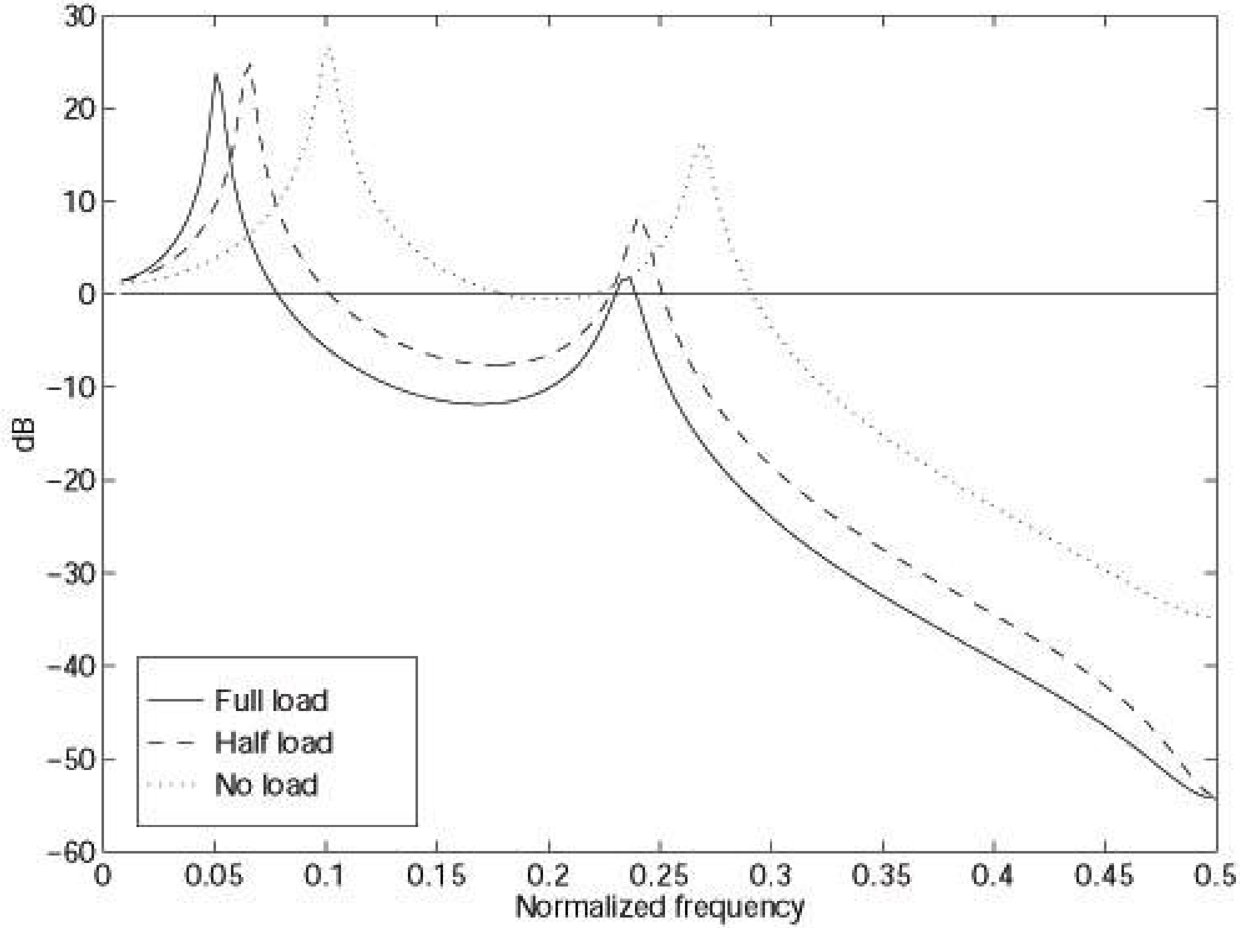
Adaptive Control of a Flexible Transmission

The flexible transmission



Adaptive Control of a Flexible Transmission

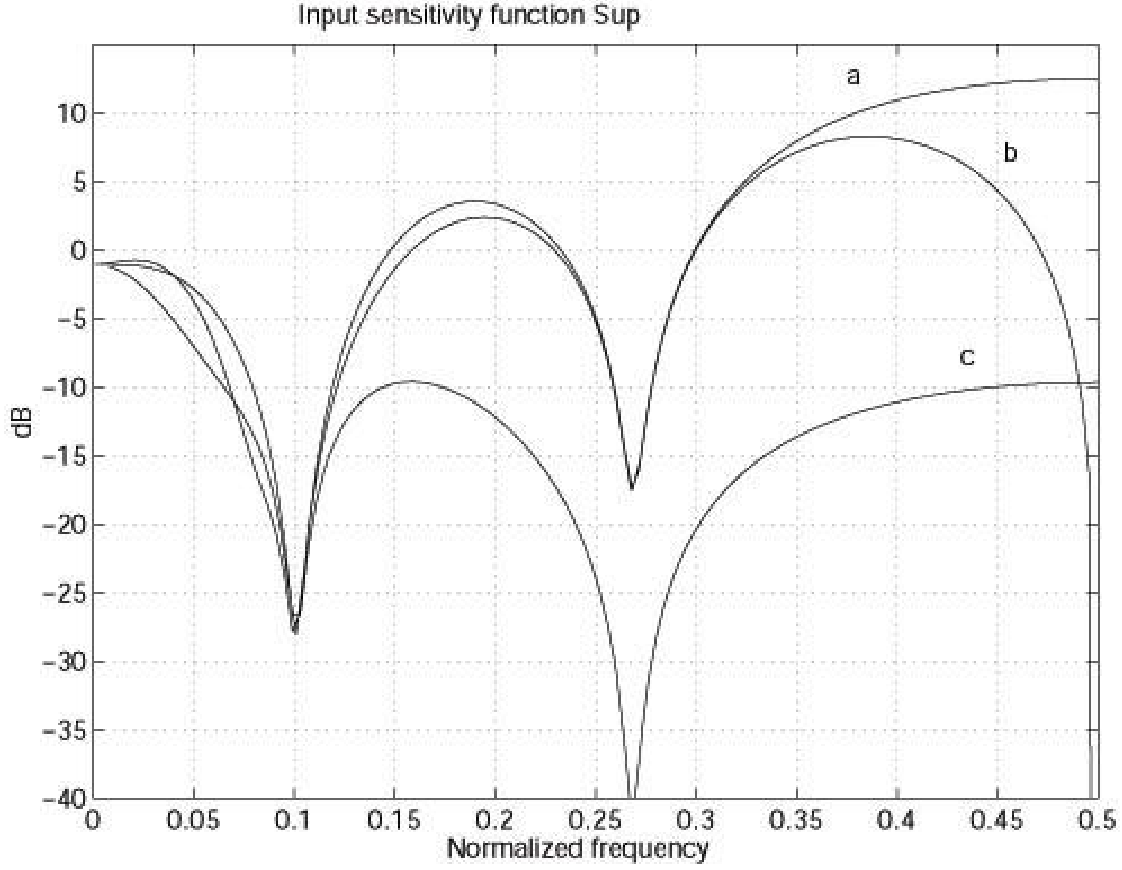
Frequency characteristics for various load



Rem.: the main vibration mode varies by 100%

Robust Control Design for Adaptive Control

(Flexible Transmission)



a) Standard pole placement (1 pair dominant poles + h.f. aperiodic poles)

b) Opening the loop at $0.5f_S$ ($H_R = 1 + q^{-1}$)

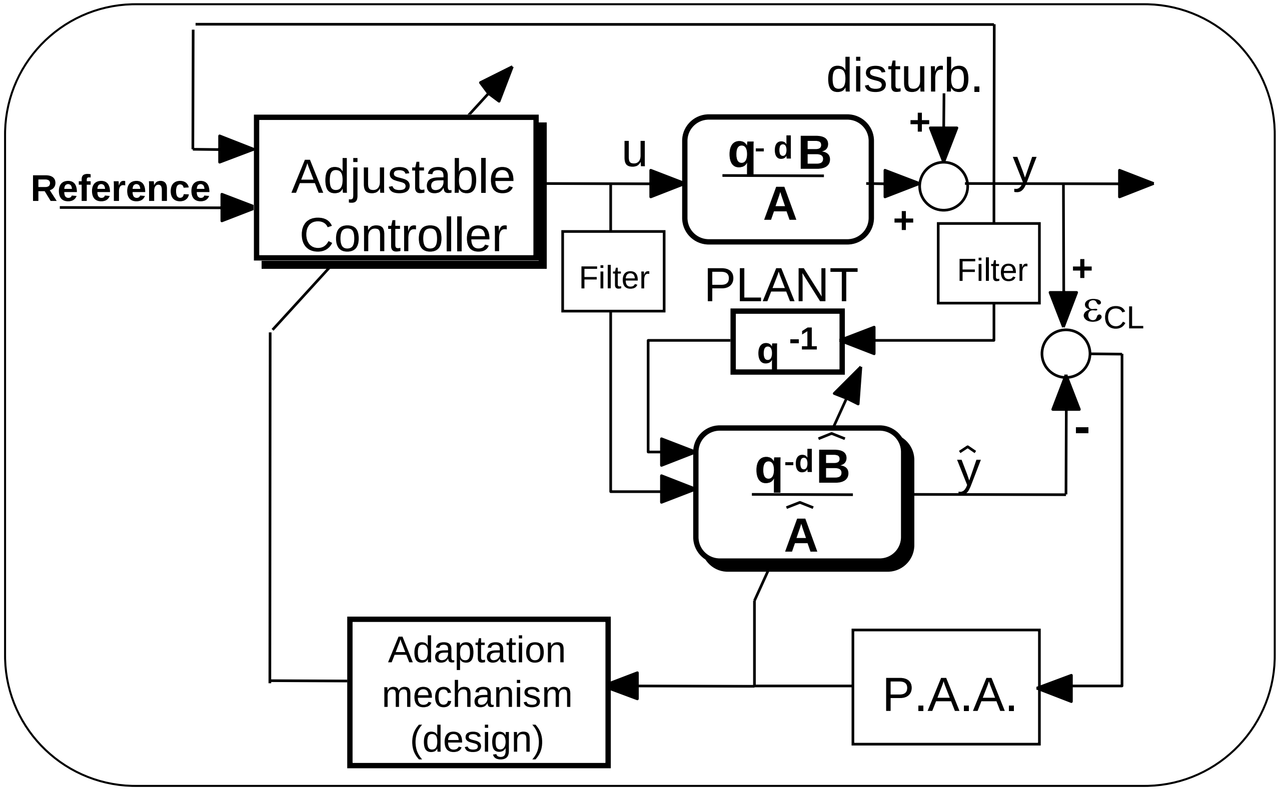
c) Auxiliary closed loop poles near high frequency plant poles

Adaptive Control

Part 7: Parameter Estimators for Adaptive Control

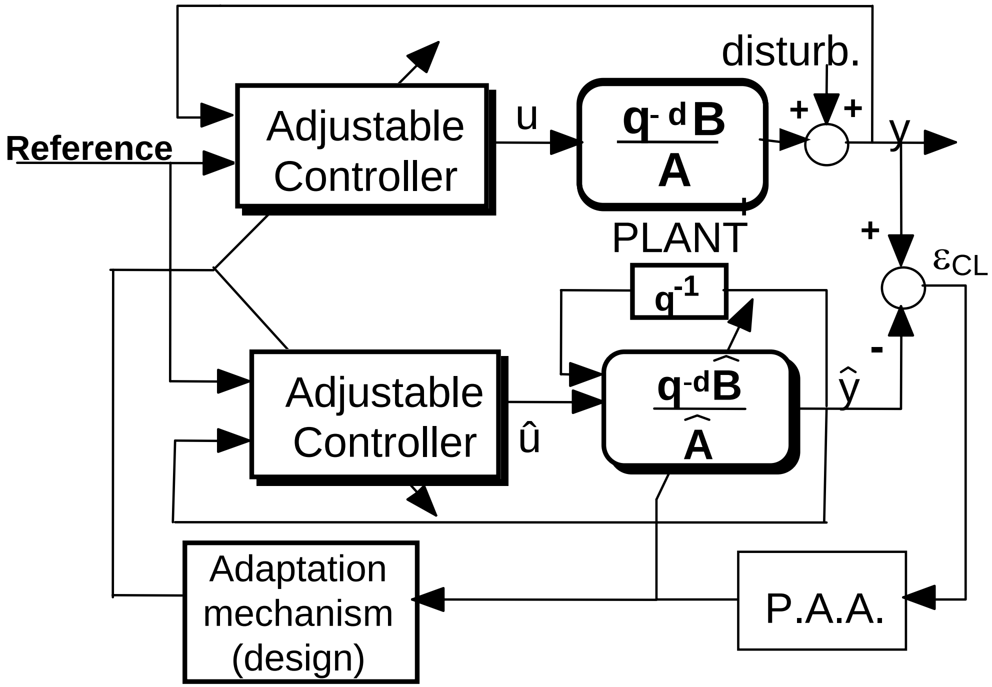
Objective : *to reduce the effect of the disturbances upon the quality of the estimation*

Classical Indirect Adaptive Control



- Uses R.L.S. type estimator (equation error)
- Sensitive to output disturbances
- Requires « adaptation freezing » in the absence of persistent excitation
- The threshold for « adaptation freezing » is problem dependent

Closed Loop Output Error Parameter Estimator for Adaptive Control

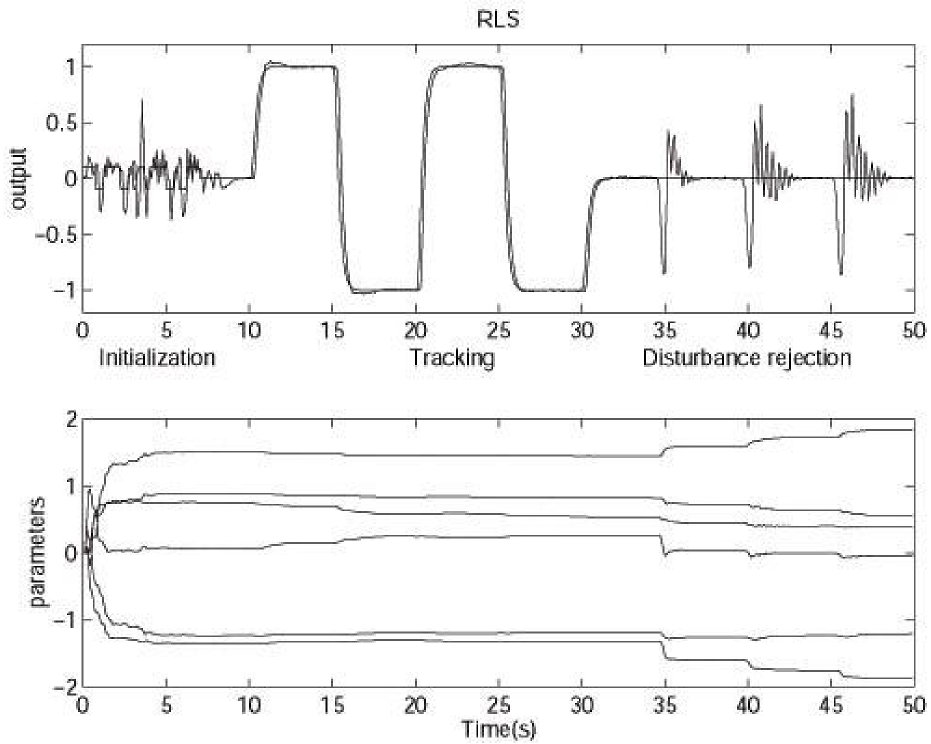


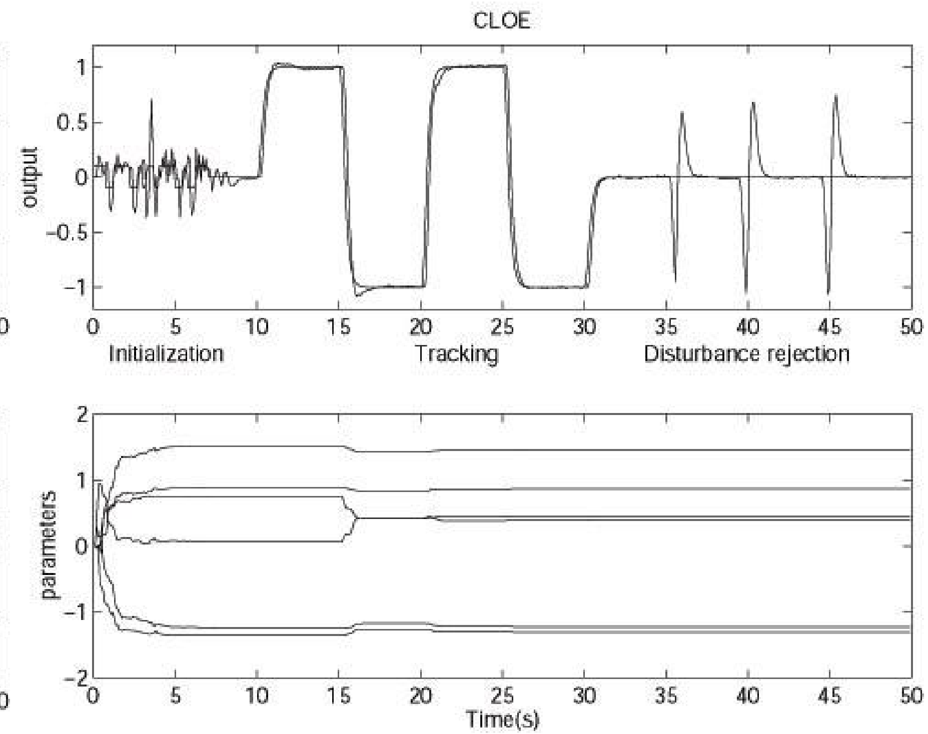
- Insensitive to output disturbances
- Remove the need for « adaptation freezing » in the absence of persistent excitation
- CLOE requires stability of the closed loop
- Well suited for « adaptive control with multiple models »

Adaptive Control – Effect of Disturbances

Classical parameter estimator
(filtered RLS)

CLOE parameter estimator



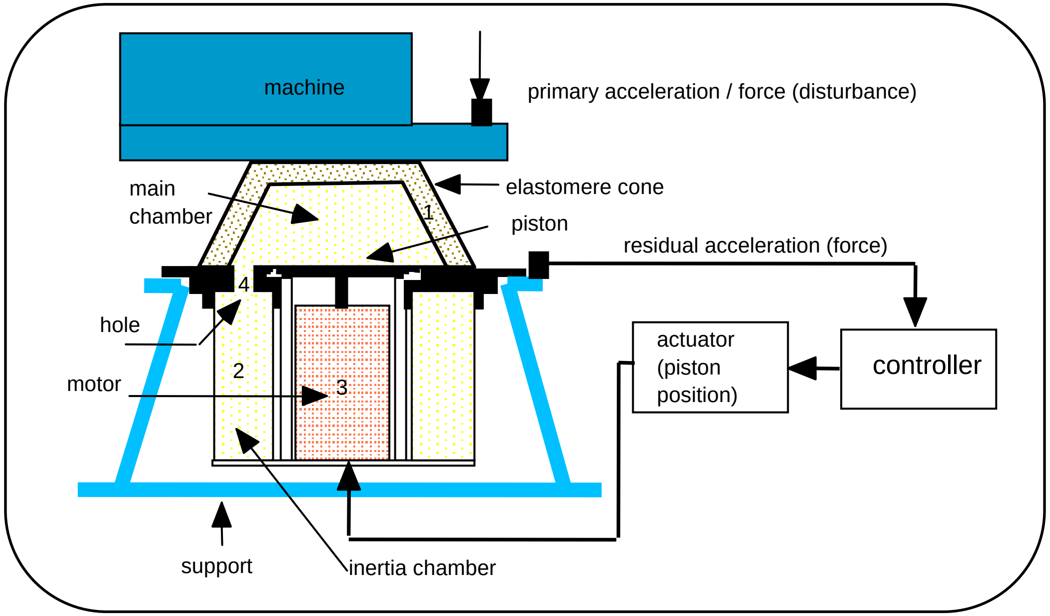


Disturbances destabilize the adaptive system when using RLS parameter estimator (in the absence of a variable reference signal)

Adaptive Regulation

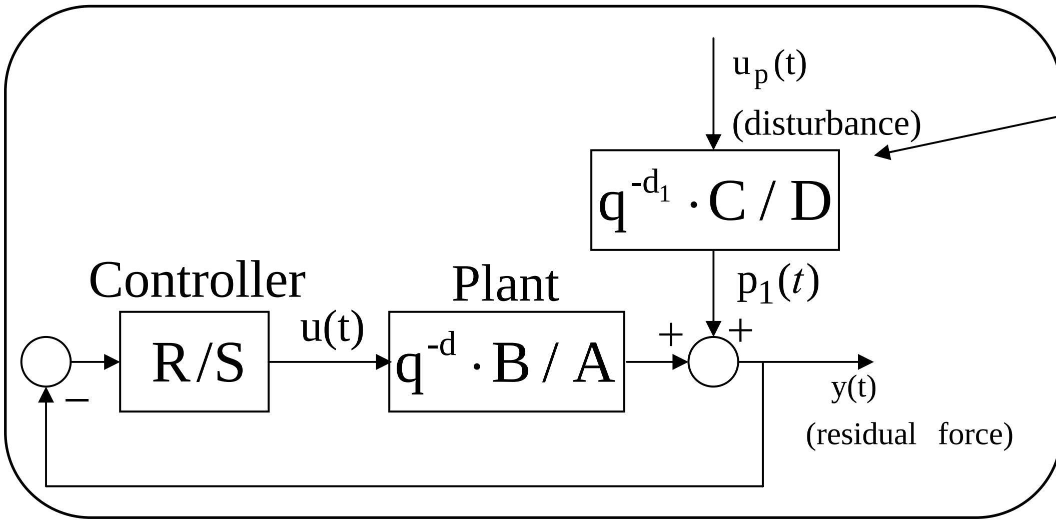
Part 8: Rejection of unknown narrow band disturbances.
Application to active vibration control

The Active Suspension System



Objective:

- **Reject the effect of unknown and variable narrow band disturbances**
- **Do not use an aditional measurement**

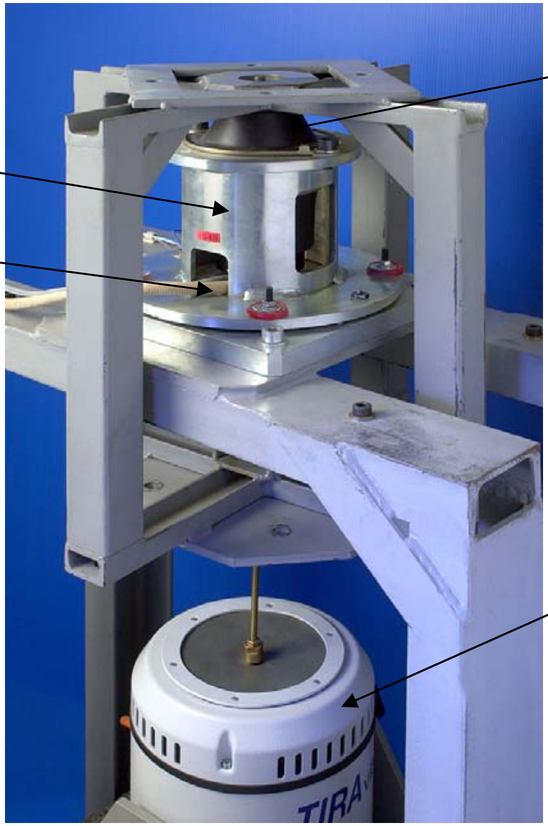


Two paths :

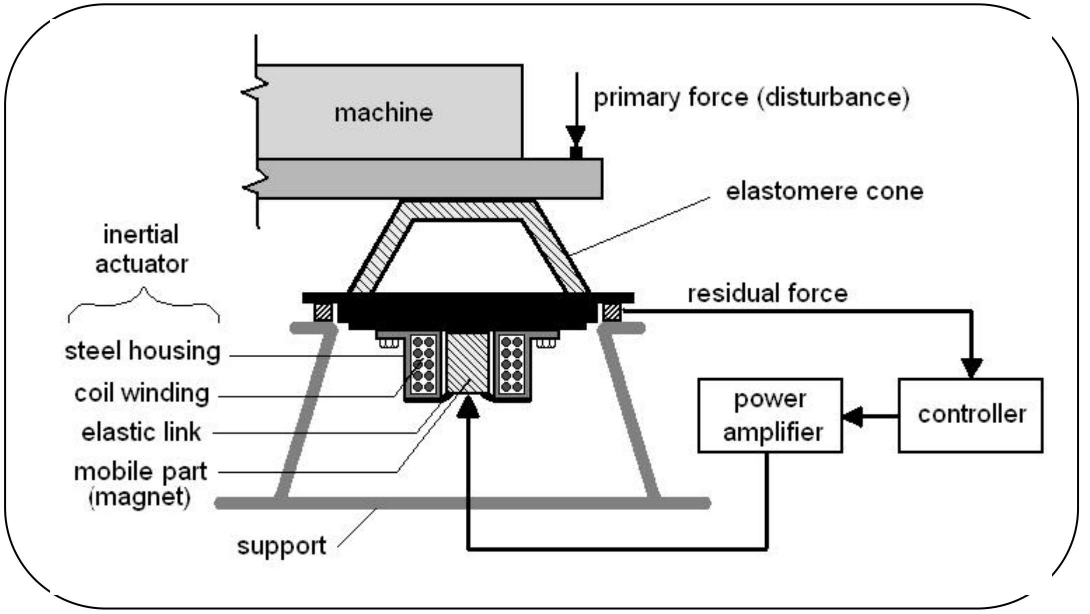
- Primary
- Secondary (double differentiator)

$$T_s = 1.25\,\mathrm{m}s$$

The Active Suspension



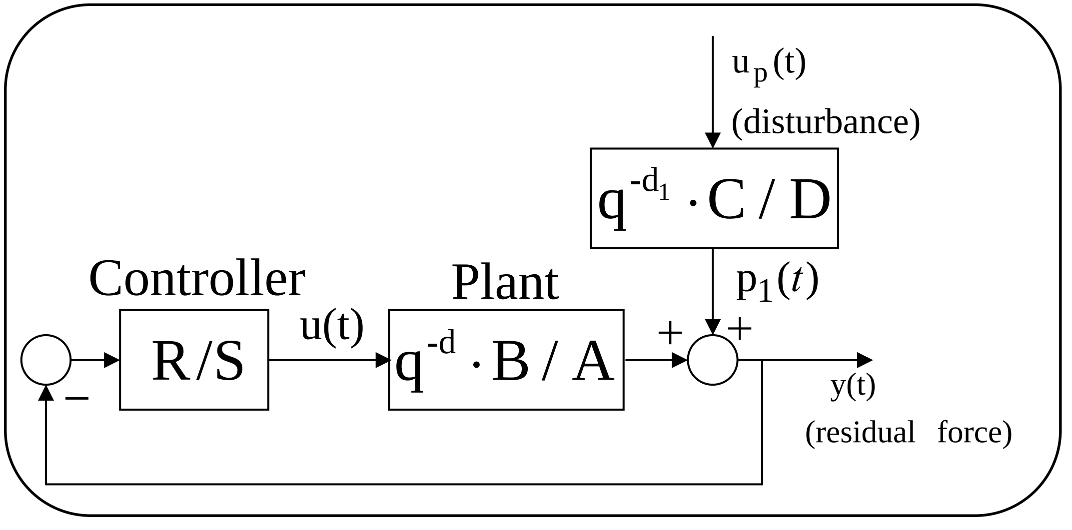
The Active Vibration Control with Inertial Actuator



Objective:

- **Reject the effect of unknown and variable narrow band disturbances**
- **Do not use an aditional measurement**

Same control approach but different technology



Two paths :

- Primary
- Secondary (double differentiator)

$$T_s = 1.25\,\mathrm{m}s$$

View of the active vibration control with inertial actuator

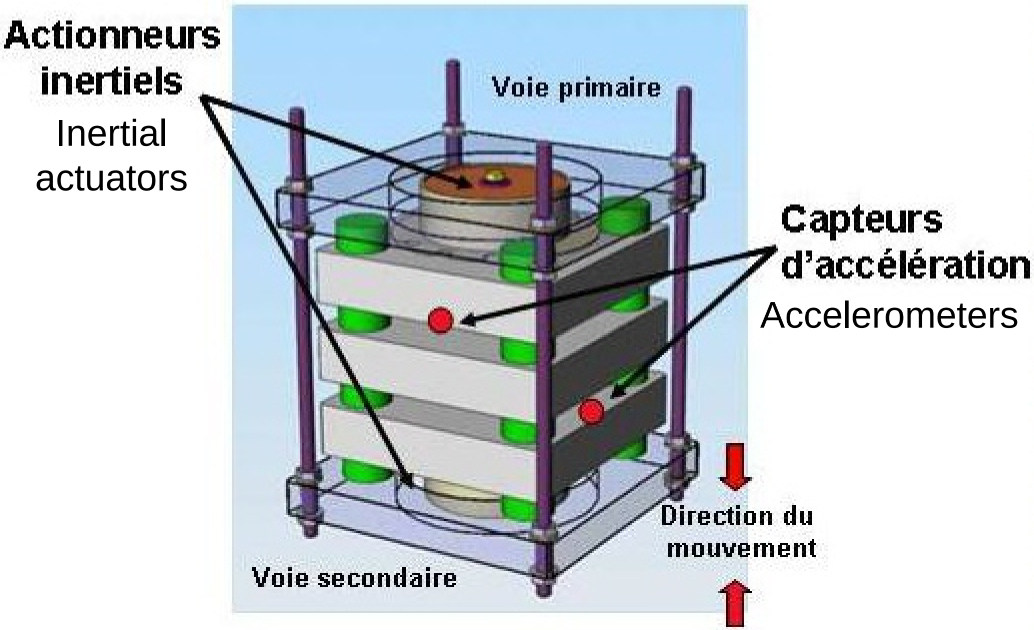
Residual
force
(acceleration)
measurement

Passive damper

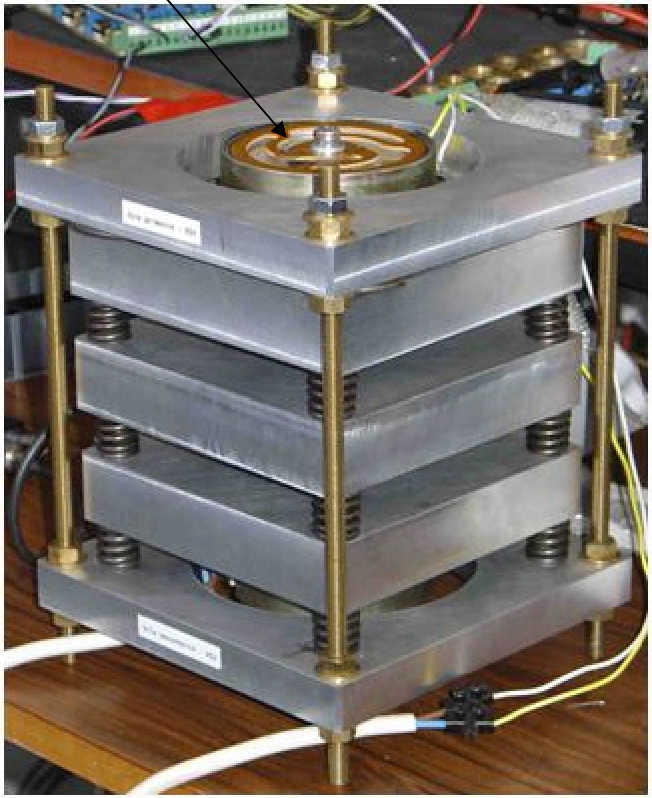
Inertial actuator
(not visible)

Primary force
(acceleration)
(the shaker)

View of a flexible controlled structure using inertial actuators



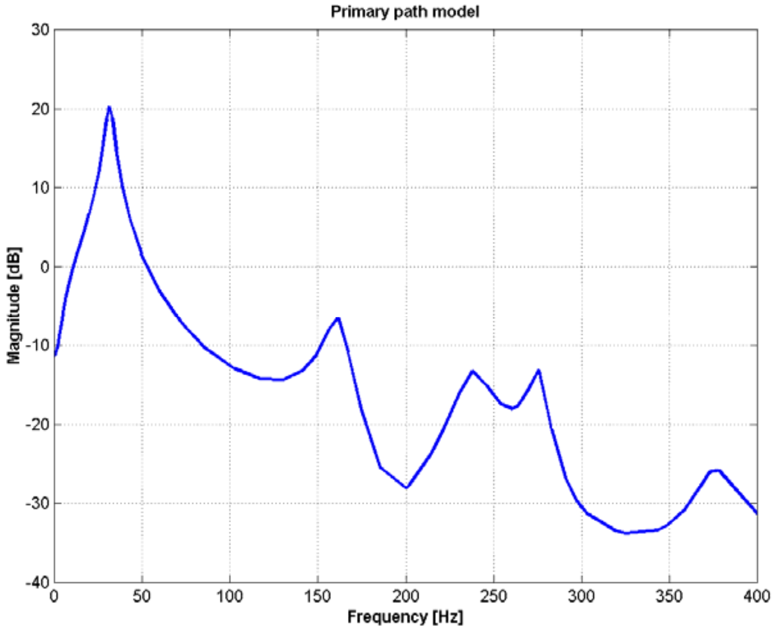
Inertial actuator



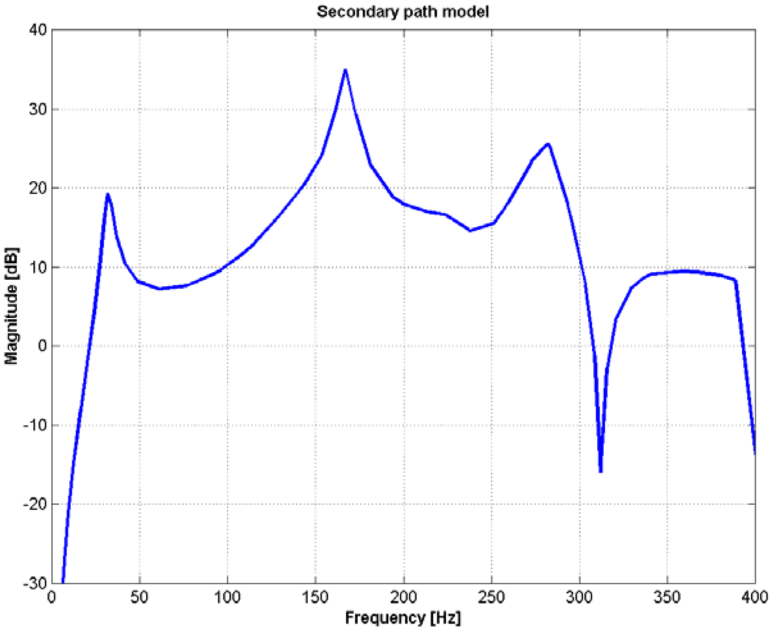
Active Suspension

Frequency Characteristics of the Identified Models

Primary path



Secondary path



$$n_A = 14\,;\, n_B = 16\,;\, d = 0$$

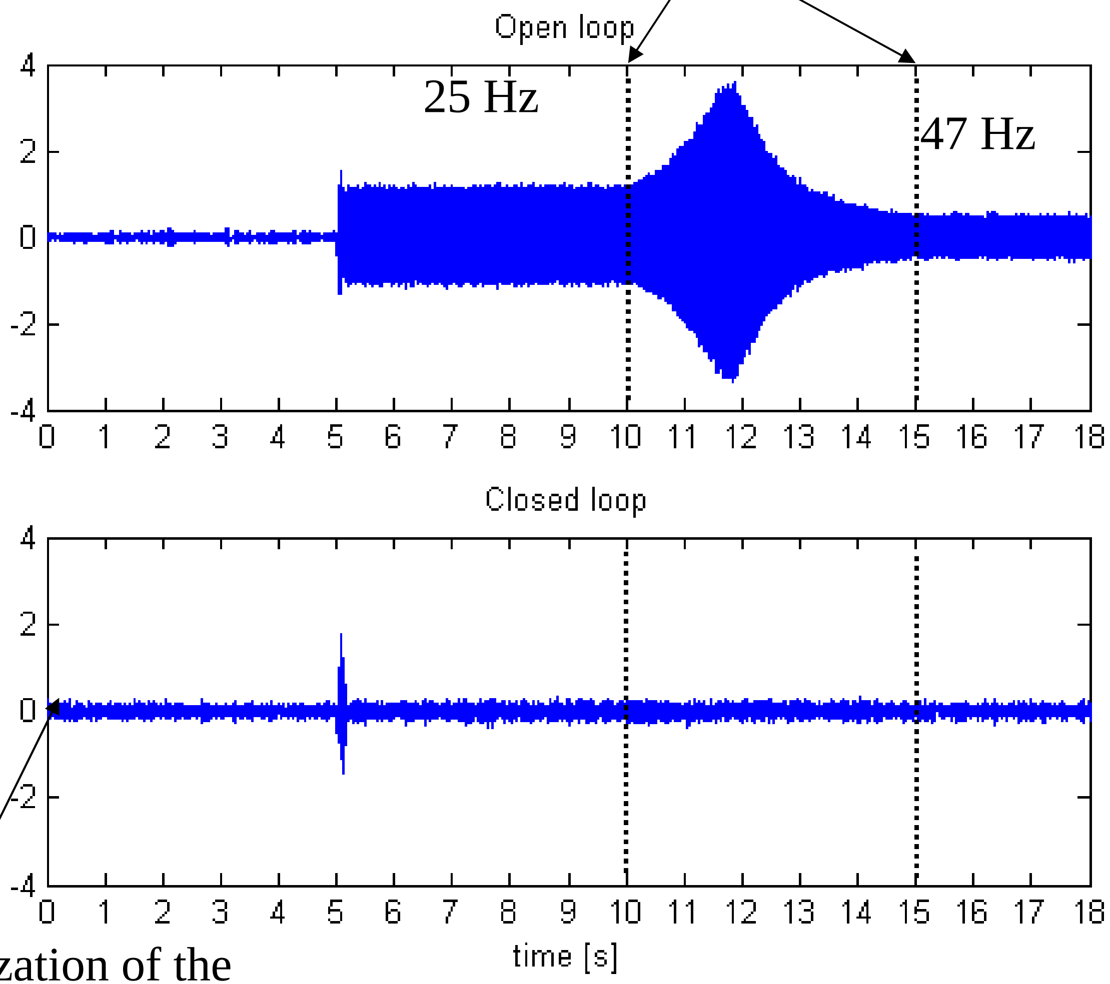
Further details can be obtained from : http://iawww.epfl.ch/News/EJC_Benchmark/

Direct Adaptive Control : disturbance rejection

Disturbance : Chirp

Open loop

Closed loop

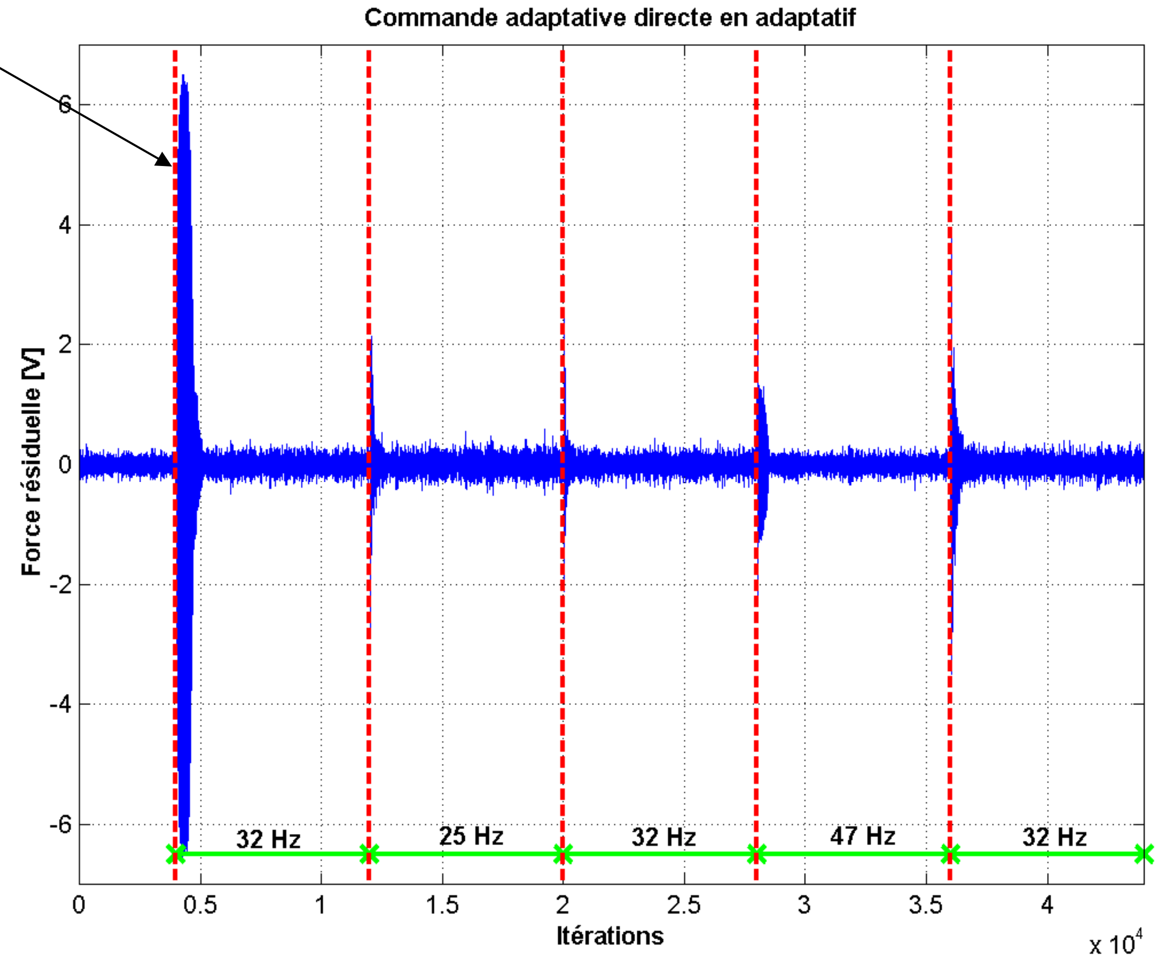


Initialization of the adaptive controller

Time Domain Results
Adaptive Operation

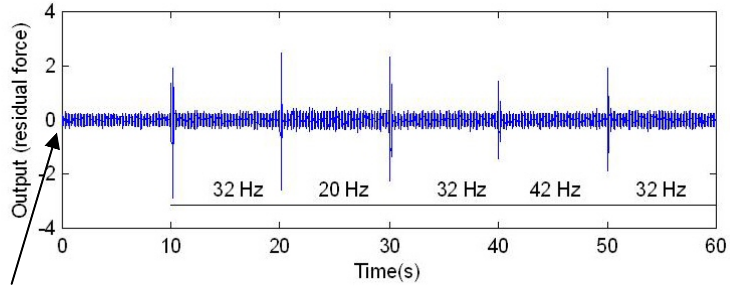
Simultaneous controller initialization and disturbance application

Direct adaptive control



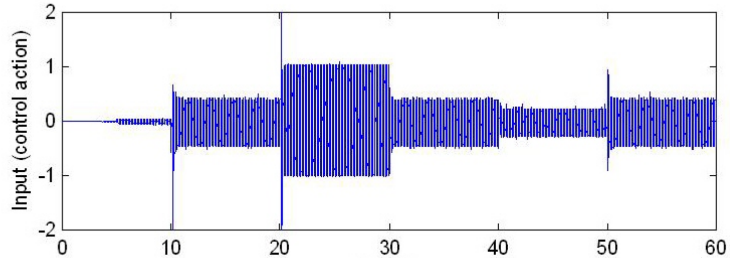
Direct Adaptive Control

Step frequency changes



Output

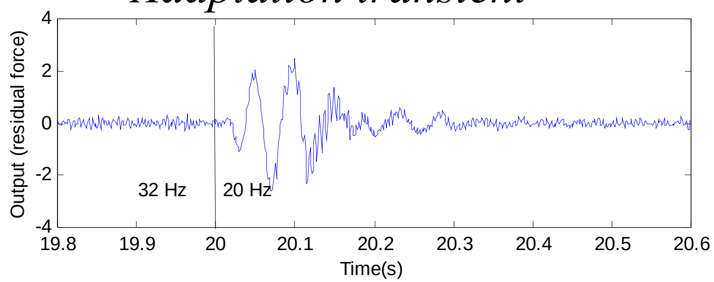
Initialization of the adaptive controller



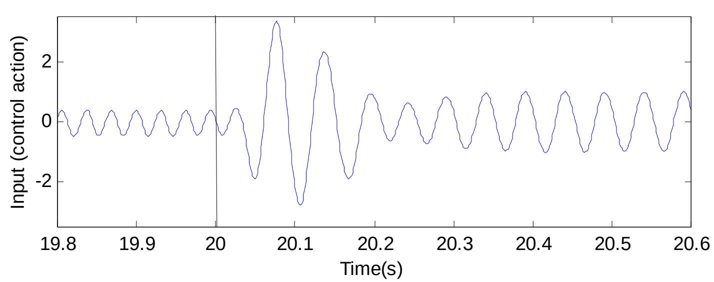
Input

Adaptation transient

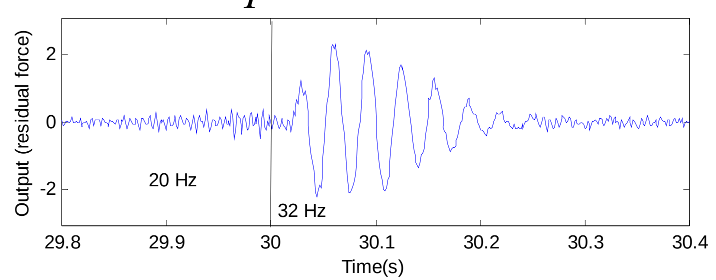
output

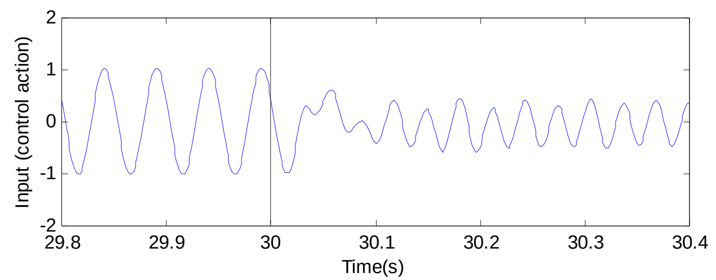


input



Adaptation transient





Rejection of unknown finite band disturbances

- **Assumption:** Plant model almost constant and known (obtained by system identification)
- **Problem:** Attenuation of unknown and/or variable stationary disturbances without using an additional measurement
- **Solution:** Adaptive feedback control
  - Estimate the model of the disturbance (indirect adaptive control)
  - Use the internal model principle
  - Use of the Youla parameterization (direct adaptive control)

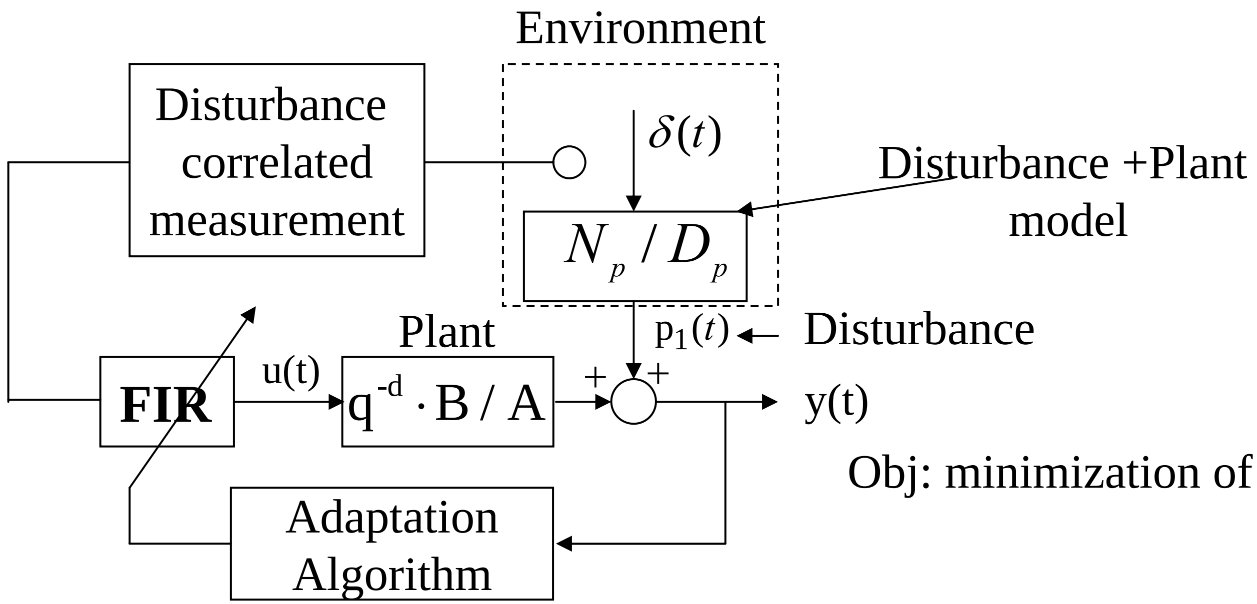
A robust control design should be considered assuming that the model of the disturbance is known

A class of applications*: suppression of unknown vibration (active vibration control)*

Attention: *The area was "dominated " by adaptive signal processing solutions (Widrow's adaptive noise cancellation) which require an additional transducer*

Remainder : *Models of stationary sinusoidal disturbances have poles on the unit circle*

Unknown disturbance rejection – classical solution



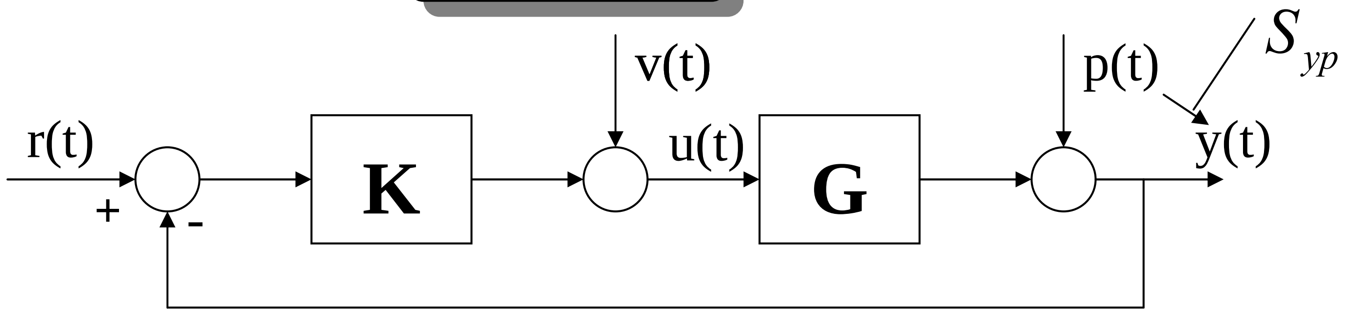
Obj: minimization of $E\{y^2\}$

Disadvantages:

- requires the use of an additional transducer
- difficult choice of the location of the transducer
- adaptation of many parameters

Not justified for the rejection of narrow band disturbances

Notations



$$G(q^{-1}) = \frac{q^{-d}B(q^{-1})}{A(q^{-1})} \qquad K(q^{-1}) = \frac{R(q^{-1})}{S(q^{-1})} = \frac{R'(q^{-1})H_R(q^{-1})}{S'(q^{-1})H_S(q^{-1})}$$

H_R and H_S are pre-specified

Output Sensitivity function :

$$S_{yp}(z^{-1}) = \frac{A(z^{-1})S'(z^{-1})H_S(z^{-1})}{P(z^{-1})}$$

Closed loop poles : $\quad P(z^{-1}) = A(z^{-1})S(z^{-1}) + z^{-d}B(z^{-1})R(z^{-1})$

The gain of S_{yp} is zero at the frequencies where $S_{yp}(e^{-j\omega})=0$ (perfect attenuation of a disturbance at this frequency)

Disturbance model

Deterministic framework

$$p(t) = \frac{N_p(q^{-1})}{D_p(q^{-1})} \cdot \delta(t): \text{ deterministic disturbance}$$

$D_p \rightarrow$ may have poles on the unit circle; $\delta(t) = \text{Dirac}$

(Sinusoid: $D_P = 1 + \alpha q^{-1} + q^{-2}; \ \alpha = -2\cos(2\pi f / f_s)$)

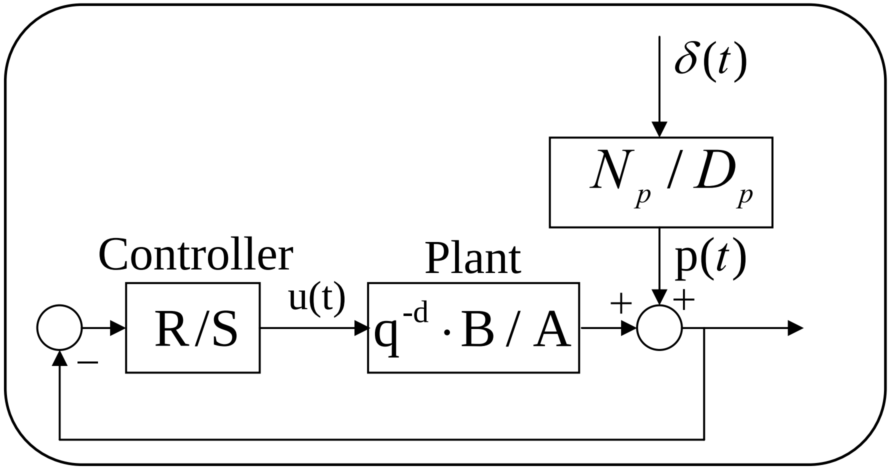
Stochastic framework

$$p(t) = \frac{N_p(q^{-1})}{D_p(q^{-1})} \cdot e(t): \text{stochastic disturbance}$$

$e(t) =$ Gaussian white noise sequence $(0, \sigma)$

$D_p \rightarrow$ may have poles on the unit circle

Closed loop system. Notations



$$p(t) = \frac{N_p(q^{-1})}{D_p(q^{-1})} \cdot \delta(t) : \text{deterministic disturbance}$$

$D_p \rightarrow$ poles on the unit circle; $\delta(t) = \text{Dirac}$

Controller :

$$R(q^{-1}) = R'(q^{-1}) \cdot H_R(q^{-1});$$

$$S(q^{-1}) = S'(q^{-1}) \cdot H_S(q^{-1}).$$

Internal model principle: $H_S(z^{-1}) = D_p(z^{-1})$

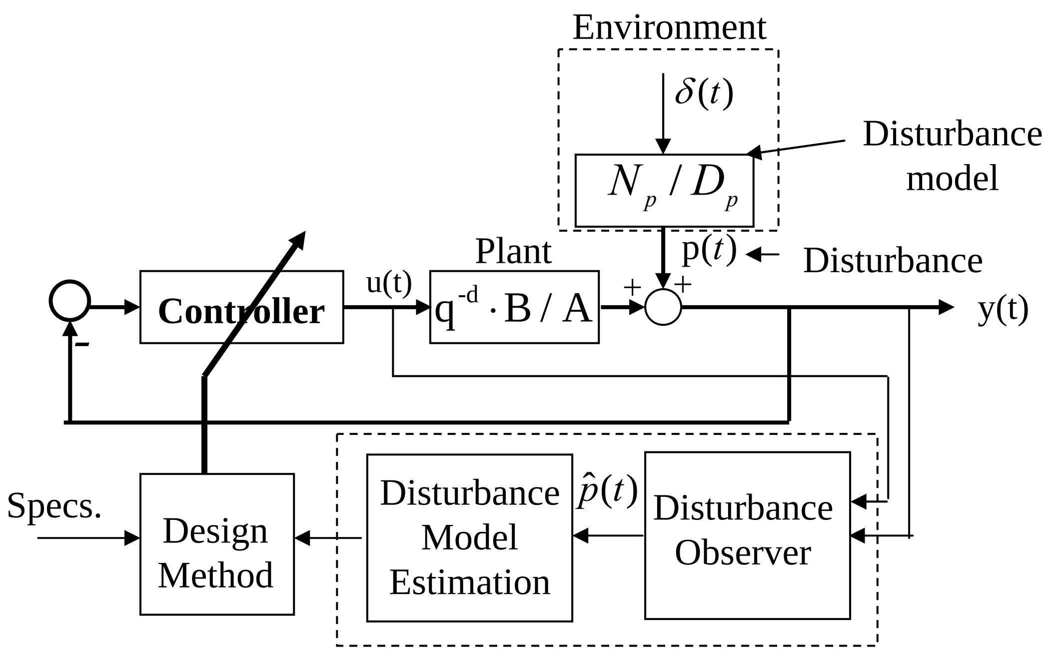
$$\text{Output} : y(t) = \frac{A(q^{-1})S(q^{-1})}{P(q^{-1})} \cdot p(t) = S_{yp}(q^{-1}) \cdot p(t) \longrightarrow y(t) = \frac{A(q^{-1})H_S(q^{-1})S'(q^{-1})N_p(q^{-1})}{P(q^{-1})} \cdot \frac{1}{D_p(q^{-1})} \cdot \delta(t)$$

$$\text{CL poles} : P(q^{-1}) = A(q^{-1})S(q^{-1}) + z^{-d}B(q^{-1})R(q^{-1})$$

Indirect adaptive regulation

Two-steps methodology:

1. Estimation of the disturbance model, $D_p(q^{-1})$
2. Computation of the controller, imposing $H_S(q^{-1}) = \hat{D}_p(q^{-1})$



It can be time consuming (if the plant model B/A is of large dimension)

Indirect adaptive control

Step I : Estimation of the disturbance model

ARMA identification (Recursive Extended Least Squares)

Step II: Computation of the controller

Solving Bezout equation (for S' and R)

$$H_S = \hat{D}_p$$

$$A\hat{D}_p S' + q^{-d} BR = P$$

$$S = \hat{D}_p S'$$

Remark:

It is time consuming for large dimension of the plant model

Internal model principle and Q-parametrization)

Central contr: $[R_0(q^{-1}),S_0(q^{-1})]$.

CL poles: $P(q^{-1})=A(q^{-1})S_0(q^{-1})+q^{-d}B(q^{-1})R_0(q^{-1})$.

Control: $S_0(q^{-1})\, u(t) = -R_0\,(q^{-1})\, y(t)$

Q-parametrization :

$\boldsymbol{R(z^{1})=R_0(q^{-1})+A(q^{-1})Q(q^{-1});}$

$\boldsymbol{S(q^{-1})=S_0(z^{-1})-q^{-d}B(q^{-1})Q(q^{-1}).}$

$$Q(q^{-1}) = q_0 + q_1 q^{-1} + \ldots . q_{n_Q} q^{-n_Q}$$

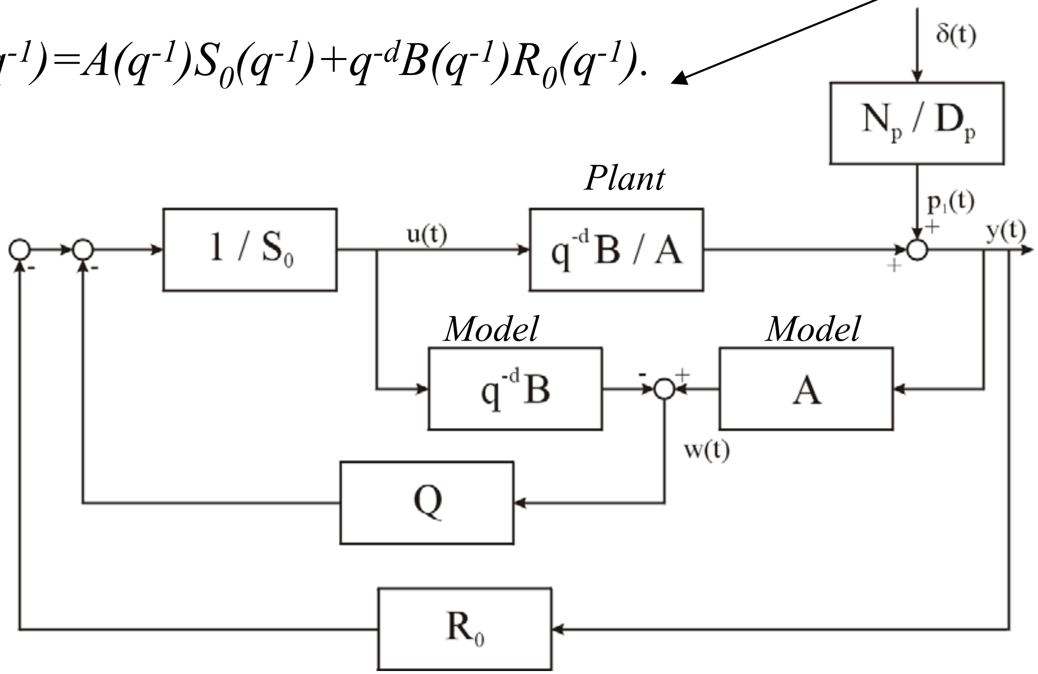
Control: $S(q^{-1})u(t) = -R(q^{-1})y(t)$

$$S_0(q^{-1})\, u(t) = -R_0\,(q^{-1})\, y(t) - Q\,(q^{-1})\, w(t),$$

where $w(t) = A\,(q^{-1})\, y(t) - q^{-d}B\,(q^{-1})\, u(t)$.

CL poles: $P(q^{-1})=A(q^{-1})S_0(q^{-1})+q^{-d}B(q^{-1})R_0(q^{-1})$.

The closed loop poles remain unchanged

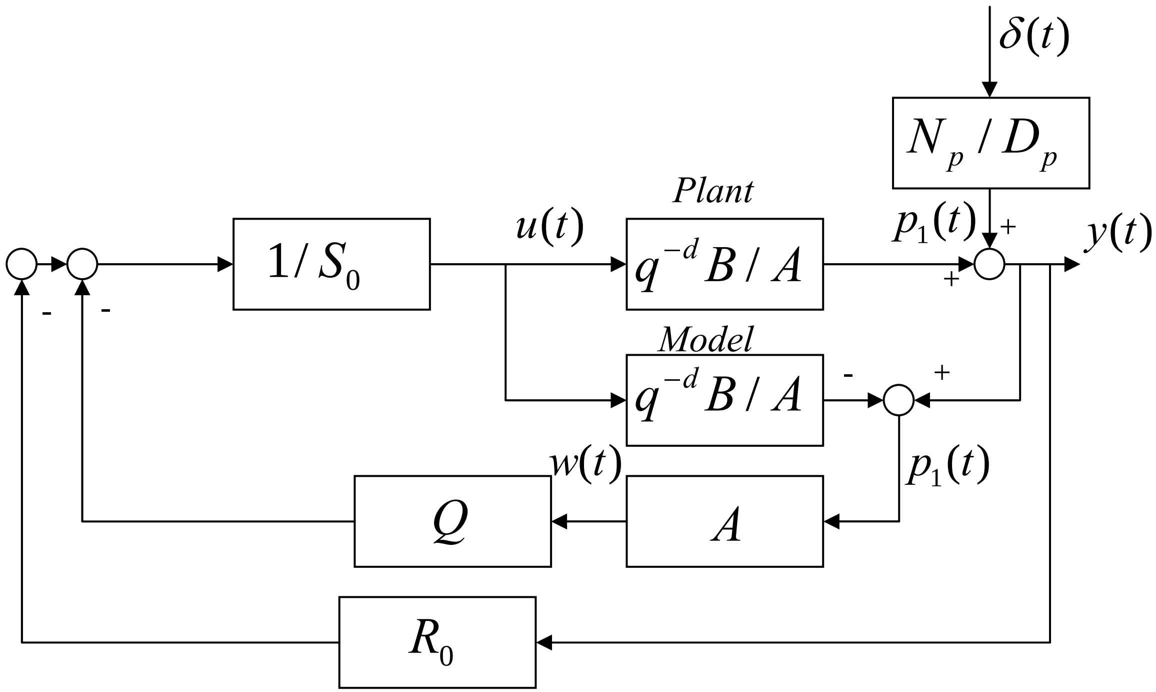


$$y(t) = \frac{q^{-d}B(q^{-1})}{A(q^{-1})} + p_1(t)$$

Model = Plant

$w=Ap_1$

Yula-Kucera parametrization
An interpretation for the case *A* asympt. stable



$$w(t)=\frac{AN_p}{D_p}\delta(t)$$

Internal model principle and Q- parameterization

Central contr: $[R_0(q^{-1}),S_0(q^{-1})]$.

CL Poles: $P(q^{-1})=A(q^{-1})S_0(q^{-1})+q^{-d}B(q^{-1})R_0(q^{-1})$.

Control: $S_0(q^{-1})\,u(t) = -R_0\,(q^{-1})\,y(t)$

Q-parameterization :

$\boldsymbol{R(z^{-1})=R_0(q^{-1})+A(q^{-1})Q(q^{-1});}$

$\boldsymbol{S(q^{-1})=S_0(z^{-1})-q^{-d}B(q^{-1})Q(q^{-1}).}$

Closed Loop Poles remain unchanged

Internal model assignment on Q *(find Q such that S contains the disturbance model):*

$$S = S_0 - q^{-d}BQ = MD_p \quad \Rightarrow \quad \textit{Solve:} \quad MD_p + q^{-d}BQ = S_0$$

? ?

Will lead also to an « indirect adaptive control solution »

BUT:

*Q can be used to "**directly**" tune the internal model without changing the closed loop poles(see next)*

Direct Adaptive Control (unknown D_p)

Hypothesis: Identified (known) plant model *(A,B,d)*.

Goal: minimize *y(t)* (according to a certain criterion).

Consider $p_1(t) = \frac{N_p(q^{-1})}{D_p(q^{-1})} \cdot \delta(t)$: deterministic disturbance. $w(t) = \frac{AN_p}{D_p}\delta(t)$

S(q^{-1})

$$y(t) = \frac{A(q^{-1})\left[S_0(q^{-1}) - q^{-d}B(q^{-1})Q(q^{-1})\right]}{P(q^{-1})} \cdot \frac{N_p(q^{-1})}{D_p(q^{-1})} \cdot \delta(t) = \frac{\left[S_0(q^{-1}) - q^{-d}B(q^{-1})Q(q^{-1})\right]}{P(q^{-1})} w(t)$$

$$w(t) = \frac{A(q^{-1})N_p(q^{-1})}{D_p(q^{-1})} \cdot \delta(t) = A(q^{-1}) \cdot y(t) - q^{-d} \cdot B(q^{-1}) \cdot u(t)$$

Define (construct): $$\varepsilon(t) = \frac{S_0(q^{-1})}{P(q^{-1})} \cdot w(t) - \frac{q^{-d}B(q^{-1})}{P(q^{-1})} Q(q^{-1}) \cdot w(t).$$

Define $\varepsilon^0(t+1)$ as the value of $y(t+1)$ obtained with $\hat{Q}(t,q^{-1})$

$$\varepsilon^0(t+1) = \frac{S_0(q^{-1})}{P(q^{-1})} \cdot w(t+1) - \frac{q^{-d}B^*(q^{-1})}{P(q^{-1})} \hat{Q}(t,q^{-1}) \cdot w(t)$$

One can define now the *a posteriori* error (using $\hat{Q}(t+1,q^{-1})$) as:

$$\varepsilon(t+1)=\frac{S_0(q^{-1})}{P(q^{-1})}\cdot w(t+1)-\frac{q^{-d}B^*(q^{-1})}{P(q^{-1})}\hat{Q}(t+1,q^{-1})\cdot w(t) \qquad (*)$$

We need to express $\varepsilon(t)$ as:

$$\varepsilon(t+1)=\left[Q(q^{-1})-\hat{Q}(t+1,q^{-1})\right]f(t)=\left[\theta-\hat{\theta}(t+1)\right]^T\Phi(t)$$

$$\theta^T=\left[q_0,q_1,\ldots\right]; \quad \Phi^T(t)=\left[f(t),f(t-1),..\right]$$

Using: $MD_p+q^{-d}BQ=S_0$

$$\frac{S_0(q^{-1})}{P(q^{-1})}w(t+1)=Q\frac{q^{-d}B*(q^{-1})}{P(q^{-1})}w(t)+\frac{M(q^{-1})D_P((q^{-1})}{P(q^{-1})}w(t+1)$$

(*) becomes: *Vanishing term*

Leads to a direct adaptive control

$$\boxed{\varepsilon(t+1)=[Q(q^{-1})-\hat{Q}(t+1,q^{-1})]\cdot\frac{q^{-d}B^*(q^{-1})}{P(q^{-1})}\cdot w(t)+v(t+1)}$$

Details: $v(t+1)=\dfrac{M(q^{-1})D_p(q^{-1})}{P(q^{-1})}w(t+1)=\dfrac{M(q^{-1})A(q^{-1})N_p(q^{-1})}{P(q^{-1})}\delta(t+1)$

$$\frac{q^{-d}B^*(q^{-1})}{P(q^{-1})}\cdot w(t)=\frac{q^{-d}B(q^{-1})}{P(q^{-1})}\cdot w(t+1)$$

The Algorithm

measured

$$w(t+1) = A(q^{-1})y(t+1) - q^{-d}B^*(q^{-1})u(t);\ \ (B(q^{-1})u(t+1) = B^*(q^{-1})u(t))$$

define :

$$\boxed{\varepsilon^\circ(t+1) = \frac{S_0(q^{-1})}{P(q^{-1})}w(t+1) - \hat{Q}(t,q^{-1})\frac{q^{-d}B(q^{-1})}{P(q^{-1})}w(t+1).}$$

$$w_1(t+1) = \frac{S_0(q^{-1})}{P(q^{-1})}\cdot w(t+1); \quad w_2(t) = \frac{q^{-d}B^*(q^{-1})}{P(q^{-1})}\cdot w(t);$$

$$\hat{\theta}^T(\mathrm{t}) = [\hat{\mathrm{q}}_0(t) \quad \hat{\mathrm{q}}_1(t)]; \quad \phi^T(t) = [w_2(t) \quad w_2(t-1)], \quad (\text{for } \ n_{D_P} = 2 \ \text{ since } n_Q = n_{D_P}\text{-1})$$

A priori adaptation error : $\boxed{\varepsilon^0(t+1) = w_1(t+1) - \hat{\theta}^T(t)\phi(t)}$

A posteriori adaptation error : $\varepsilon(t+1) = w_1(t+1) - \hat{\theta}^T(t+1)\phi(t)$

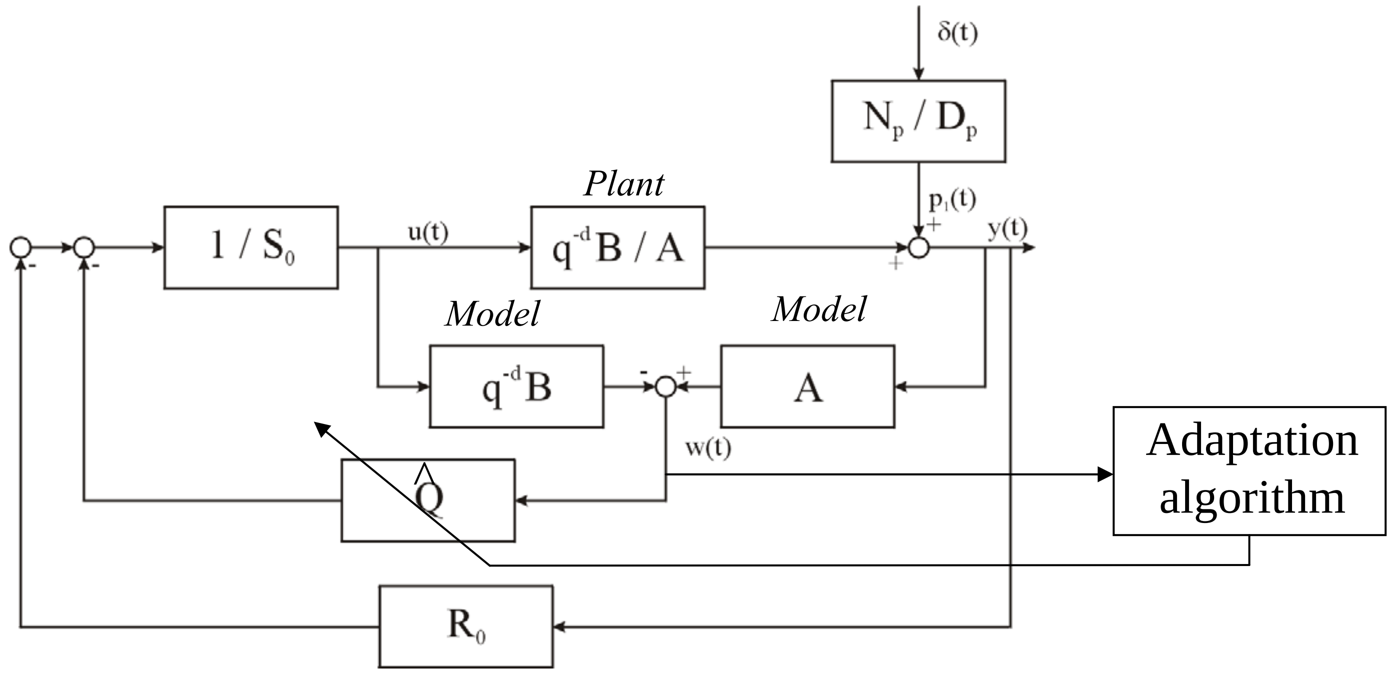
Parameter adaptation algorithm:

$$\begin{cases} \hat{\theta}(\mathrm{t}+1) = \hat{\theta}(\mathrm{t}) + \mathrm{F}(\mathrm{t}+1)\phi(\mathrm{t})\varepsilon^0(\mathrm{t}+1); \\ F^{-1}(t+1) = \lambda_1(t)F^{-1}(t) + \lambda_2(t)\phi(t)\phi^T(t). \end{cases}$$

Various choices possible for λ_1 and λ_2 *which define the adaptation gain time profile*

(For a stability proof see Automatica, 2005, no.4 pp. 563-574)

Direct adaptive rejection of unknown disturbances



- *The order of the Q polynomial depends upon the order of the disturbance model denominator (D_P) and not upon the complexity of the plant model*
- *Less parameters to estimate then for the identification of the disturbance model*
- *Operation in "self –tuning" mode (constant unknown disturbance) or "adaptive" mode (time varying unknown disturbance)*

Further experimental results on the active suspension

Comparison between direct/indirect adaptive control

Real-time results – Active Suspension (continuation)

Narrow band disturbances = variable frequency sinusoid $\Rightarrow n_Q = 1$
Frequency range: 25 ÷ 47 Hz

Evaluation of the two algorithms in **real-time**

Nominal controller $[R_0(q^{-1}),S_0(q^{-1})]$: $n_{R0}=14$, $n_{S0}=16$

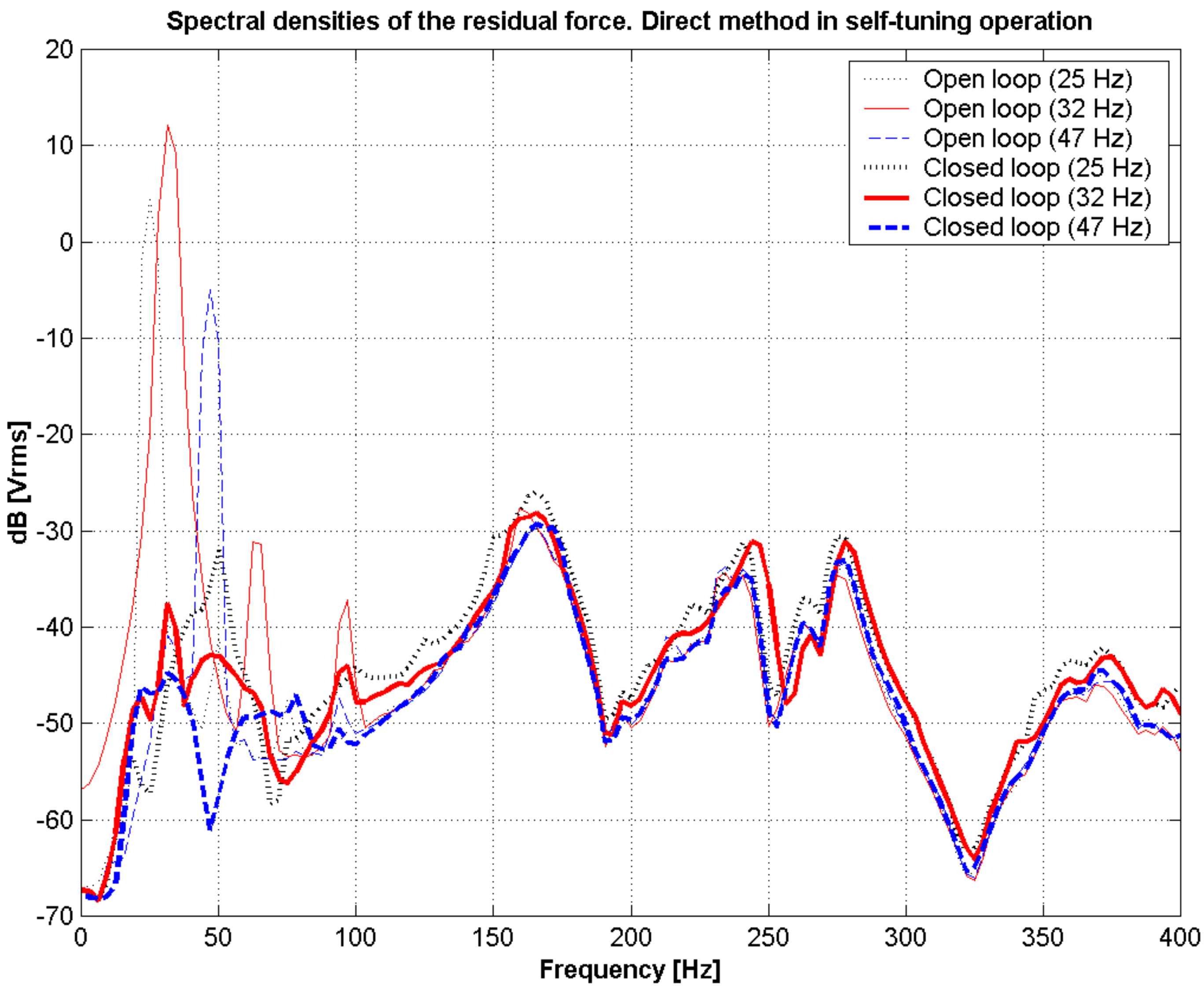
Implementation protocol 1: Self-tuning

- The algorithm stops when it converges and the controller is applied.
- It restarts when the variance of the residual force is bigger than a given threshold.
- As long as the variance is not bigger than the threshold, the controller is constant.

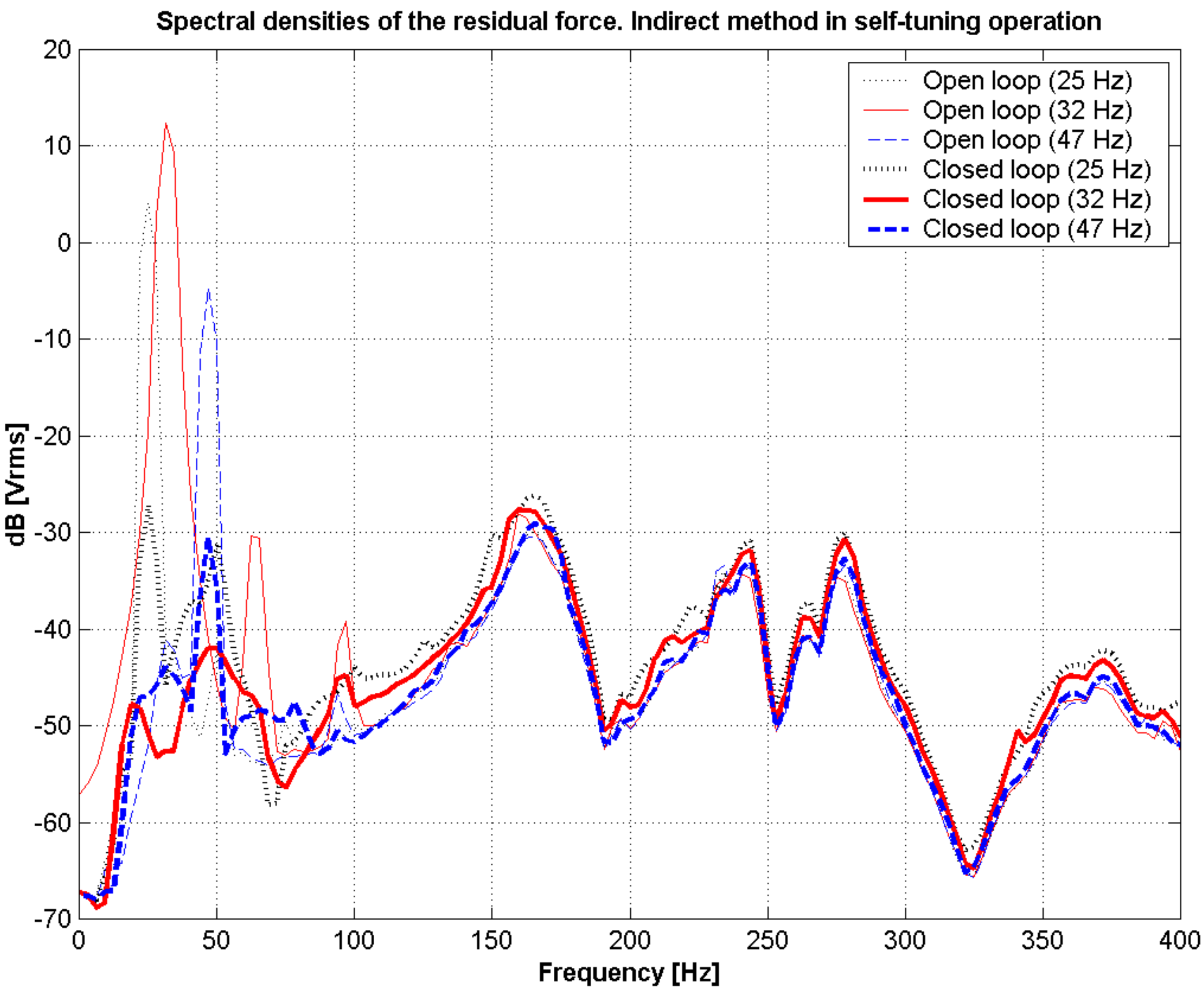
Implementation protocol 2: Adaptive

- The adaptation algorithm is continuously operating
- The controller is updated at each sample

Frequency domain results – direct adaptive method

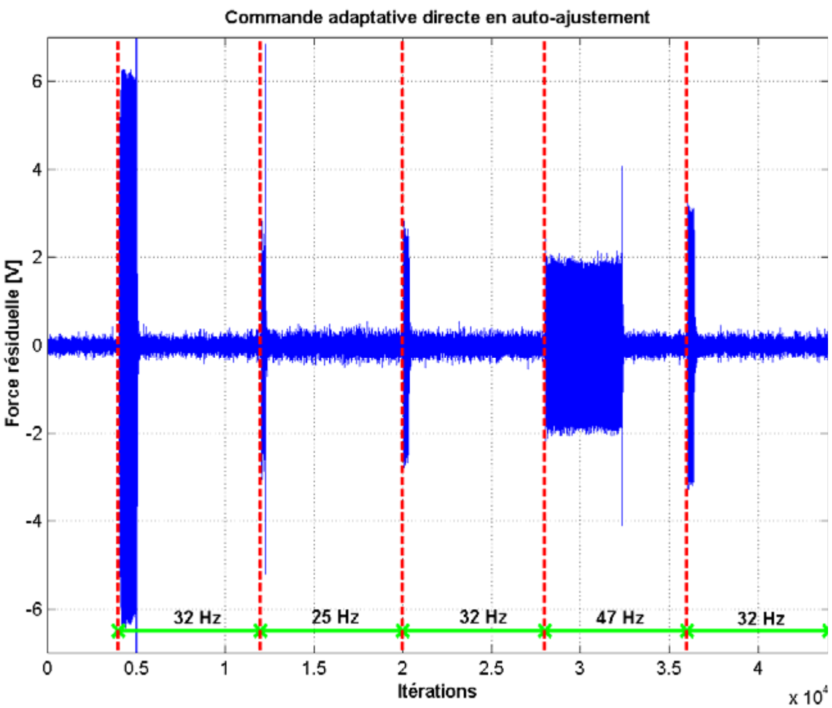


Frequency domain results – indirect adaptive method

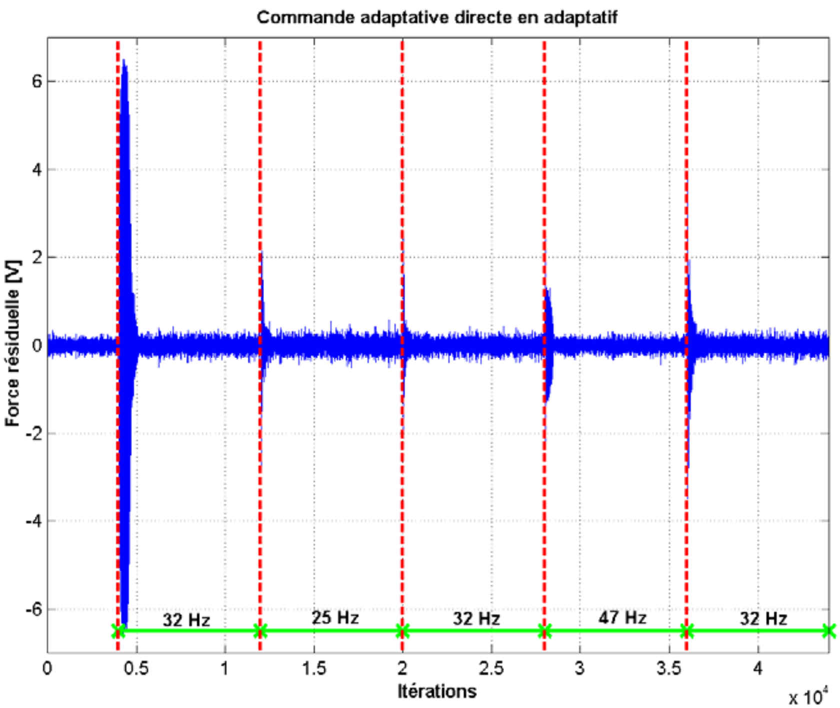


Direct Adaptive Control

Self-tuning Mode



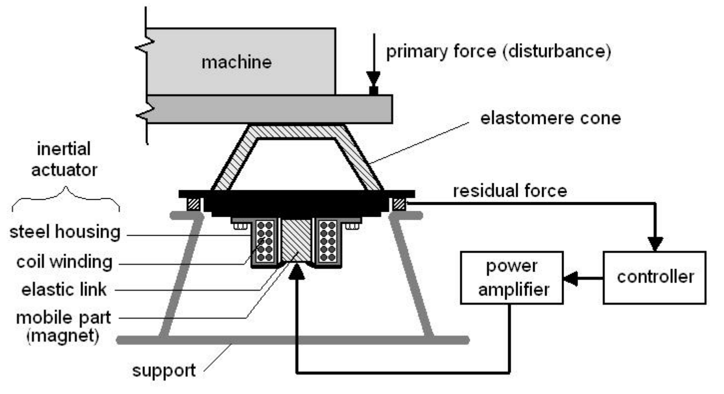
Adaptive Mode



- *Direct adaptive control in adaptive mode operation gives better results than direct adaptive control in self-tuning mode*
- *Direct adaptive control leads to a much simpler implementation and better performance than indirect adaptive control*

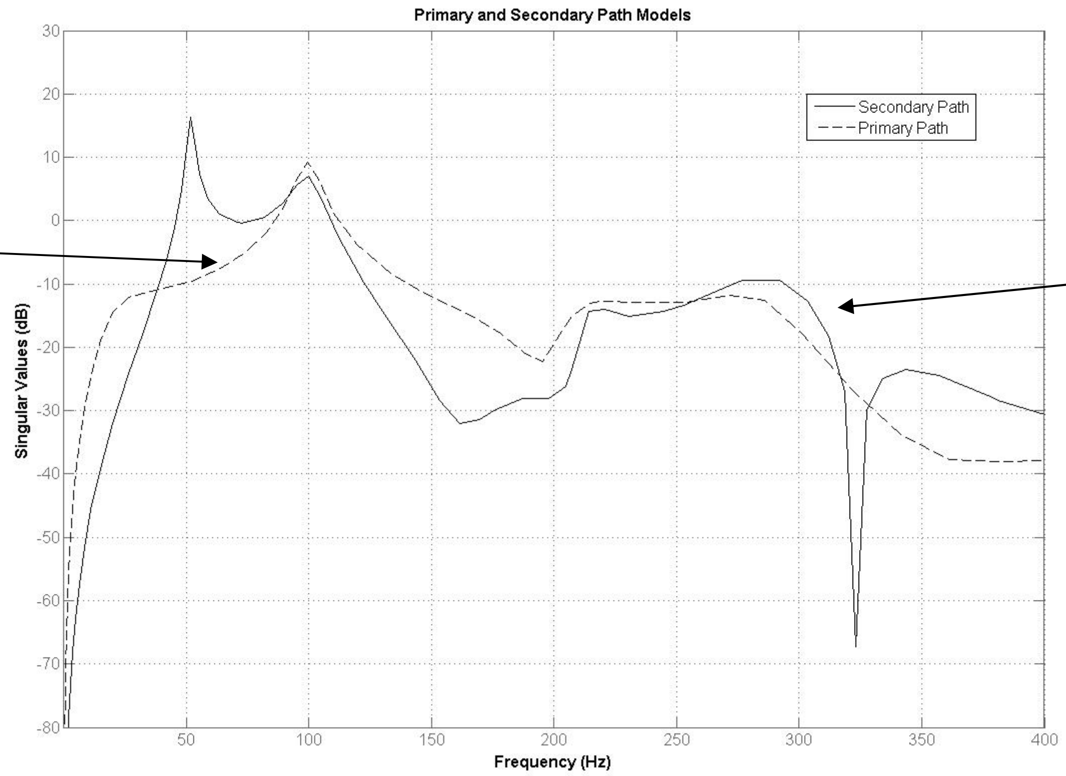
Active vibration control using an inertial actuator
Real-time results

Rejection of two simultaneous sinusoidal disturbances



Frequency Characteristics of the Identified Models

Primary path

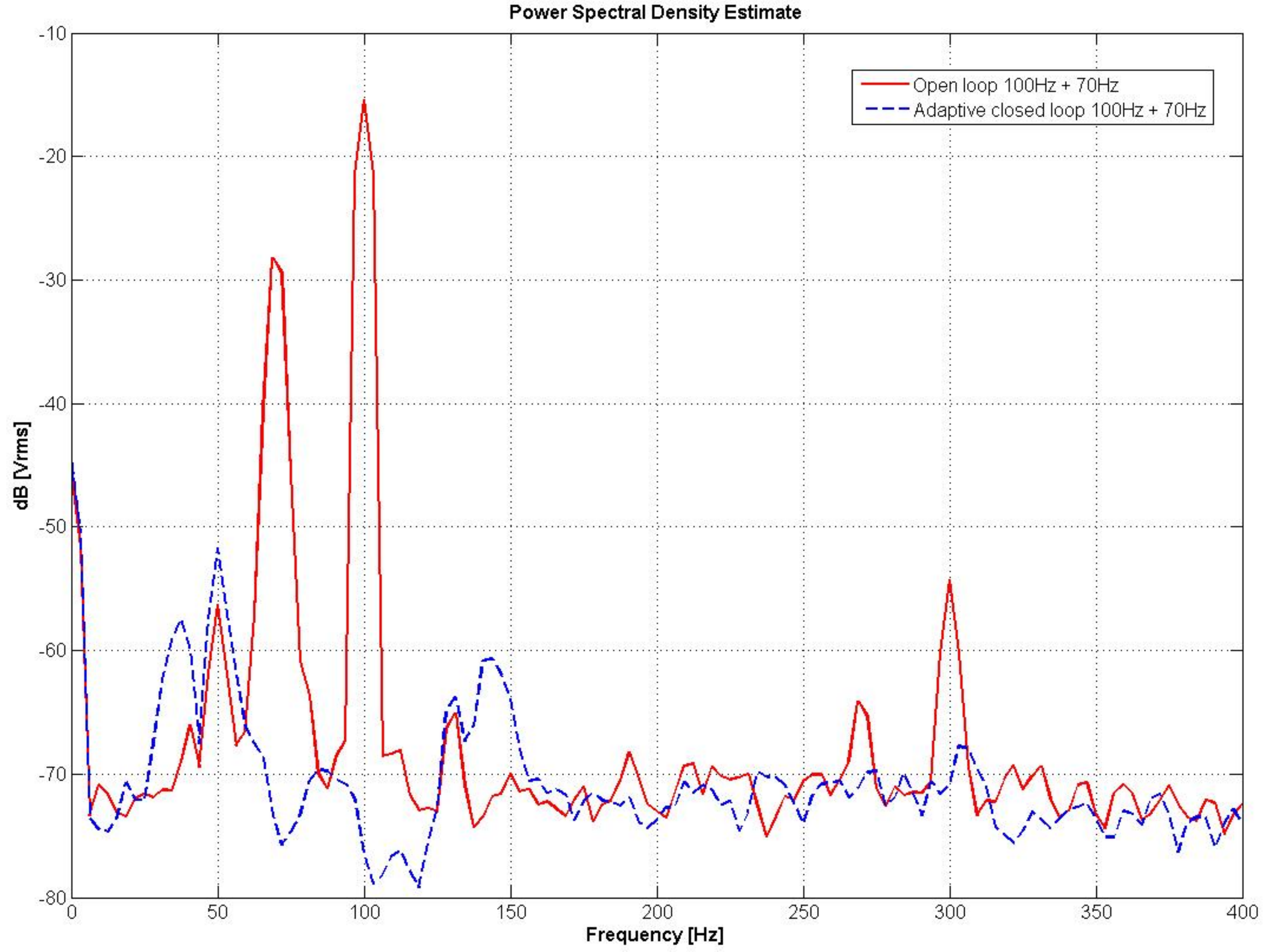


Secondary path

Complexity of secondary path: $n_A = 10 \, ; n_B = 12 \, ; d = 0$

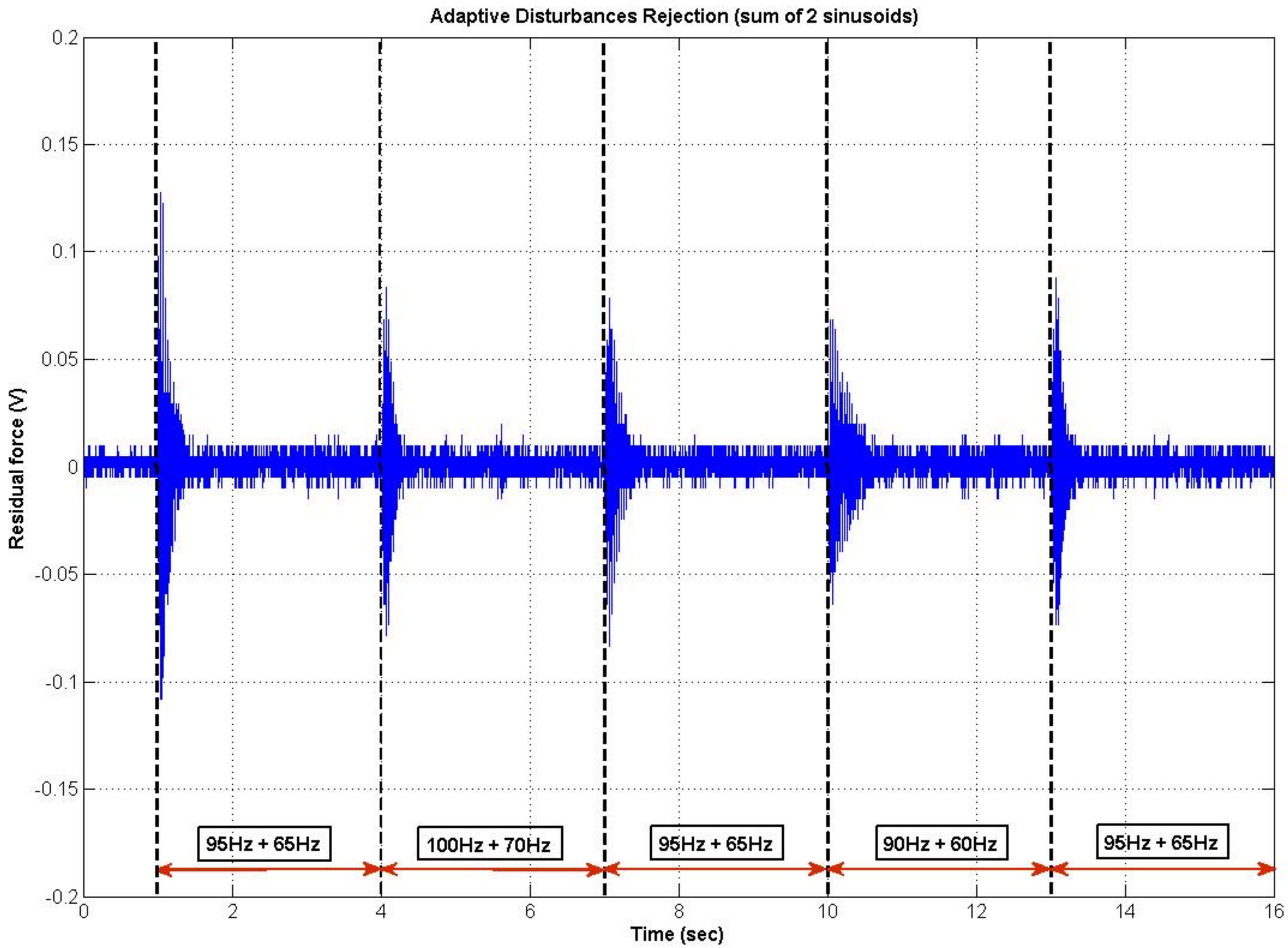
Frequency domain results – direct adaptive method

Rejection of two simultaneous sinusoidal disturbances



Time Domain Results – Direct adaptive control
Adaptive Operation

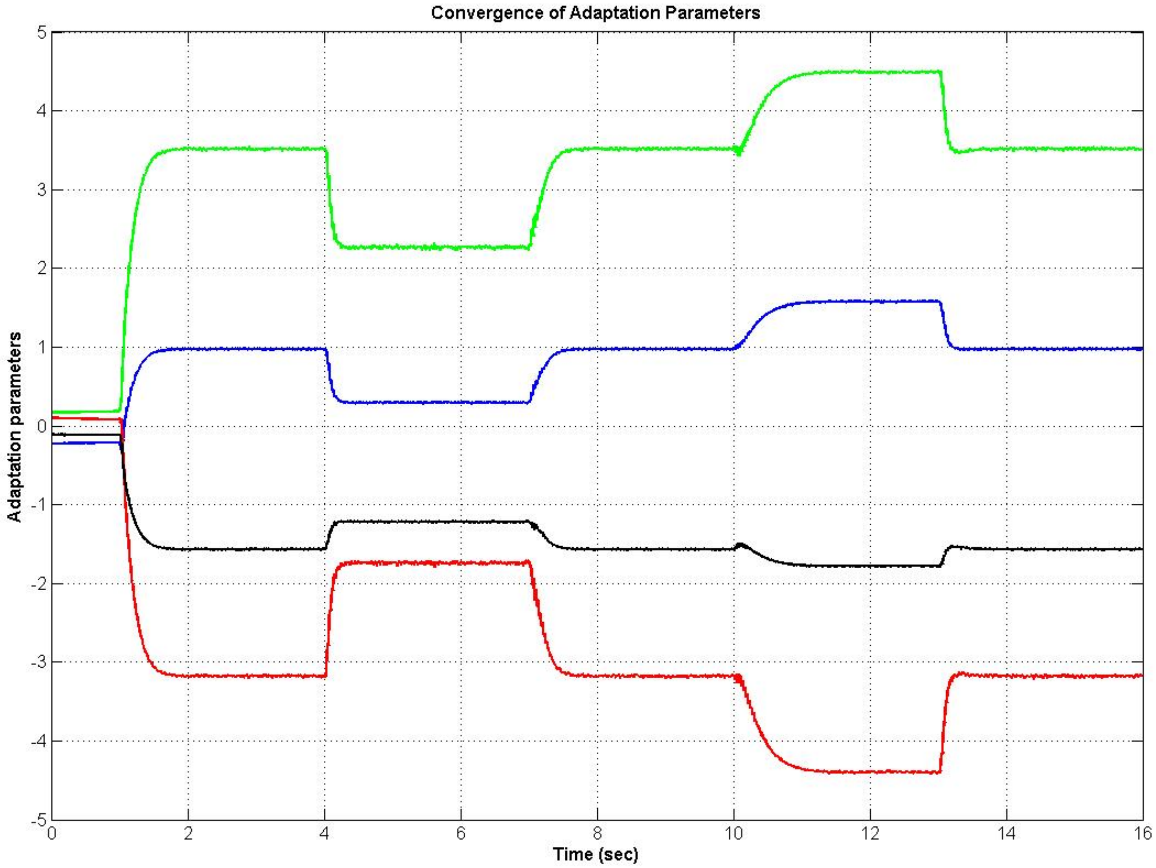
Simultaneous rejection of two time varying sinusoidal disturbances



Output

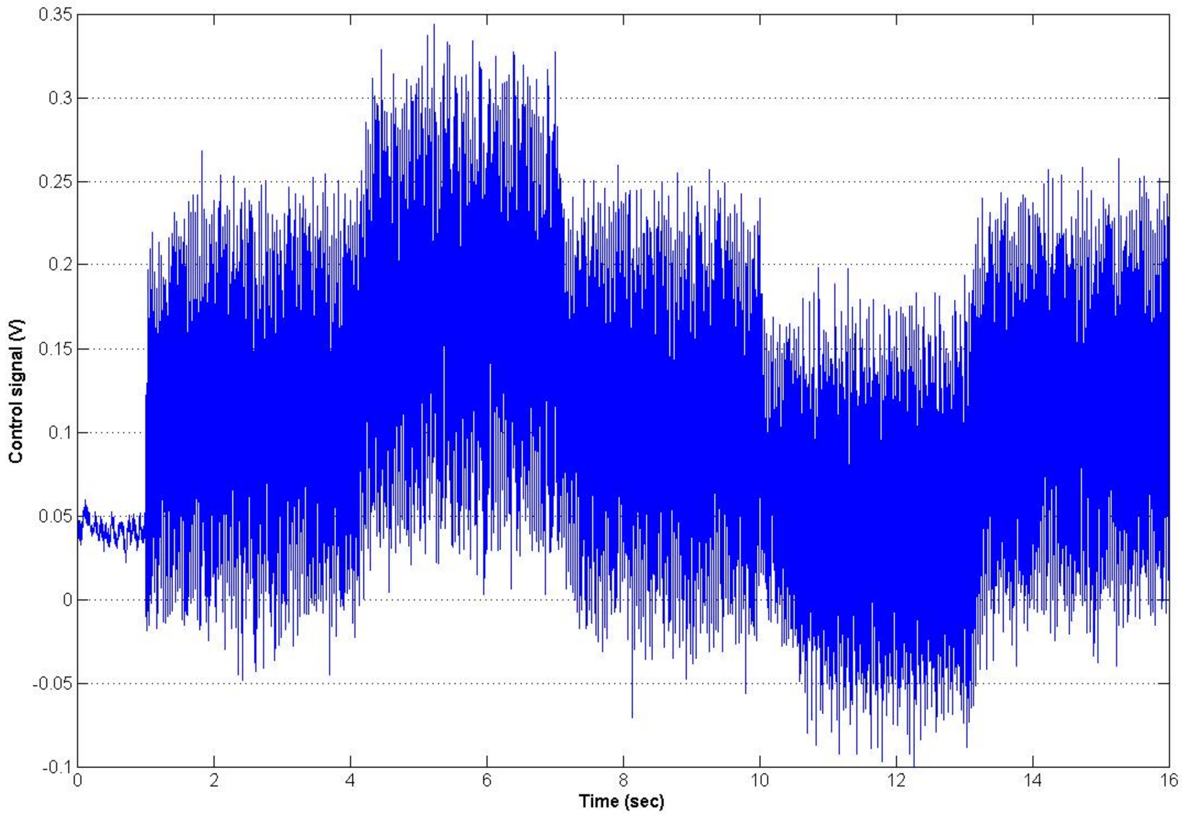
Time Domain Results – Direct adaptive control
Evolution of the Q parameters

Simultaneous rejection of two time varying sinusoidal disturbances



Time Domain Results – Direct adaptive control
Evolution of the control input

Simultaneous rejection of two time varying sinusoidal disturbances

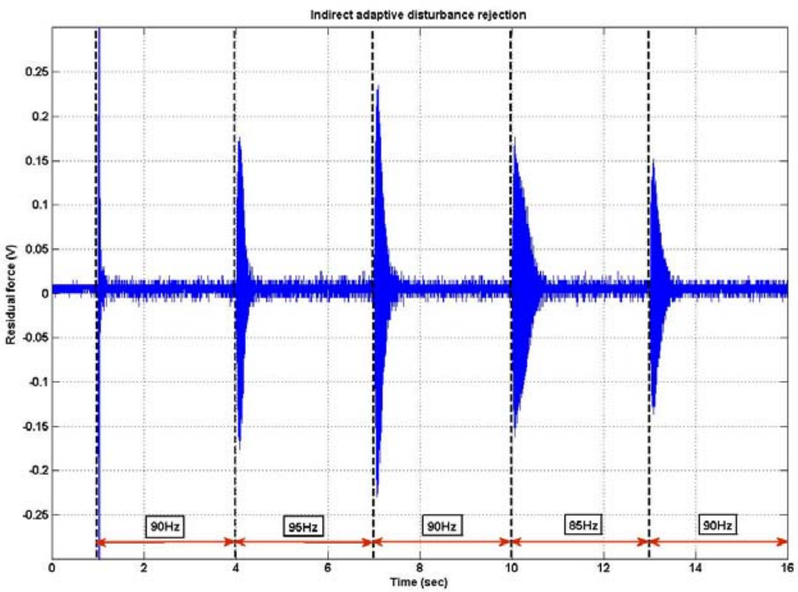


Comparison direct/indirect adaptive regulation

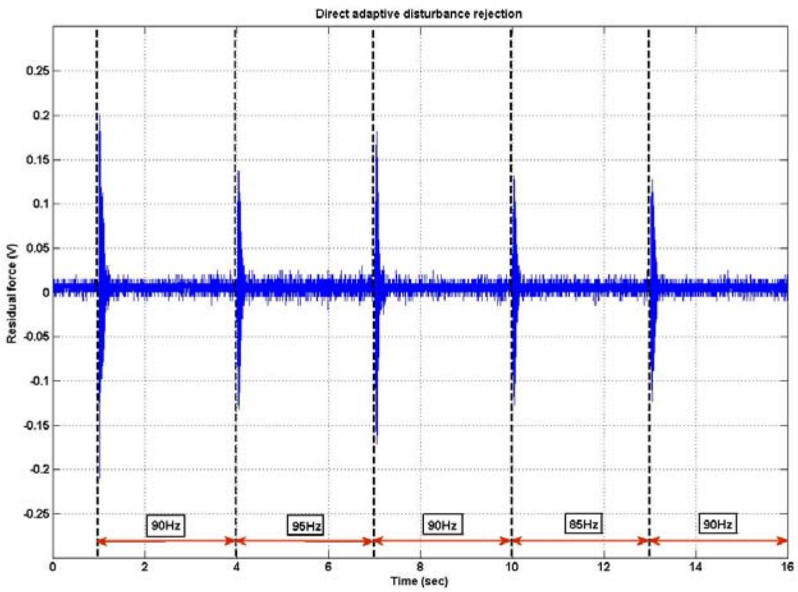
Time domain results – Adaptive regime

Active vibration control using an inertial actuator

Indirect adaptive method



Direct adaptive method



***Direct** adaptive control leads to a much simpler implementation and better performance than **Indirect** adaptive control*

Conclusions

-Using internal model principle, adaptive feedback control solutions can be provided for the rejection of unknown disturbances

-Both direct and indirect solutions can be provided

-Two modes of operation can be used : self-tuning and adaptive

-Direct adaptive control is the simplest to implement

-Direct adaptive control offers better performance

-The methodology has been extensively tested on:

- active suspension system
- active vibration control with an inertial actuator