New adaptive feedforward compensation algorithms for active vibration control with mechanical coupling.

Theory and applications

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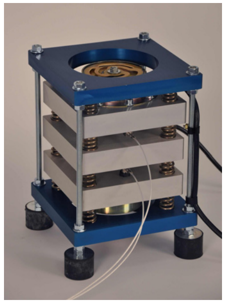

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Attenuation (cancellation) of disturbances

- Adaptive feedforward compensation of disturbances has a long history
- Noise and Vibration cancellation were the driving forces

Lets forget for the moment the history and ask the question:

Why we need adaptive feedforward compensation?

Attenuation (cancellation) of disturbances

How about using feedback?

- The performance of disturbance attenuation is limited by the Bode integral
- Only narrow or limited band disturbances can be significantly attenuated by feedback

For “wide band“ disturbances one needs to use:
feedforward compensation

Why it should be adaptive?

Because the characteristics of the disturbance are unknown and time varying

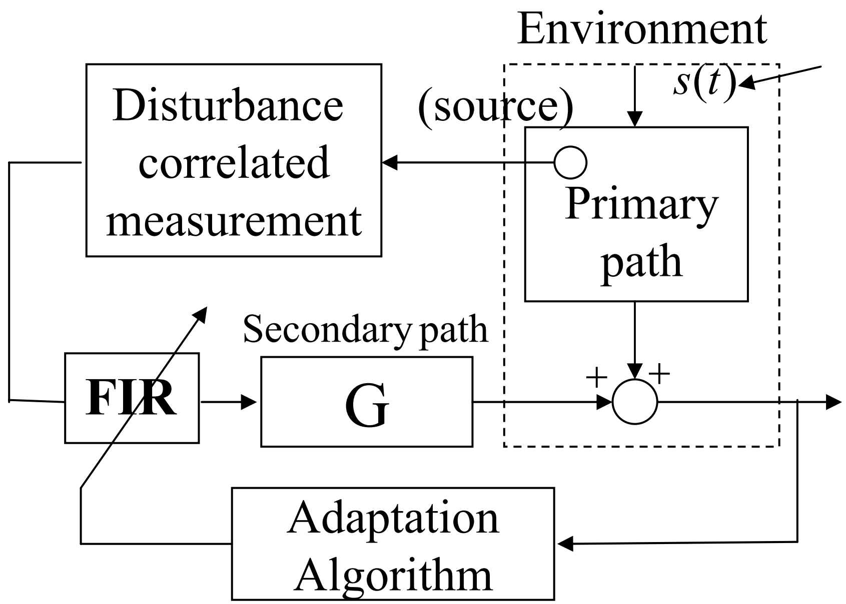
What is the price to pay?

- One needs a correlated measurement with the disturbance.
- This implies:

1) Hardware (additional transducer)
2) A good location of the transducer (may require a study).

Adaptive Feedforward Compensation

An "idyllic" scheme for adaptive feedforward disturbance compensation



Obj:- minimization of $E\{\chi^2\}$
- stability of the system

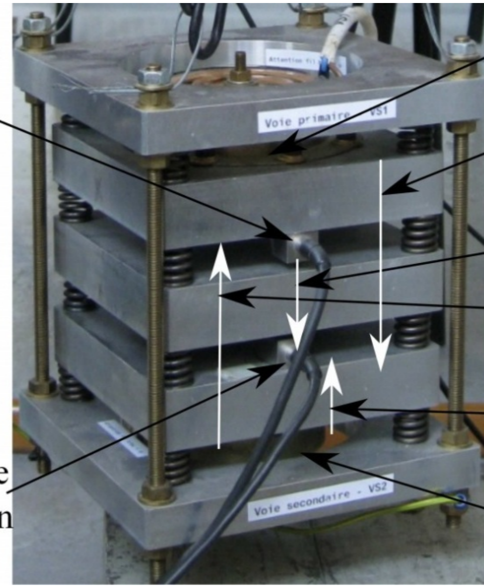
The reality is different

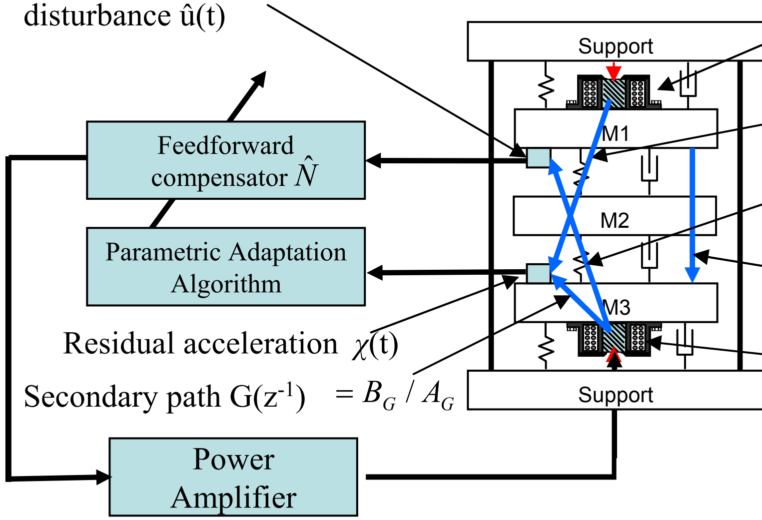
- The compensator system (secondary path) acts upon the source
- There is an **internal positive feedback** (physical feedback)
- FIR compensators are not very efficient

Outline

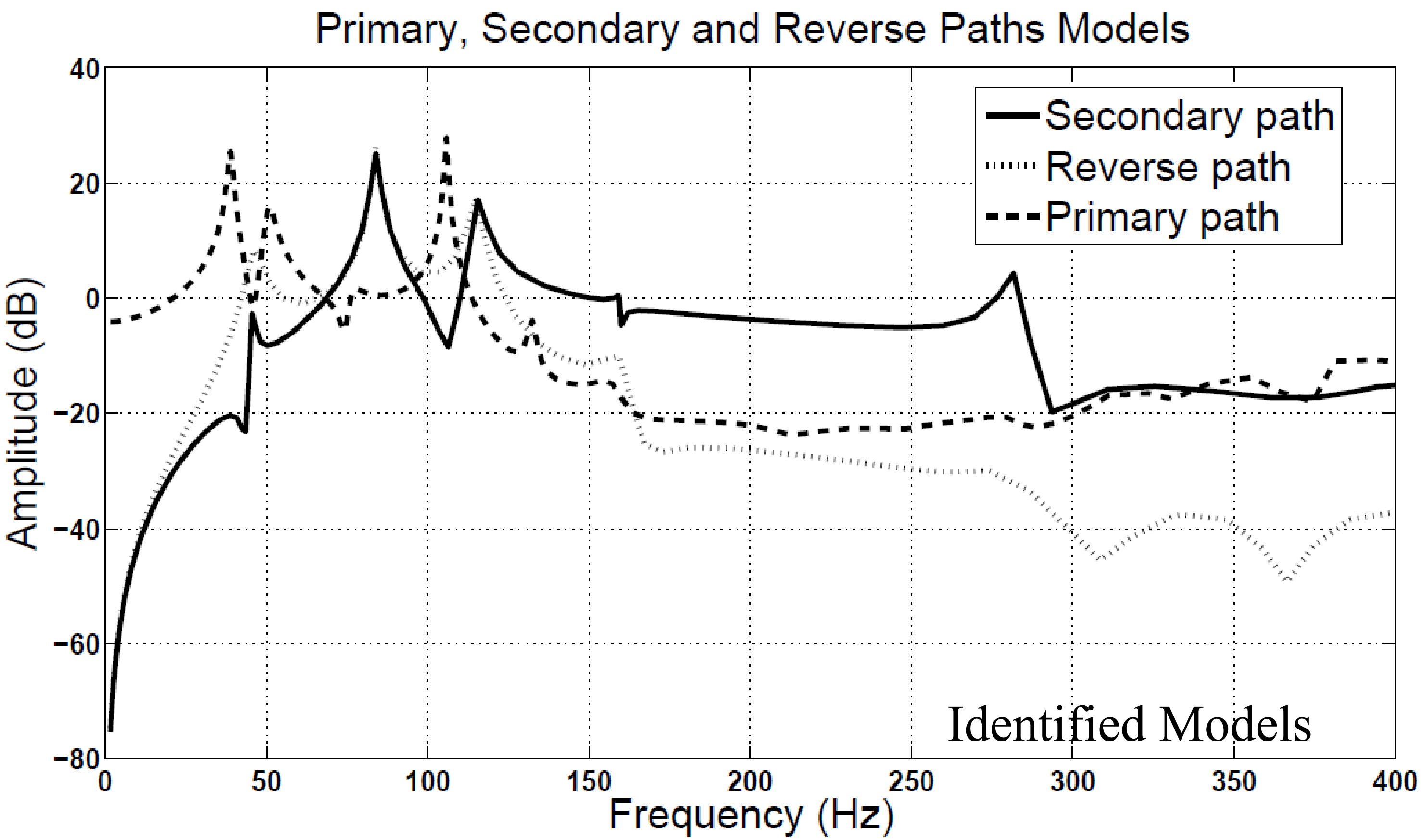
- Why we need “adaptive feedforward compensation”?
- A basic configuration
- The inherent positive internal feedback
- An active vibration control system
- Background
- Problem formulation
- Adaptive IIR feedforward compensators
- Experimental results and comparisons with existing algorithms
- Youla Kucera parametrized adaptive feedforward compensators
- Experimental results and comparison with adaptive IIR
- How about adding feedback?
- Experimental results
- Concluding comments

An active vibration control system using an inertial actuator





Frequency characteristics



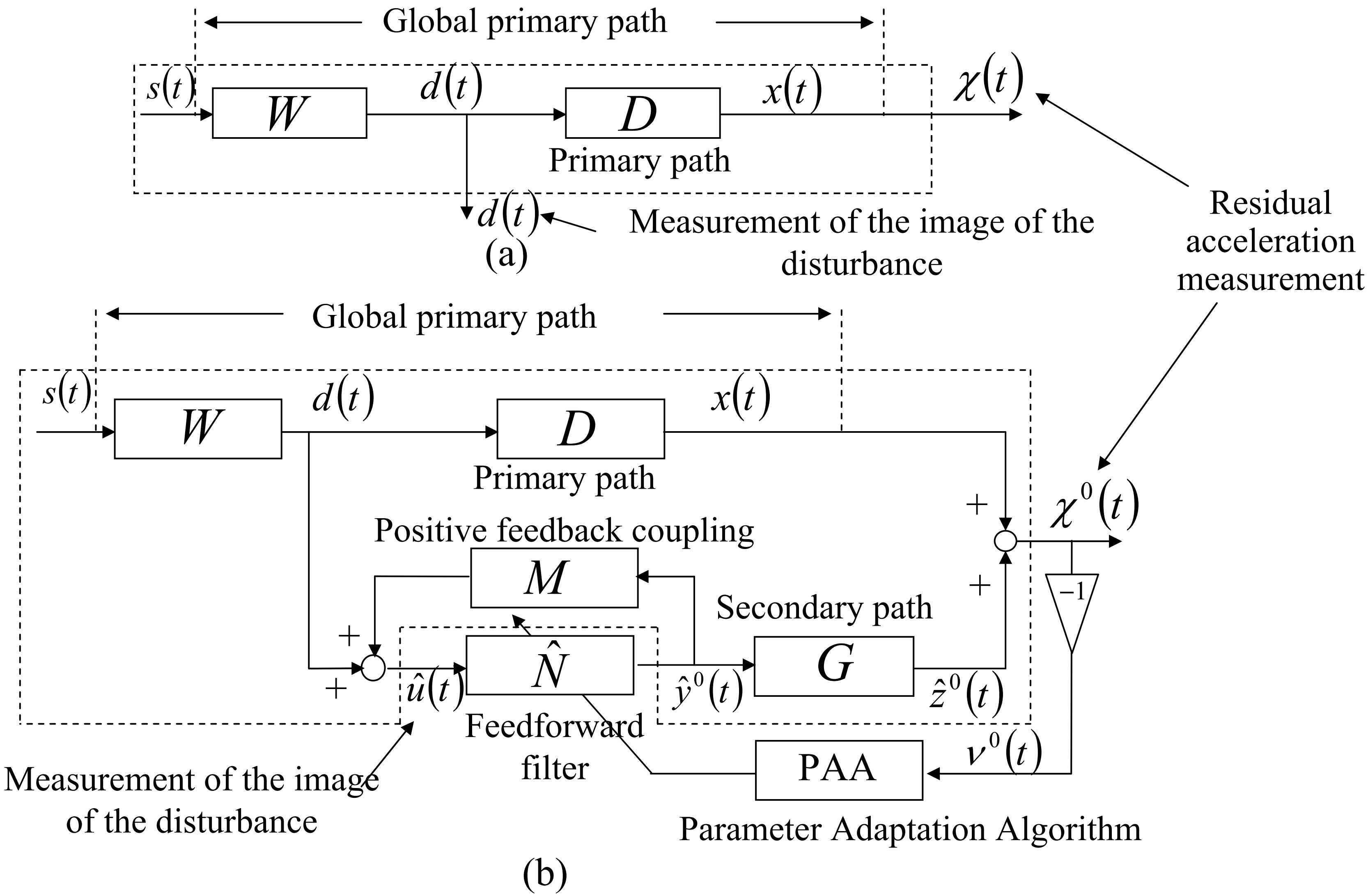
Complexity of the models:

$$n_{B_D} = 26, \quad n_{A_D} = 26; \quad n_{B_G} = 17, \quad n_{A_G} = 15; \quad n_{B_M} = 16, \quad n_{A_M} = 16;$$

primary — secondary — reverse

Rem: secondary path has complex zeros at 108 Hz where primary path has a resonance

IIR Adaptive feedforward disturbance compensation



Background

- The adaptive feedforward compensation algorithms have been developed assuming no « internal positive coupling »
- The « internal positive coupling » has been recognized after 1995
- Ad-hoc solutions have been proposed. Not validated by practice

Directions of research:

- *Analysis of existing algorithms (Fu-LMS) in presence of internal feedback (Wang-Ren(1999), Fraanje(2003) - convergence but not stability analysis)*
- *Development of dedicated adaptation algorithms for IIR feedforward compensators (stability approach)(Johnson-Jacobson(2001), Landau et al (2011))*
- *Use of Youla Kucera parametrization around a stabilizing fix controller (Callafon-Zeng (2006) - MBD, not continuous adaptation, no stability analysis, Landau et al, (2011a,b) – stable adaptive YK_FIR and YK_IIR)*

The Problem

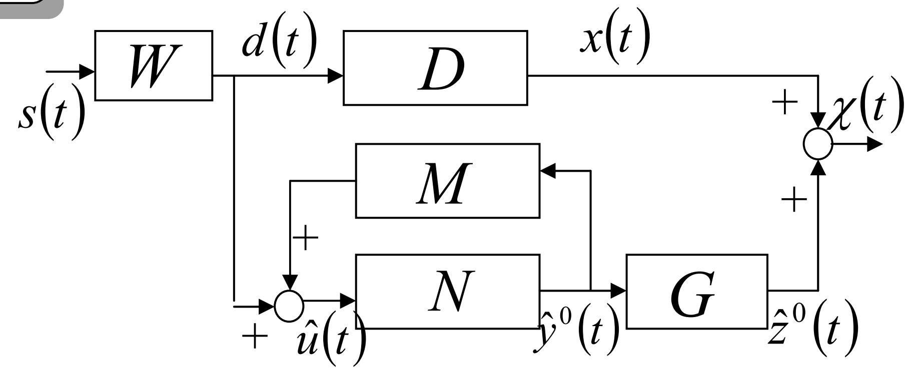
The adaptive feedforward compensator should minimize the effect of the disturbance while simultaneously assuring the stability of the internal positive feedback loop.

Two modes of operation →
- **Self-tuning** (adaptation gain vanishes)
- **Adaptive** (non vanishing adaptation gain)

How to solve the problem:

1) Design the largest family of stable adaptive algorithms for feedforward compensation in the presence of the internal positive feedback loop
2) Find among this family the algorithms which provide the best performance

Hypotheses



Case I (basic case)
H1: disturbance *d(t)* is bounded
H2: Perfect matching condition

Exists $N(z^{-1}) = \frac{R(z^{-1})}{S(z^{-1})}$ such that $\frac{GN}{1-NM} = \frac{G \cdot A_M R}{A_M S - B_M R} = -D$

and the characteristic polynomial of the internal feedback loop:

$$P(z^{-1}) = A_M(z^{-1})S(z^{-1}) - B_M(z^{-1})R(z^{-1})$$

is Hurwitz

H3: Measurement noise on the residual error is neglected (deterministic context)
H4: The primary path model D is unknown and constant

Case II : Hypotheses H2 and H3 are removed

IIR Feedforward - Basic equations

Feedforward compensator : $\hat{N}(t,q^{-1}) = \hat{R}(t,q^{-1}) / \hat{S}(t,q^{-1})$

Define:

$$\hat{\theta}^T(t) = \left[\hat{s}_1(t),\ldots,\hat{s}_{n_S}(t),\hat{r}_0(t),\ldots,\hat{r}_{n_R}(t)\right]$$

$$\varphi^T(t) = \left[-\hat{y}(t),\ldots,-\hat{y}(t-n_S+1),\hat{u}(t+1),\ldots,\hat{u}(t-n_R+1)\right]$$

Compensator output:

$$\hat{y}^\circ(t+1) = \hat{y}\left(t+1/\hat{\theta}(t)\right) = \hat{\theta}^T(t)\varphi(t) \qquad \hat{y}(t+1) = \hat{y}\left(t+1/\hat{\theta}(t+1)\right) = \hat{\theta}^T(t+1)\varphi(t)$$

Measurement (residual acceleration): $\chi^0(t+1)$

A priori adaptation error: $\nu^0(t+1) = \nu(t+1/\hat{\theta}(t)) = -\chi^0(t+1)$

A posteriori adaptation error: $\nu(t+1) = \nu(t+1/\hat{\theta}(t+1))$

Filtered observation vector: $\varphi_f(t) = L(q^{-1})\varphi(t)$

$$\nu(t+1) = \frac{A_M(q^{-1})G(q^{-1})}{P(q^{-1})L(q^{-1})}\left[\theta - \hat{\theta}(t+1)\right]^T \varphi_f(t)$$

Parametric adaptation algorithms

Measurement (residual acceleration): $\chi^0(t+1)$

A priori adaptation error: $\nu^0(t+1) = -\chi^0(t+1)$

Parametric adaptation algorithm: $\hat{\theta}(t+1) = \hat{\theta}(t) + F(t)\varphi_f(t)\nu(t+1)$

A posteriori adaptation error: $\nu(t+1) = \dfrac{\nu^0(t+1)}{1+\varphi_f^T(t)F(t)\varphi_f(t)}$

Adaptation gain: $F(t+1)^{-1} = \lambda_1(t)F(t)^{-1} + \lambda_2(t)\varphi_f(t)\varphi_f(t)^T$

$\lambda_1(t)$ and $\lambda_2(t)$ define the adaptation gain profile

$0 < \lambda_1(t) \le 1 \quad ; \quad 0 \le \lambda_2(t) < 2 \quad ; \quad F(0) > 0$

with: $\varphi_f(t) = L(q^{-1})\varphi(t)$

Choice of filter L: (for stability)

$(\mathrm{I}): L = G; \quad (\mathrm{II}): L = \hat{G}; \quad (\mathrm{III}): L = \dfrac{\hat{A}_M}{\hat{P}}\hat{G}$

Best performance (III)

Stability and Convergence condition -IIR

$$H'\left(z^{-1}\right)=H\left(z^{-1}\right)-\frac{\lambda_2}{2}=\text{Strictly Positive Real }(S.P.R.).\qquad (*)$$

for Alg. III: $$H\left(z^{-1}\right)=\frac{\hat{P}A_M G}{P\hat{A}_M\hat{G}}$$

const. adapt. gain: $\lambda_2=0$ time varying. adapt. gain: $\lambda_2\le 2$

$$P\left(z^{-1}\right)=A_M\left(z^{-1}\right)\hat{S}\left(z^{-1}\right)-B_M\left(z^{-1}\right)\hat{R}\left(z^{-1}\right);\ \hat{P}\left(z^{-1}\right)=\hat{A}_M\left(z^{-1}\right)\hat{S}\left(z^{-1}\right)-\hat{B}_M\left(z^{-1}\right)\hat{R}\left(z^{-1}\right)$$

A_M, B_M and G are constant and very good estimations are available

Condition (*) for λ_2=1 becomes: $$\left|\left(\frac{A_M}{\hat{A}_M}\cdot\frac{\hat{P}}{P}\cdot\frac{G}{\hat{G}}\right)^{-1}-1\right|<1 \text{ for all } \omega$$

Rem: Alg III needs initialization with Alg II in order to get a first estimation of $\hat{P}$

An interpretation of the SPR condition

For constant adaptation gain the SPR condition on *H'(=H in this case)* implies that the angle between the inverse of the true gradient (which can not be computed) and the direction of correction (*defined by* φ_f) is less than 90° in all directions. (see Appendix)

For time varying adaptation gains a similar interpretation holds

Case II : non perfect matching + measurement noise

Measurement noise

In the presence of measurement noise independent of the disturbance, convergence occurs under same conditions as for stability in the deterministic context

Non perfect matching

- Boundedness of the variables can be guaranteed under mild hypotheses
- The approximation in the frequency domain is given by:

$$\hat{\theta}^* = \arg\min_{\hat{\theta}} \int_{-\pi}^{\pi} \left[\left|S_{NM}\right|^2 \left|N - \hat{N}\right|^2 \left|S_{\hat{N}M}\right|^2 \left|G\right|^2 \Phi_d(\omega) + \Phi_w(\omega) \right] d\omega$$

With : $S_{NM} = \frac{1}{1 - NM}$; $S_{\hat{N}M} = \frac{1}{1 - \hat{N}M}$

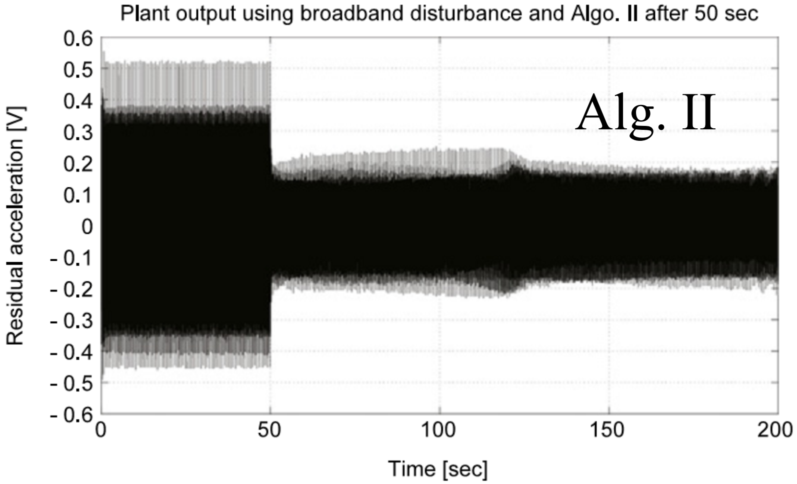
$\Phi_d = PSD\ of\ d$

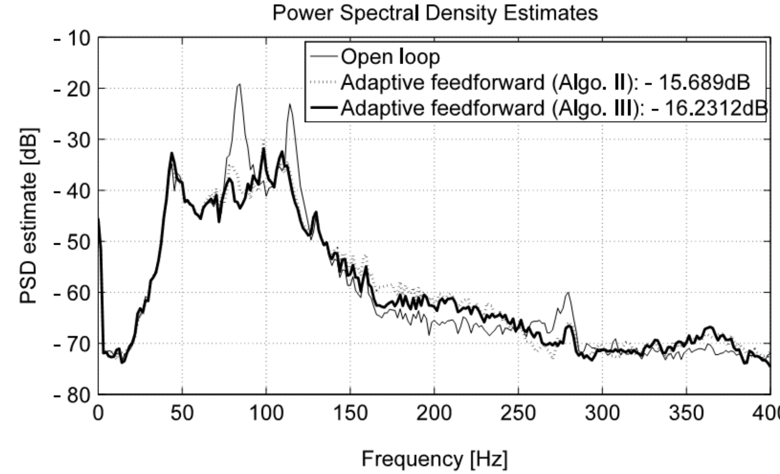
$\Phi_w = PSD\ of\ meas.\ noise$

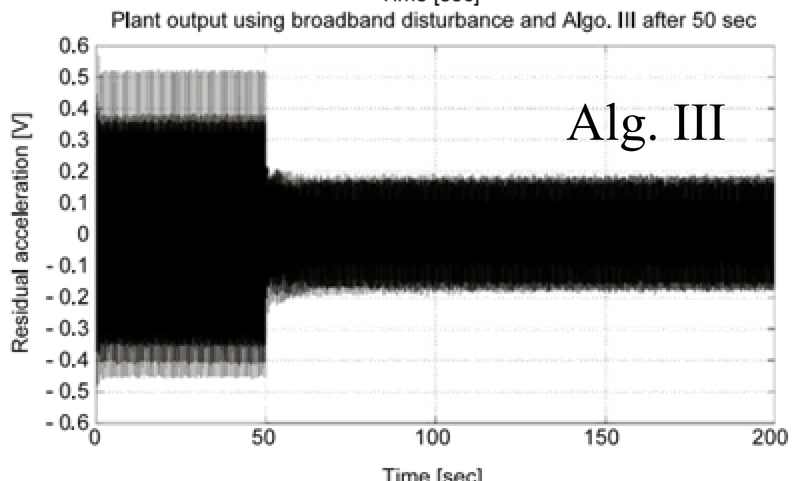
- *Good approximation of the optimal compensator N will be obtained in the frequency regions where Φ_d is significant and G has high gain.*
- *Further wheighting is introduced by the sensitivity function of the internal loop*

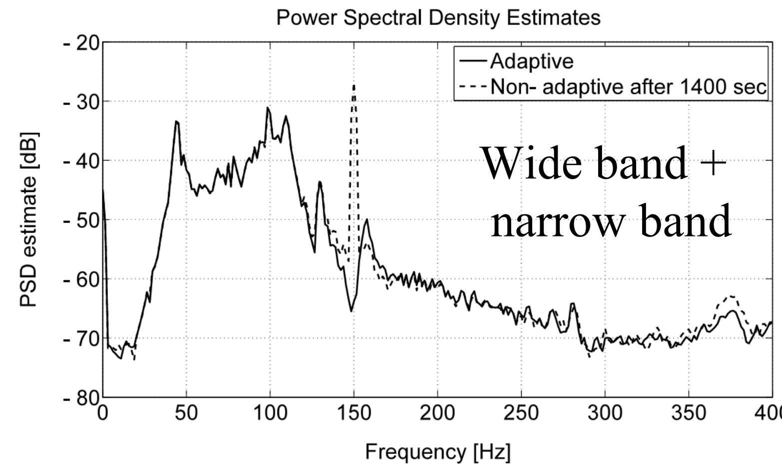
Adaptive IIR feedforward compensator – Experimental results

Time varying matrix adaptation gain (decreasing gain + constant trace)







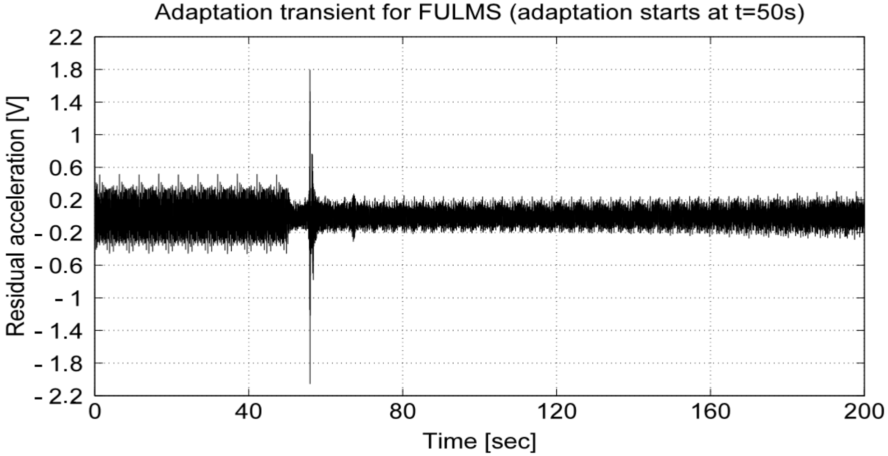


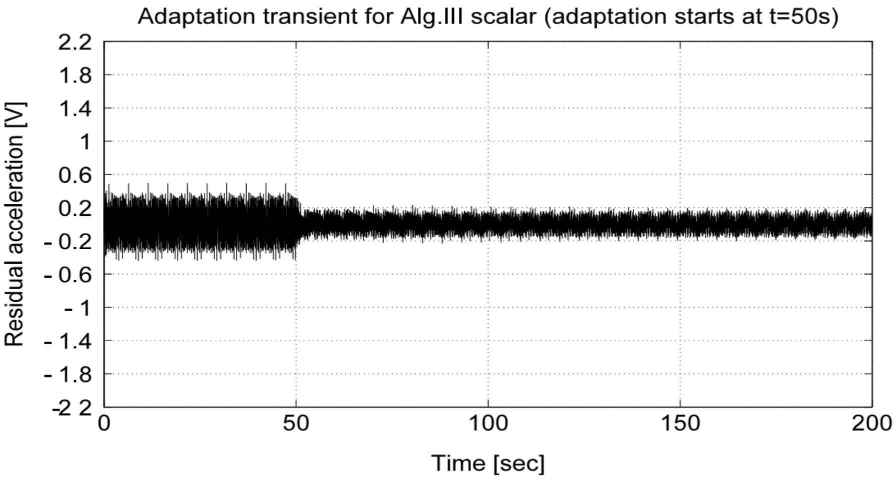
Influence of the number of parameters upon the global attenuation.

Number of parameters	20	32	40
Global attenuation (db)	16.23	16.49	16.89

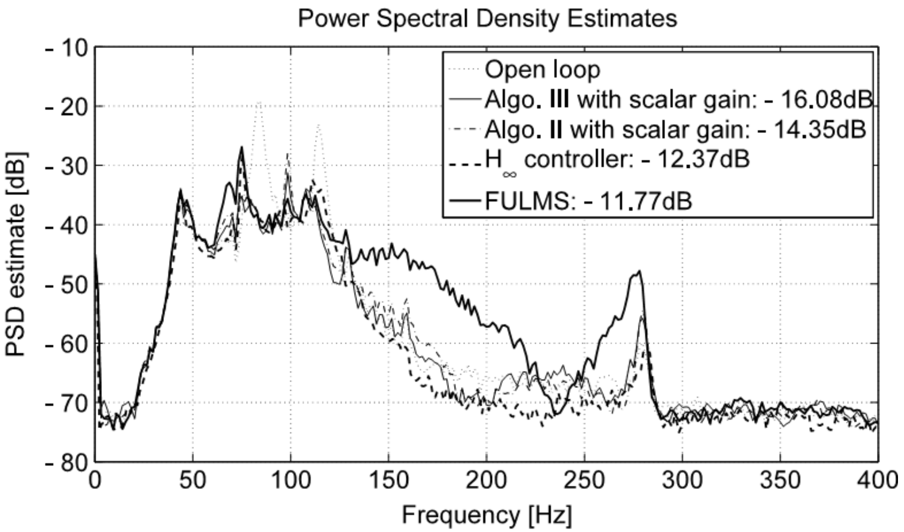
Adaptive IIR feedforward compensator – Experimental results

Scalar adaptation gain – comparison with FULMS

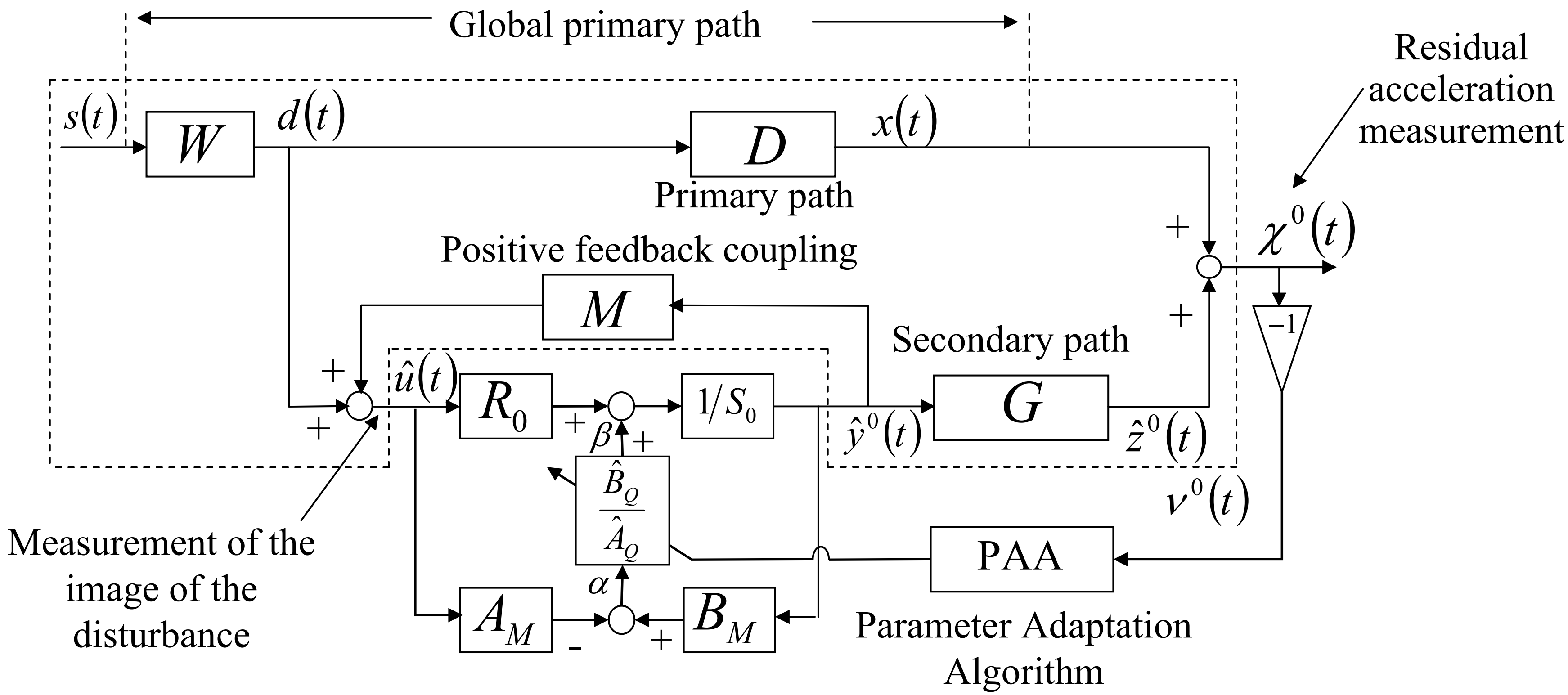




Instability occurs on a long run



IIR Filter + Adaptive YK_IIR filter feedforward compensation



$\hat{A}_Q \equiv 1,$
$\hat{B}_Q = \hat{Q}$
YK_FIR

IIR Filter + Adaptive Youla Kucera (IIR) – Basic equations

Optimal Q filter: $$Q\left(z^{-1}\right)=\frac{B_Q\left(z^{-1}\right)}{A_Q\left(z^{-1}\right)}=\frac{b_0^Q+b_1^Q z^{-1}+\ldots+b_{nB_Q}^Q z^{-nB_Q}}{1+a_1^Q z^{-1}+\ldots+a_{nA_Q}^Q z^{-nA_Q}}$$

$$\hat{R}\left(q^{-1}\right)=\hat{A}_Q\left(q^{-1}\right)R_0\left(q^{-1}\right)-A_M\left(q^{-1}\right)\hat{B}_Q\left(t,q^{-1}\right);\quad \hat{S}\left(q^{-1}\right)=\hat{A}_Q\left(q^{-1}\right)S_0\left(q^{-1}\right)-B_M\left(q^{-1}\right)\hat{B}_Q\left(t,q^{-1}\right)$$

Perfect matching → $$\frac{G\cdot A_M\cdot\left(R_0A_Q-A_MB_Q\right)}{A_Q(A_MS_0-B_MR_0)}=-D$$

Characteristic polynomial of the internal feedback loop:

$$P\left(z^{-1}\right)=A_Q\left[A_M\left(z^{-1}\right)S_0\left(z^{-1}\right)-B_M\left(z^{-1}\right)R_0\left(z^{-1}\right)\right]$$

YK_FIR
$A_Q=1$

Define: $$\hat{\theta}^T(t)=\left[\hat{b}_0^Q(t),\ldots,\hat{b}_{n_{B_Q}}^Q(t),\hat{a}_1^Q(t),\ldots,\hat{a}_{n_{A_Q}}^Q(t)\right]$$

$$\hat{\alpha}(t+1)=B_M\hat{y}(t+1)-A_M\hat{u}(t+1);\quad \hat{\beta}(t)=S_0\hat{y}(t)-R_0\hat{u}(t)$$

with $$\varphi^T(t)=\left[\hat{\alpha}(t+1),\ldots,\hat{\alpha}\left(t-n_{B_Q}+1\right),-\hat{\beta}(t),\ldots,-\hat{\beta}\left(t-n_{A_Q}\right)\right]$$

Compensator output:

$$\hat{y}^0(t+1)=-S_0^*\hat{y}(t)+R_0\hat{u}(t+1)+\hat{\theta}^T(t)\varphi(t)$$

$$\hat{y}(t+1)=-S_0^*\hat{y}(t)+R_0\hat{u}(t+1)+\hat{\theta}^T(t+1)\varphi(t)$$

Parametric adaptation algorithms (PAA)

Measurement (residual acceleration): $\chi^0(t+1)$

A priori adaptation error: $\nu^0(t+1) = -\chi^0(t+1)$

Parametric adaptation algorithm: $\hat{\theta}(t+1) = \hat{\theta}(t) + F(t)\varphi_f(t)\nu(t+1)$

A posteriori adaptation error: $\nu(t+1) = \dfrac{\nu^0(t+1)}{1+\varphi_f^T(t)F(t)\varphi_f(t)}$

Adaptation gain: $F(t+1)^{-1} = \lambda_1(t)F(t)^{-1} + \lambda_2(t)\varphi_f(t)\varphi_f(t)^T$

$\lambda_1(t)$ and $\lambda_2(t)$ define the adaptation gain profile: $0 < \lambda_1(t) \le 1 \;;\; 0 \le \lambda_2(t) < 2 \;;\; F(0) > 0$

with: $\varphi_f(t) = L(q^{-1})\varphi(t)$

Choice of filter L: (for stability)

$(\mathrm{I}): L = G; \quad (\mathrm{II}): L = \hat{G};$

$(\mathrm{III}): L = \dfrac{\hat{A}_M}{\hat{P}}\hat{G}$ ← Best performance

$$\nu(t+1) = \frac{A_M(q^{-1})}{P(q^{-1})L(q^{-1})}G(q^{-1})\left[\theta - \hat{\theta}(t+1)\right]^T \varphi_f(t)$$

Stability and Convergence condition

$$H\left(z^{-1}\right)-\frac{\lambda_2}{2}=\text{Strictly Positive Real } (S.P.R.).$$

for Alg. III: $H\left(z^{-1}\right)=\frac{\hat{P}A_M G}{P\hat{A}_M\hat{G}}$ (for IIR ,YK_FIR, YK_IIR)

const. adapt. gain: $\lambda_2 = 0$ time varying. adapt. gain: $\lambda_2 \leq 2$

Adaptive IIR $P\left(z^{-1}\right)=A_M\left(z^{-1}\right)\hat{S}\left(z^{-1}\right)-B_M\left(z^{-1}\right)\hat{R}\left(z^{-1}\right)$

IIR + adaptiveYK_FIR $P\left(z^{-1}\right)=A_M\left(z^{-1}\right)S_0\left(z^{-1}\right)-B_M\left(z^{-1}\right)R_0\left(z^{-1}\right)$

constant and known

IIR + adaptiveYK_IIR $P\left(z^{-1}\right)=\hat{A}_Q\left[A_M\left(z^{-1}\right)S_0\left(z^{-1}\right)-B_M\left(z^{-1}\right)R_0\left(z^{-1}\right)\right]$

A_M, B_M and G are constant and very good estimations are available

Theoretical comparative analysis

Adaptive IIR, Fix IIR+adaptive YK_FIR, Fix IIR+adaptive YK_IIR,
*same **Strictly Positive Real** condition for stability and convergence*

Differences

- **The poles of the internal loop**

IIR: unknown, can be very close to the unit circle
IIR+YK_FIR: assigned and fixed
IIR+YK_IIR: partly assigned and fixed, the remaining can be bounded

- **The SPR condition** (to be achieved by filtering)

IIR: one needs an estimation of the poles to build the filter III
IIR+YK_FIR: the poles for the filter III are known from the begining
IIR+YK_IIR: some poles are known from the beginning, the other are directly obtained from the PAA

- **Use of a model based designed compensator for initialization**

IIR: difficult (dimension problem)
IIR+ YK_FIR or +YK_IIR: easy (it is the central stabilizing controller)

Summary of experimental results

Disturbance : wide band disturbance

Comparable performance both in frequency and in time domains

Global attenuation

Number of adjustable parameters	0	8	16	32	40
Global attenuation-IIR (db)	-			**16.49**	16.89
Global att.-YK_FIR/H_∞ (db)	14.70	15.4	15.6	**16.52**	16.03
Global att.-YK_FIR/PP (db)	4.61	14.69	**15.89**	15.7	15.33
Global att.-YK_IIR/H_∞ (db)	14.70	**16.53**	16.47		
Global att.-YK_IIR/PP (db)	4.61	15.53	**16.21**		

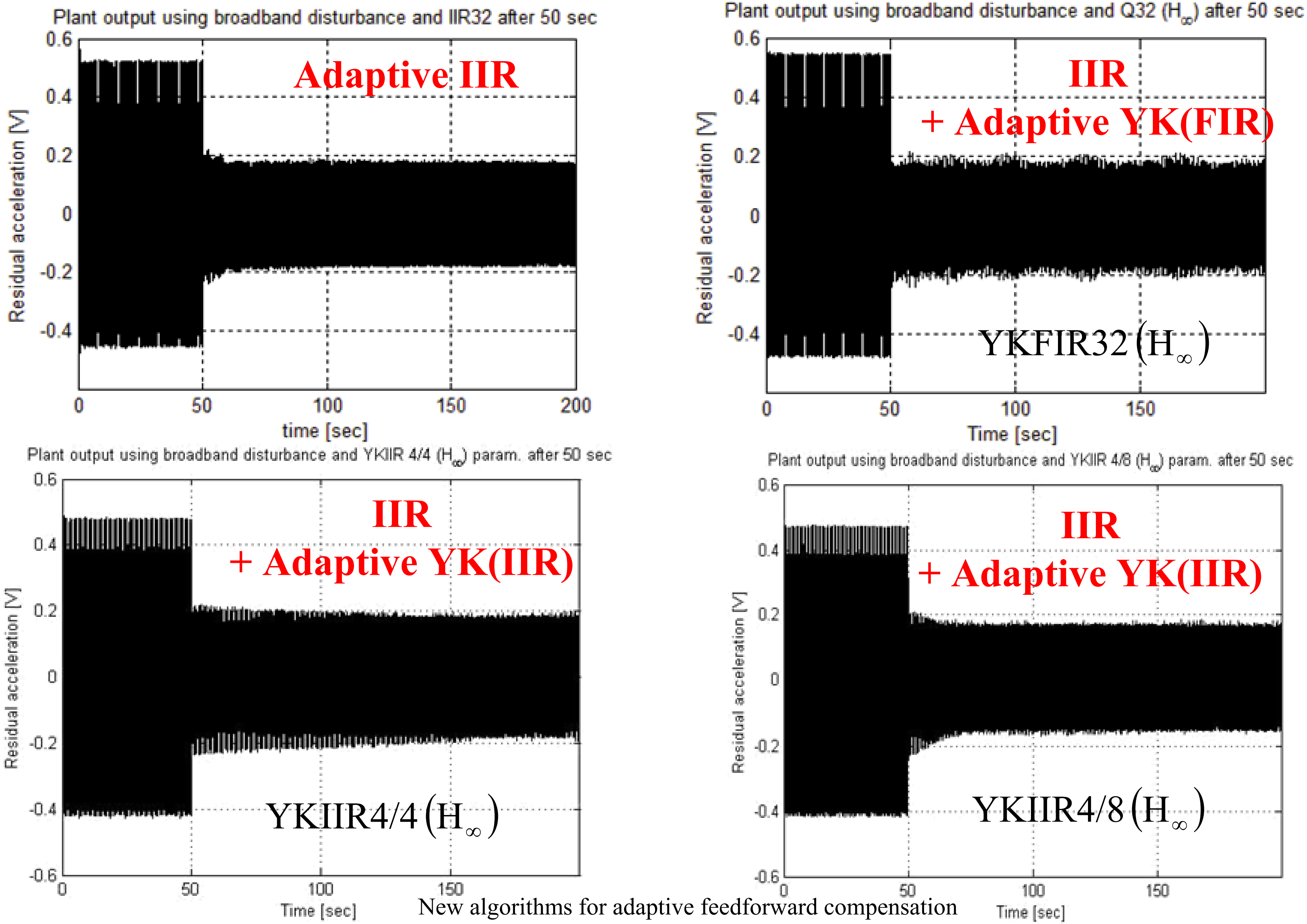
H_∞ = H_∞ MBD central controller

PP = Pole Placement central controller

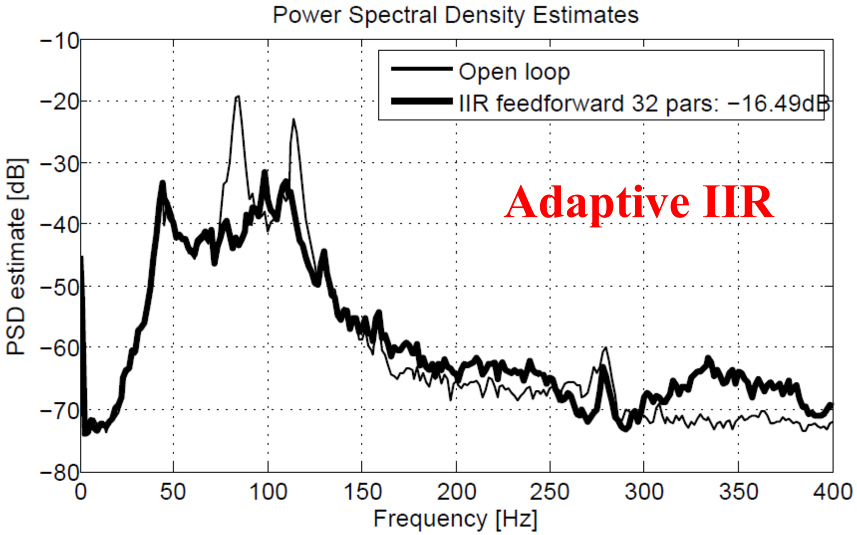
Model Based Design requires identification of the disturbance and of the primary path

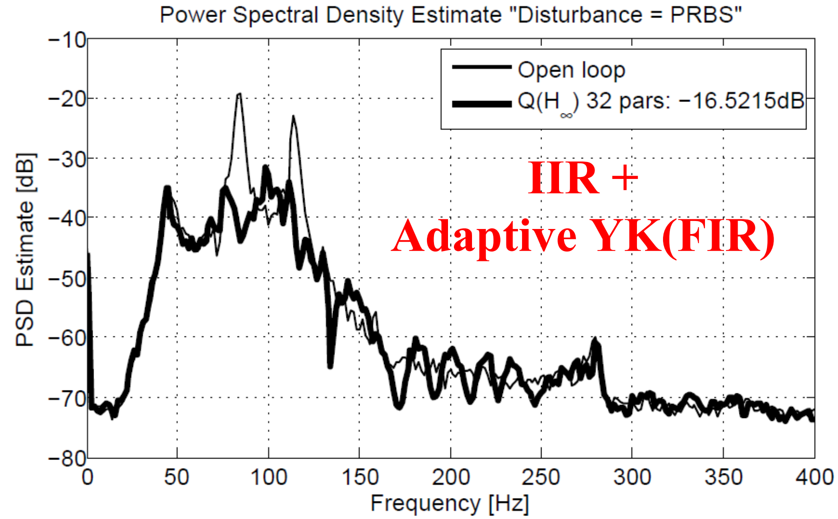
For IIR+YK_F(I)IR , performance depends upon the central controller

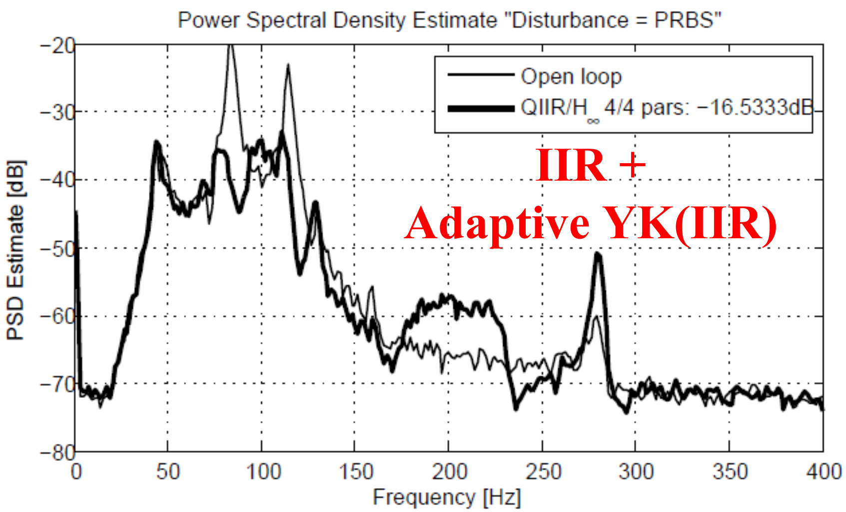
Time domain results (32 parameters)



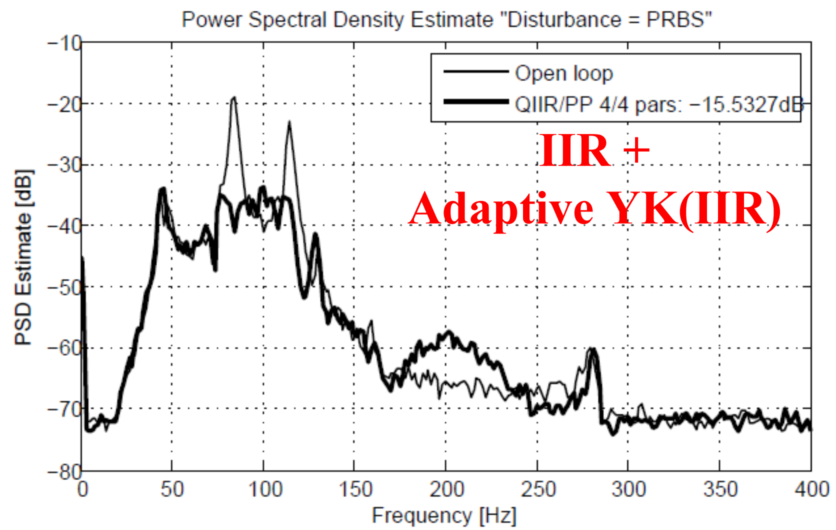
Frequency domain results (32 parameters)







YKIIR4/4 (H_∞)



YKIIR4/4 (PP)

Concluding Remarks

- Adaptive IIR, Fix IIR + adaptive YK_FIR and Fix IIR + adaptive YK_FIR have close performances
- The performances of the Fix IIR + adaptive YK_FIR depends upon the performance of the central controller.
- The performance of the Fix IIR + adaptive YK_IIR is less dependent upon the performance of the central controller
- Fix IIR + adaptive YK_FIR allows the easiest implementation of the algorithm (the filter for SPR condition)
- **Fix IIR + adaptive YK_IIR offers the best ratio performance/nb. of adaptive parameters**

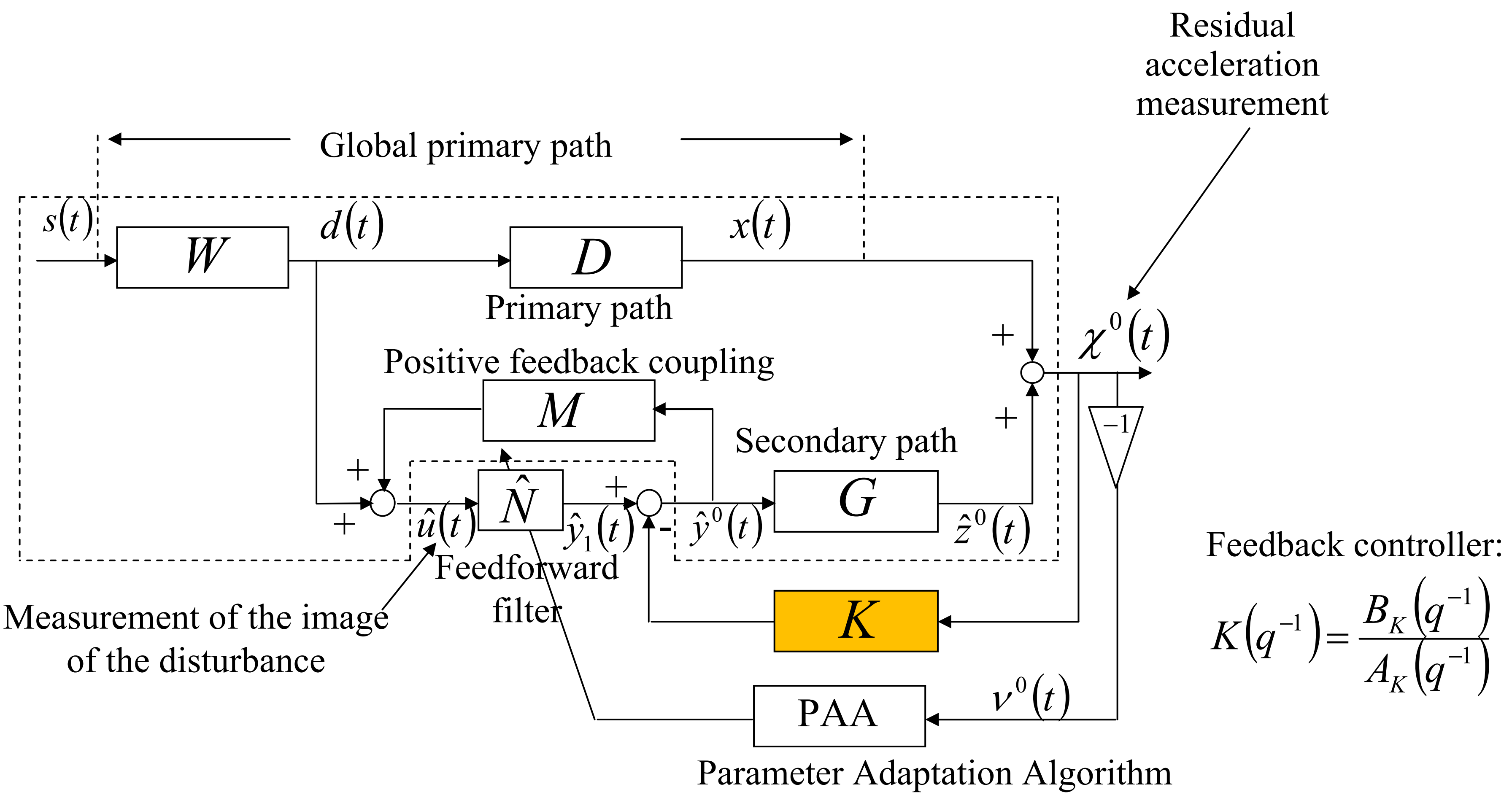
Adaptive IIR fedforward compensation + feedback controller

The "logical" approach in active vibration control (AVC):
Do as much as possible by feedback (limitations due to Bode Integral) and then add feedforward compensation of disturbance

The "reality":
The developments in the field of AVC started by using feedforward compensation and only recently the interest of adding feedback has been recognized (Eyzmailzadeh et al 2002 Ray et al 2006, de Callafon 2010)

No reference available for the analysis of the algorithms for adaptive feedforward compensation:
- in the presence of a feedback controller
- in the joint presence of a feedback controller and of an internal positive feedback coupling

Adaptive IIR feedforward compensation + feedback controller



Stability and Convergence condition

Same structure of the adaptation algorithm but different filtering

Choice of filter L: $(\mathrm{I}): L = G; \quad (\mathrm{II}): L = \dfrac{\hat{G}}{1+\hat{G}K};$ $\quad (\mathrm{III}): L = \dfrac{\hat{A}_M \hat{A}_G \hat{A}_K}{\hat{P}_{fb-ff}} \hat{G}$ ← Best performance

$$H(z^{-1}) - \frac{\lambda_2}{2} = \text{Strictly Positive Real } (S.P.R.).$$

const. adapt. gain: $\lambda_2 = 0$ time varying. adapt. gain: $\lambda_2 \le 2$

for Alg. III: $$H(z^{-1}) = \frac{\hat{P}_{fb-ff} A_M A_G G}{P_{fb-ff} \hat{A}_M \hat{A}_G \hat{G}} = \frac{\hat{P}_{fb-ff} A_M B_G}{P_{fb-ff} \hat{A}_M \hat{B}_G}$$

$$\hat{P}_{fb-ff}(z^{-1}) = \hat{A}_M \hat{S}\left[\hat{A}_G A_K + \hat{B}_G B_K\right] - \hat{B}_M \hat{R} A_K \hat{A}_G$$

- $M(A_M, B_M)$ and $G(A_G, B_G)$ are constant and very good estimations are available
- Implementation of alg. III requires an estimation of S and R (one runs alg. II during an initialization horizon)
- It is possible to continuously update $\hat{S}$ and $\hat{R}$ in the filter

Summary of experimental results

Disturbance : wide band disturbance

Global attenuation

	Feedback only	Feedforward only (H_{inf})	Adaptive Feedforward only	Feedback & Adaptive Feedforward	Feedforward (H_{inf}) & Feedback
Attenuation	-14.40 dB	-14.70 dB	-16.23 dB	-20.53 dB	-18.42 dB

Feedback controller: nB_K=16, nA_K=18 (Pole placement with sensitivity functions shaping)

Adaptive IIR feedforward compensator: 20 adjustable parameters (n_R=10, n_S=9)

Attention:
The design of the fixed H_{inf} feedforward compensator (last column) requires the knowledge of the disturbance characteristics (unknown and time varying in practice) and of the model of the primary path (not required by the adaptive algorithms)

Concluding Remarks

- Adding feedback to adaptive feedforward compensation improves significantly the performance of AVC
- The stability conditions for the adaptive feedforward algorithms are drastically changed by adding feedback
- The filter used in the adaptive feedforward algorithms depends upon the parameters of the feedback controller

Future work:
Combining adaptive feedforward compensation with adaptive feedback regulation

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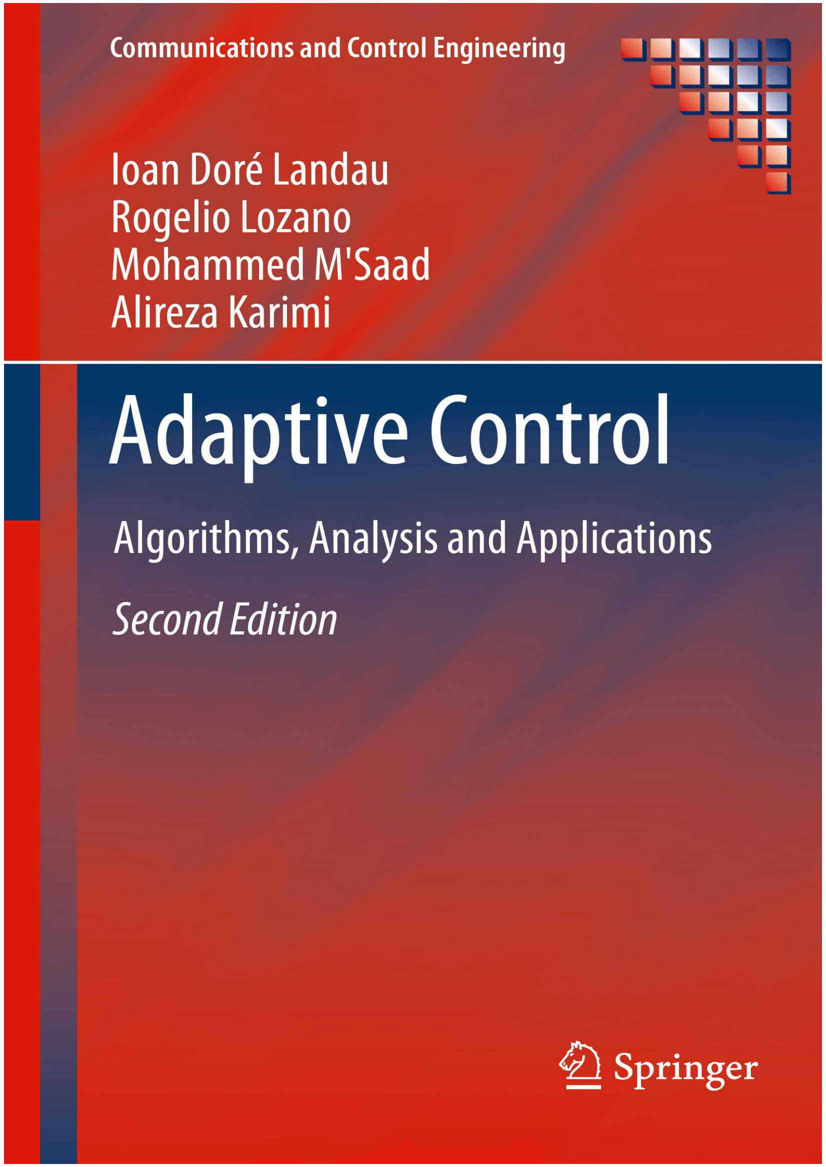
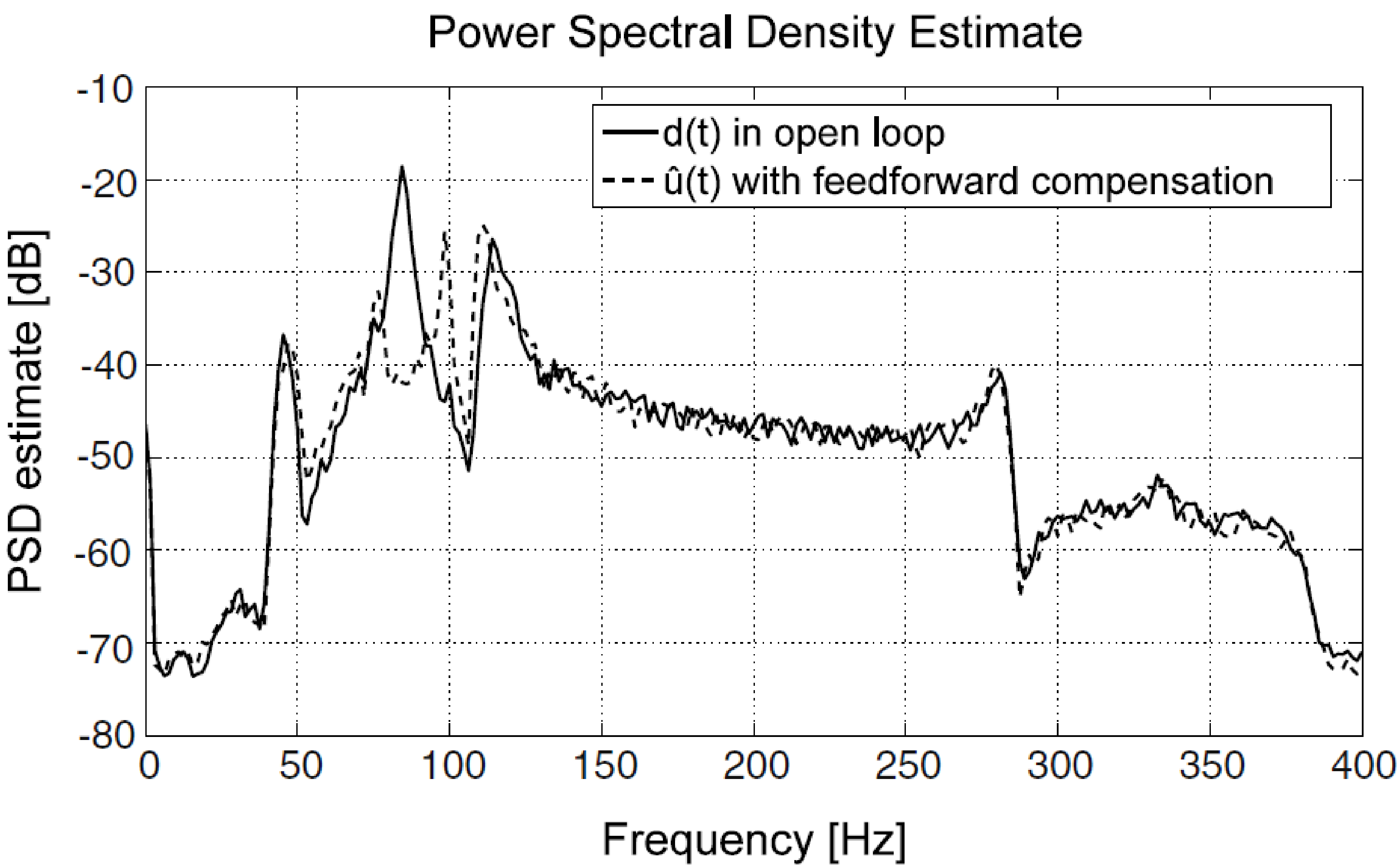


http://www.landau-adaptivecontrol.org/

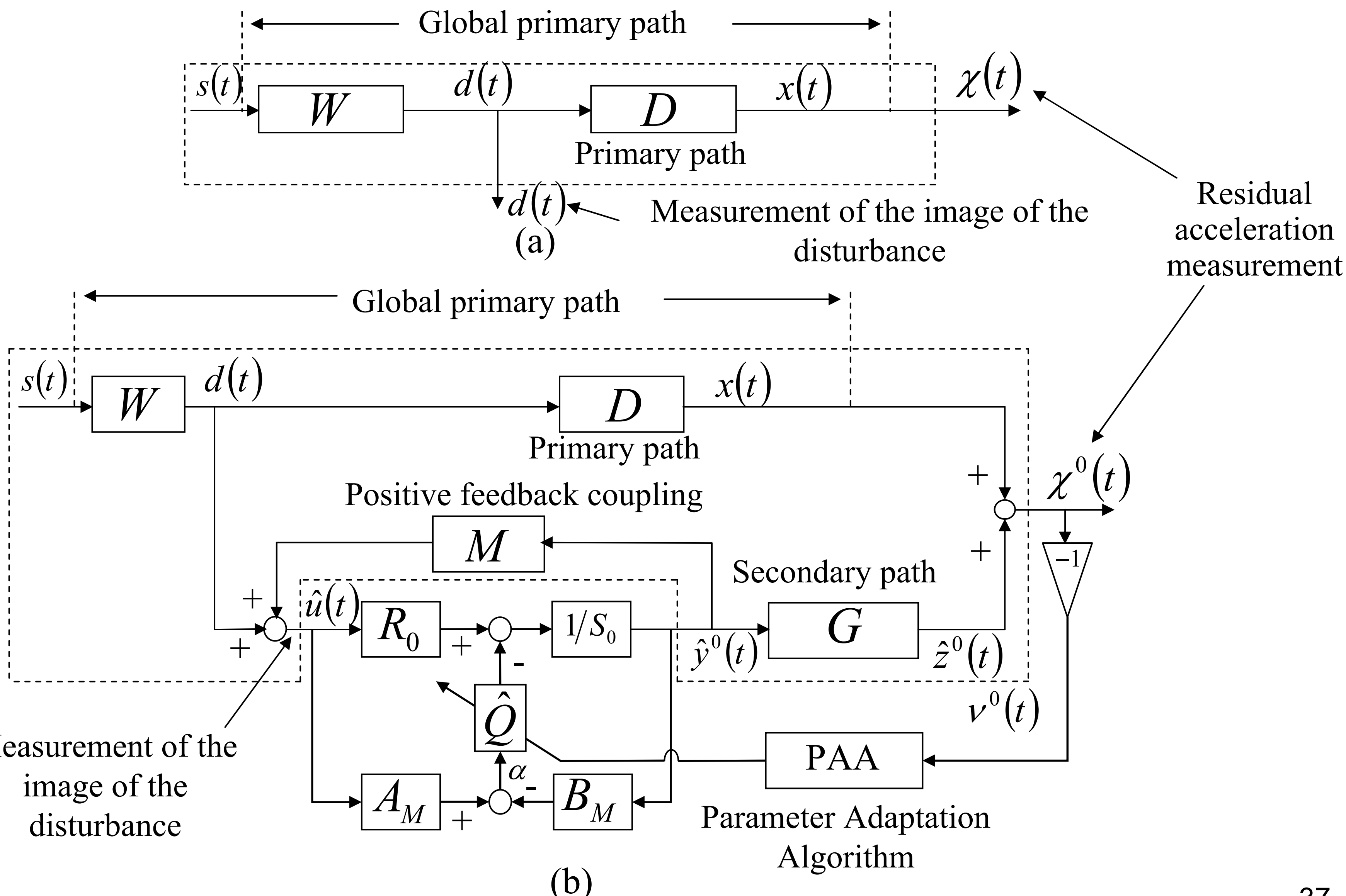
Appendix

Influence of the physical feedback upon the source d(t)



The influence is significant and can not be neglected

IIR Filter + Adaptive YK (FIR) filter feedforward compensation



New algorithms for adaptive feedforward compensation

IIR Filter + Adaptive Youla Kucera (FIR) – Basic equations

$$\hat{R}\left(q^{-1}\right)=R_0\left(q^{-1}\right)-A_M\left(q^{-1}\right)\hat{Q}\left(t,q^{-1}\right);\quad \hat{S}\left(q^{-1}\right)=S_0\left(q^{-1}\right)-B_M\left(q^{-1}\right)\hat{Q}\left(t,q^{-1}\right)$$

Optimal Q filter: $Q\left(z^{-1}\right)=q_0+q_1z^{-1}+\ldots+q_{n_Q}z^{-n_Q}$

Perfect matching → $\dfrac{G\cdot A_M\cdot\left(R_0-A_MQ\right)}{A_MS_0-B_MR_0}=-D$

Characteristic polynomial of the internal feedback loop:

$P\left(z^{-1}\right)=A_M\left(z^{-1}\right)S_0\left(z^{-1}\right)-B_M\left(z^{-1}\right)R_0\left(z^{-1}\right)$ (remains unchanged)

Define: $\theta^T(t)=\left[\hat{q}_0(t)\ldots\hat{q}_{n_Q}(t)\right];\quad \varphi^T(t)=\left[\hat{\alpha}(t+1)\ldots\hat{\alpha}\left(t-n_Q+1\right)\right]$

with $\hat{\alpha}(t+1)=B_M\hat{y}(t+1)-A_M\hat{u}(t+1)$

Compensator output: $\hat{y}^0(t+1)=-S_0^*\hat{y}(t)+R_0\hat{u}(t+1)+\hat{\theta}^T(t)\varphi(t)$

$$\hat{y}(t+1)=-S_0^*\hat{y}(t)+R_0\hat{u}(t+1)+\hat{\theta}^T(t+1)\varphi(t)$$

IIR Feedforward+ Feedback - Basic equations (1)

Feedforward compensator : $$\hat{N}(t,q^{-1}) = \frac{\hat{R}(t,q^{-1})}{\hat{S}(t,q^{-1})}$$

Perfect matching (optimal) → $$\frac{GN}{1-NM} = \frac{G \cdot A_M R}{A_M S - B_M R} = -D$$

The algorithms have been developed under "perfect matching" assumption and analyzed in the context of non perfect matching

Characteristic polynomial of the internal "positive" feedback loop:

$$P(z^{-1}) = A_M(z^{-1})\hat{S}(z^{-1}) - B_M(z^{-1})\hat{R}(z^{-1})$$

Characteristic polynomial of the feedback loop:

$$P_{cl}(z^{-1}) = A_G(z^{-1})A_K(z^{-1}) + B_G(z^{-1})B_K(z^{-1})$$

Characteristic polynomial of the coupled feedforward-feedback loop:

$$P_{fb-ff}(z^{-1}) = A_M S[A_G A_K + B_G B_K] - B_M R A_K A_G$$

IIR Feedforward+ Feedback - Basic equations (2)

Define:

$$\hat{\theta}^T(t) = \left[\hat{s}_1(t), \ldots, \hat{s}_{n_S}(t), \hat{r}_0(t), \ldots, \hat{r}_{n_R}(t)\right]$$

$$\varphi^T(t) = \left[-\hat{y}_1(t), \ldots, -\hat{y}_1(t - n_S + 1), \hat{u}(t+1), \ldots, \hat{u}(t - n_R + 1)\right]$$

Feedforward Compensator output:

$$\hat{y}^\circ(t+1) = \hat{y}\left(t+1 / \hat{\theta}(t)\right) = \hat{\theta}^T(t)\varphi(t) \qquad \hat{y}(t+1) = \hat{y}\left(t+1 / \hat{\theta}(t+1)\right) = \hat{\theta}^T(t+1)\varphi(t)$$

Measurement (residual acceleration): $\chi^0(t+1)$

A priori adaptation error: $\nu^0(t+1) = \nu(t+1 / \hat{\theta}(t)) = -\chi^0(t+1)$

A posteriori adaptation error: $\nu(t+1) = \nu(t+1 / \hat{\theta}(t+1))$

Filtered observation vector: $\varphi_f(t) = L(q^{-1})\varphi(t)$

$$\nu(t+1) = \frac{A_M(q^{-1})A_G(q^{-1})A_K(q^{-1})}{P_{fb-ff}(q^{-1})L(q^{-1})} G(q^{-1})\left[\theta - \hat{\theta}(t+1)\right]^T \varphi_f(t)$$

Parametric adaptation algorithms (PAA)

Measurement (residual acceleration): $\chi^0(t+1)$

A priori adaptation error: $\nu^0(t+1) = -\chi^0(t+1)$

Parametric adaptation algorithm: $\hat{\theta}(t+1) = \hat{\theta}(t) + F(t)\varphi_f(t)\nu(t+1)$

A posteriori adaptation error: $\nu(t+1) = \dfrac{\nu^0(t+1)}{1+\varphi_f^T(t)F(t)\varphi_f(t)}$

Adaptation gain: $F(t+1)^{-1} = \lambda_1(t)F(t)^{-1} + \lambda_2(t)\varphi_f(t)\varphi_f(t)^T$

$\lambda_1(t)$ and $\lambda_2(t)$ define the adaptation gain profile

$$0 < \lambda_1(t) \le 1 \;\; ; \;\; 0 \le \lambda_2(t) < 2 \;\; ; \;\; F(0) > 0$$

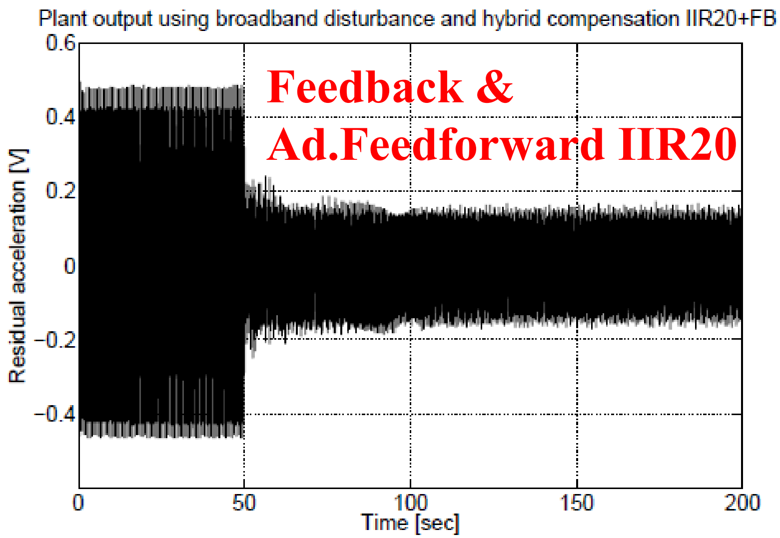
with: $\varphi_f(t) = L(q^{-1})\varphi(t)$

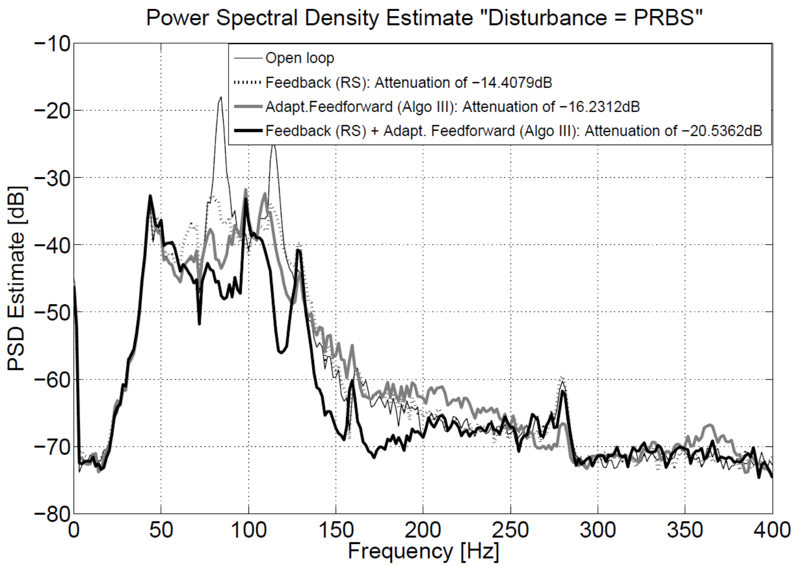
Choice of filter L: (for stability)

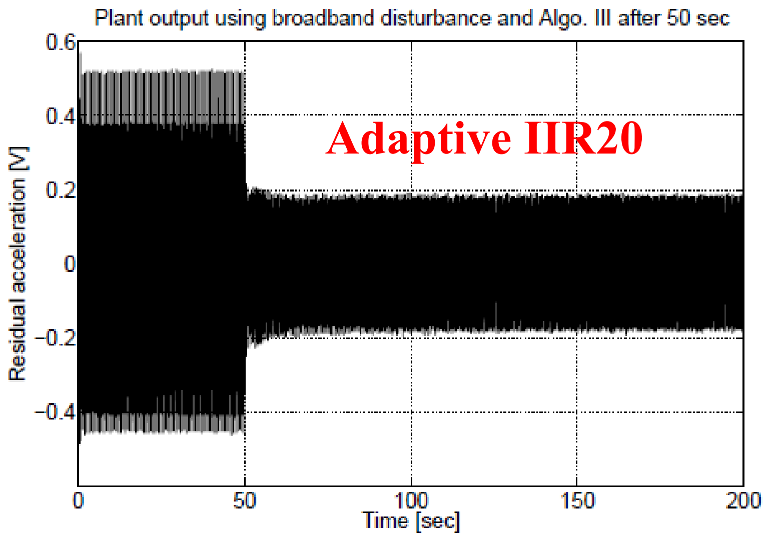
$$(\mathrm{I}): L = G; \quad (\mathrm{II}): L = \frac{\hat{G}}{1+\hat{G}K}; \quad (\mathrm{III}): L = \frac{\hat{A}_M \hat{A}_G \hat{A}_K}{\hat{P}_{fb-ff}}\hat{G}$$

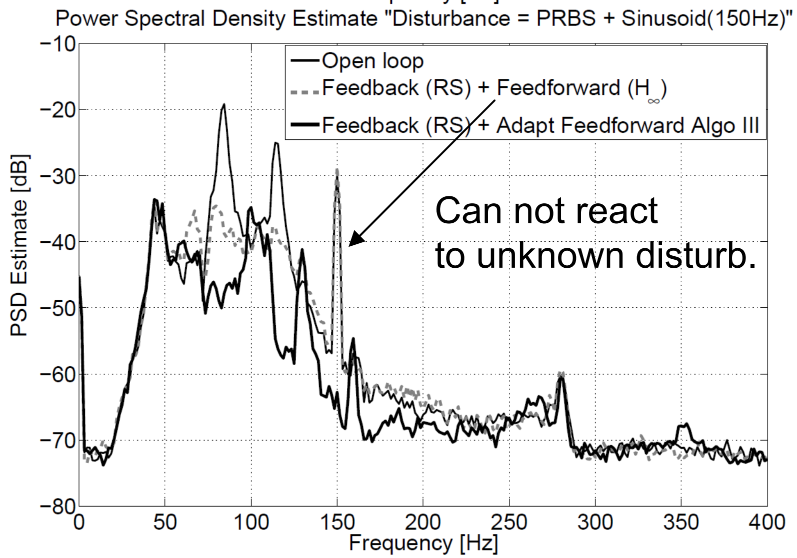
Best performance (III)

Time domain and frequency domain experimental results









Wide band disturbance + sinusoid 150 Hz

An interpretation of the SPR condition

Without filtering and neglecting the non commutativity of time arying operators

$$\nu(t+1) = \frac{A_M(q^{-1})G(q^{-1})}{P(q^{-1})}\left[\theta - \hat{\theta}(t+1)\right]^T \varphi\,(t) = \left[\theta - \hat{\theta}(t+1)\right]^T \varphi_f^{'}(t)$$

$$\varphi_f^{'}(t) = \frac{A_M(q^{-1})G(q^{-1})}{P(q^{-1})}\varphi(t)$$

Lets minimize $J(t+1) = \nu^2(t+1)$ using gradient technique

$$\frac{1}{2}\frac{\delta J(t+1)}{\delta\hat{\theta}(t+1)} = -\varphi_f^{'}(t)\nu(t+1) \qquad \hat{\theta}(t+1) = \hat{\theta}(t) + F\varphi_f^{'}(t)\nu(t+1);\ F = \alpha I,\ \alpha > 0$$

In fact one uses (alg III):

Approx. of gradient

$$\varphi_f(t) = \frac{\hat{A}_M}{\hat{P}}\hat{G}\varphi(t) \qquad \hat{\theta}(t+1) = \hat{\theta}(t) + F\overbrace{\varphi_f(t)\nu(t+1)}$$

SPR condition implies that the angle between the true gradient and the direction of correction is less than 90°

Interpretation of the SPR condition

Consider minimization of $J(t+1) = \nu^2(t+1)$ using gradient technique

$$\hat{\theta}(t+1) = \hat{\theta}(t) + F\varphi'_f(t)\nu(t+1);\ F = \alpha I,\ \alpha > 0$$

$$\varphi'_f(t) = \frac{A_M G}{P}\varphi(t)$$

Not available

In fact one uses (alg III):

$$\hat{\theta}(t+1) = \hat{\theta}(t) + F\underbrace{\varphi_f(t)\nu(t+1)}$$

Approx. of gradient

$$\varphi_f(t) = \frac{\hat{A}_M \hat{G}}{\hat{P}}\varphi(t)$$

SPR condition on *H'(=H in this case)* implies that the angle between the true gradient and the direction of correction is less than 90° in all directions

The "phase" of *H* is given by:

$$-90^\circ < \angle\frac{A_M(e^{-j\omega})G(e^{-j\omega})}{P(e^{-j\omega})} - \angle\frac{\hat{A}_M(e^{-j\omega})\hat{G}(e^{-j\omega})}{\hat{P}(e^{-j\omega})} < 90^\circ \qquad \text{at all frequencies}$$