Adaptive Regulation
Rejection of Unknown Multiple Narrow Band Disturbances
Theory and Applications

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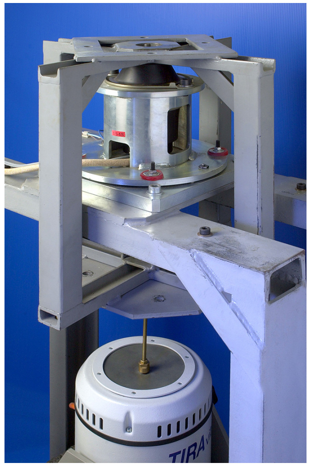

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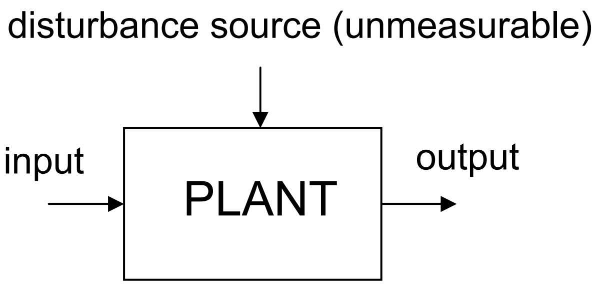
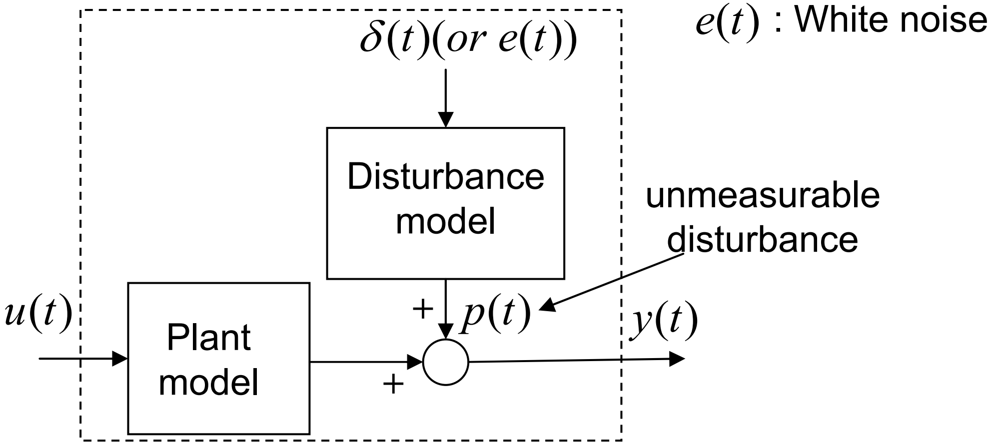
Ce qui est simple, est faux
Ce qui ne l’est pas est inutilisable

Paul Valéry
(Mauvaises pensées)

(What is simple, is false. What is not (simple) is unusable)

Adaptive Control



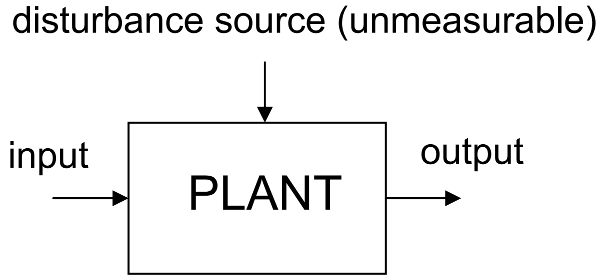


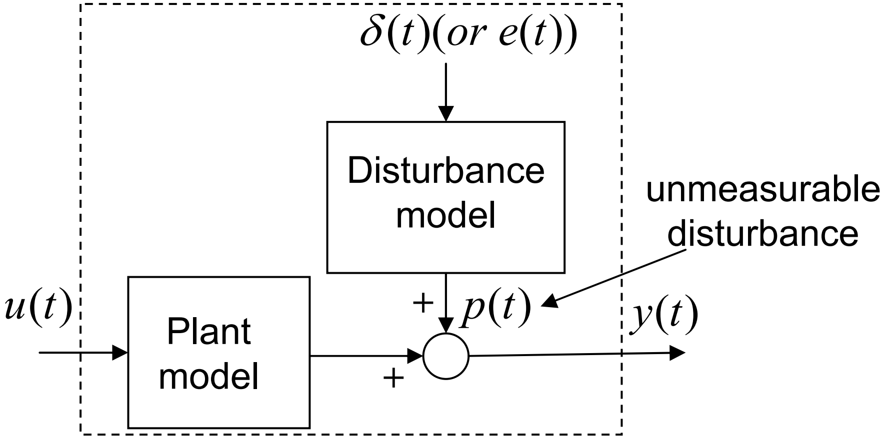
Objective : tracking/disturbance attenuation performance

- **Focus on adaptation with respect to plant model parameters variations**
- The model of the disturbance is assumed to be known and constant
- Only a level of attenuation in a frequency band is required*
- **No effort is made to simultaneously estimate the model of the disturbance**

*) Except for known DC disturbances (use of integrators)

Adaptive Regulation





Objective : Suppressing the effect of the (unknown) disturbance*

- **Focus on adaptation with respect to disturbance model parameters variations**
- Plant model is assumed to be known (a priori system identification) and almost constant
- Small plant parameters variations handled by a robust control design
- **No effort is made to simultaneously estimate the plant model**

*) Assumed to be characterized by a rational power spectrum if stationary

Adaptive Control (Regulation)

Direct adaptive control

The parameters of the controllers are directly adapted from measurement of a performance error

Indirect adaptive control

Two steps process:

1) Estimation of the plant (disturbance) model parameters
(or selection of a model) based on current measurements

2) Computation of the controller based on the identified (selected) model

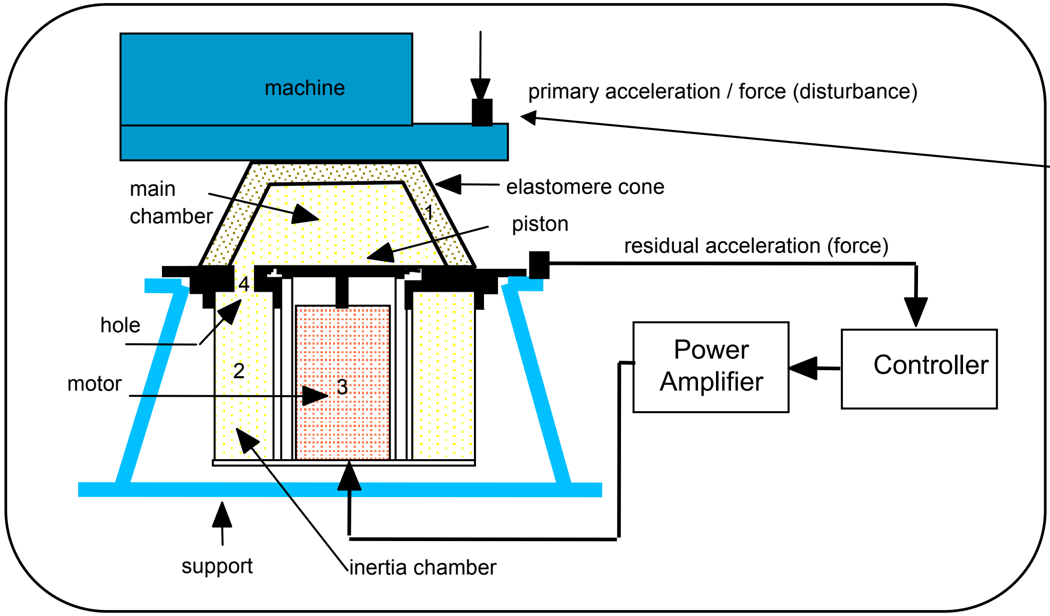
Outline

- Introduction
- Rejection of unknown narrow band disturbances in active vibration control.
  Real-time results on an active suspension
- Rejection of unknown narrow band stationary disturbances. *Basic facts*
- Adaptive control solutions (*indirect/direct*)
- Rejection of unknown **multiple** narrow band disturbances in active vibration control.
  Real-time results (inertial actuator)
- Conclusions

Rejection of unknown narrow band disturbances in active vibration control

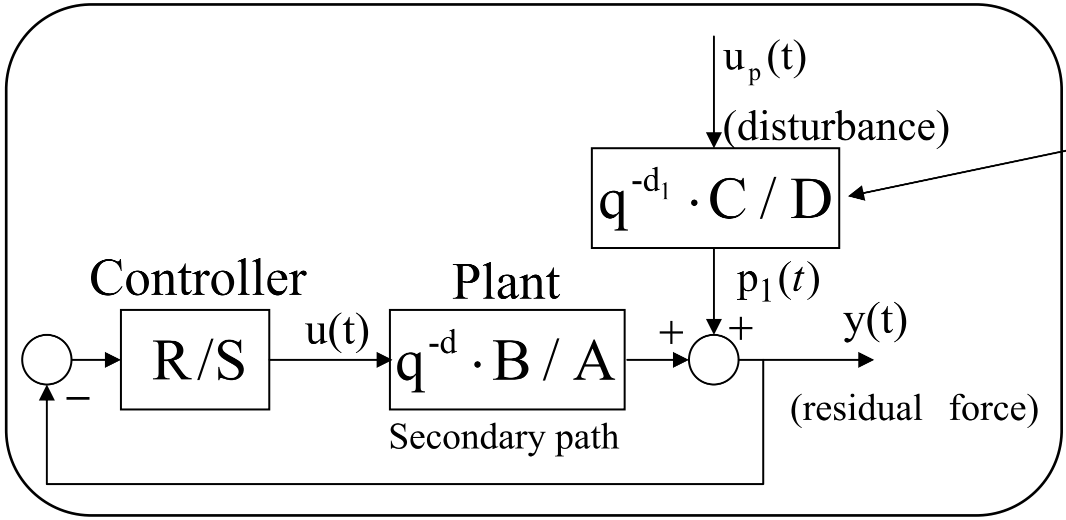
A first example

The Active Suspension System



Objective:

- **Reject the effect of unknown and variable narrow band disturbances**
- **Do not use an additional measurement**

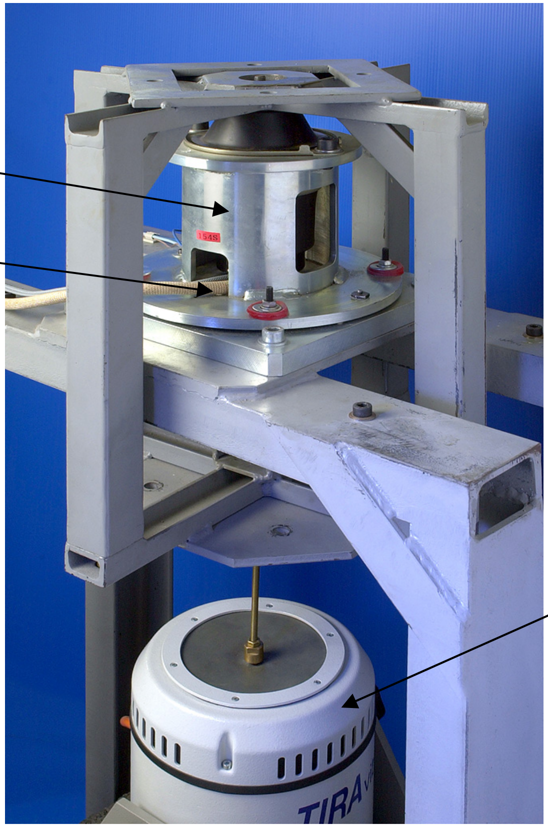


Two paths :

- Primary
- Secondary (double differentiator)

$$T_s = 1.25\,\text{ms}\ (f_s = 800 Hz)$$

The Active Suspension



Active suspension

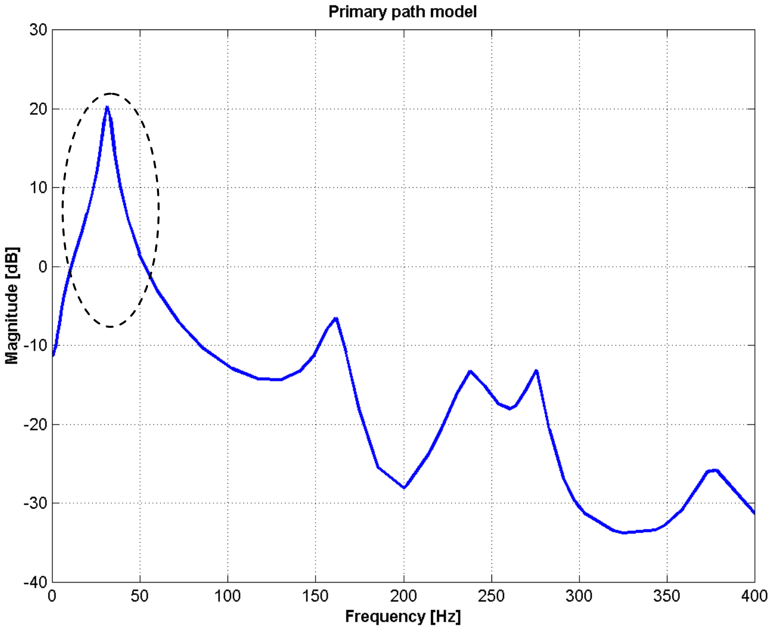
Residual force (acceleration) measurement

Primary force (acceleration) (the shaker)

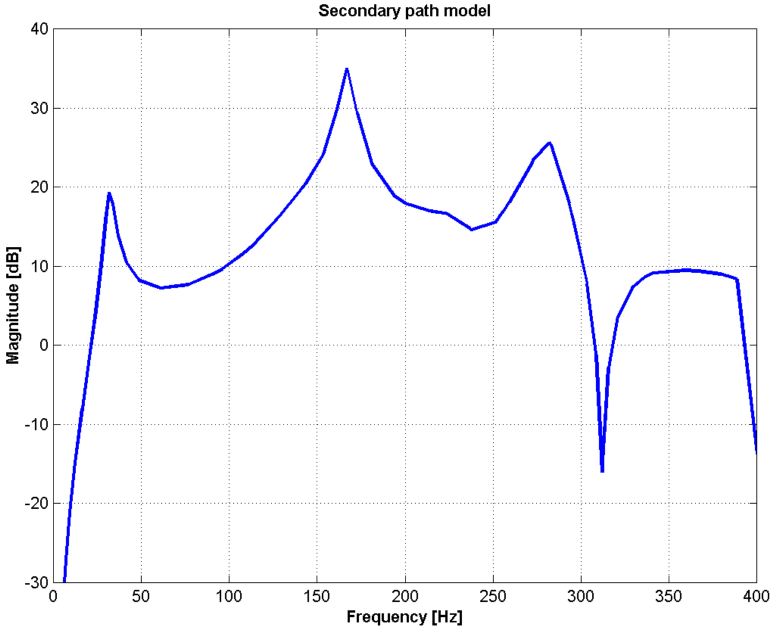
Active Suspension

Frequency Characteristics of the Identified Models

Primary path



Secondary path

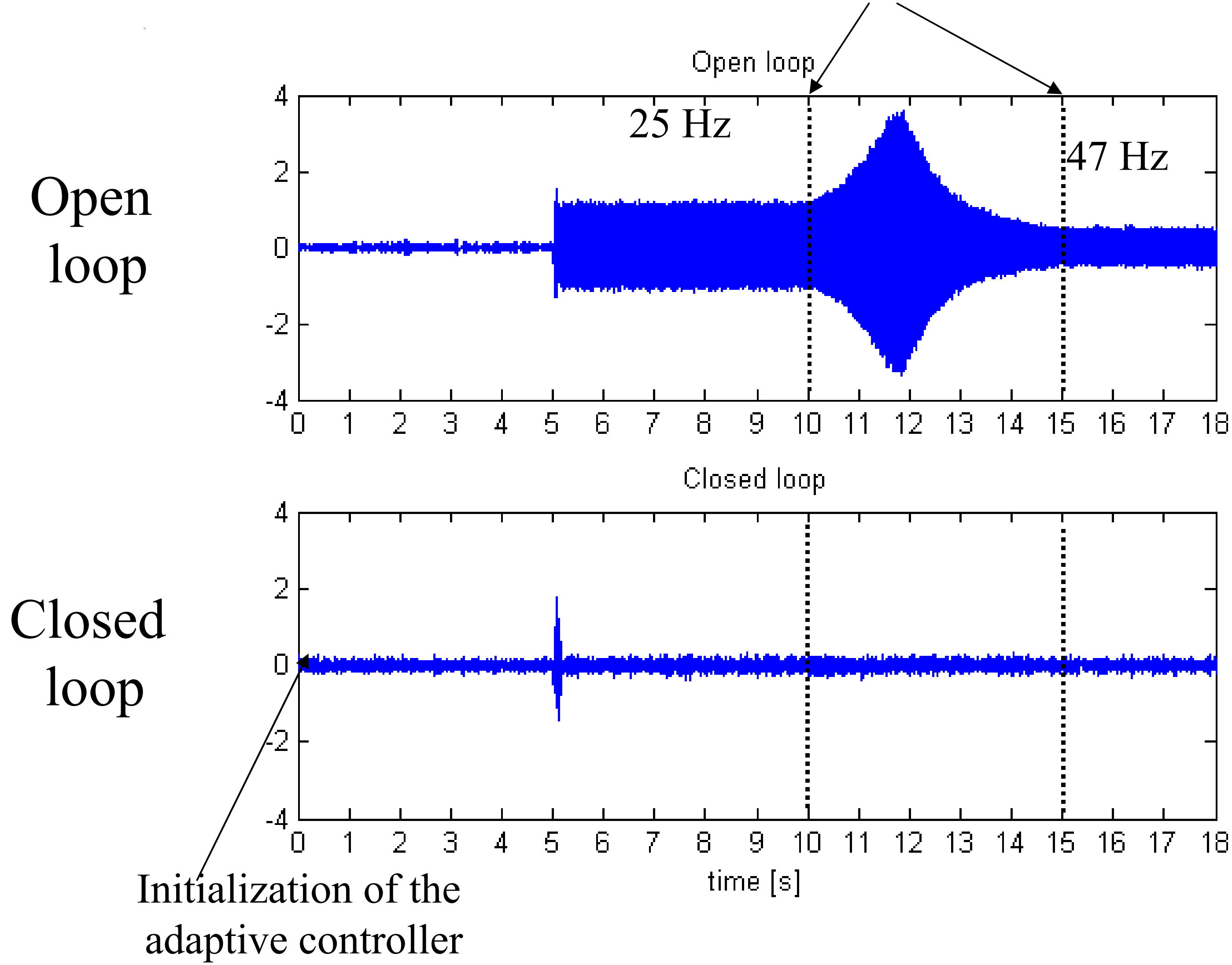


$$n_A = 14 \, ; n_B = 16 \, ; d = 0$$

Further details can be obtained from : http://iawww.epfl.ch/News/EJC_Benchmark/

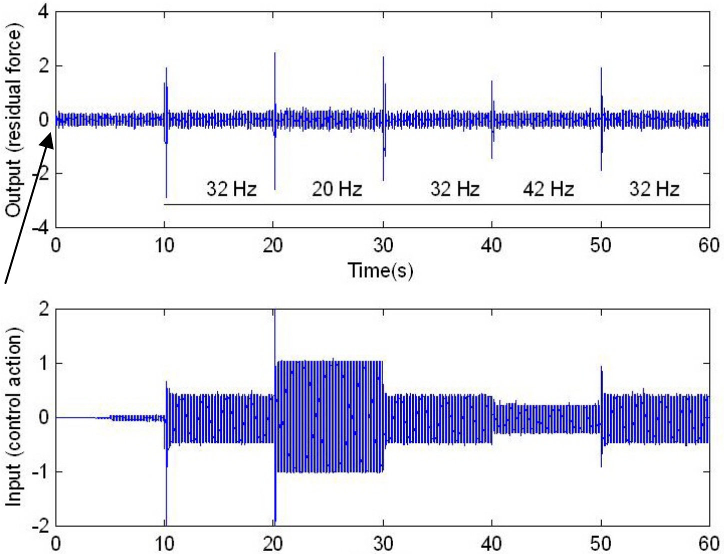
Direct Adaptive Control : disturbance rejection

Disturbance : Chirp



Direct Adaptive Control

Step frequency changes

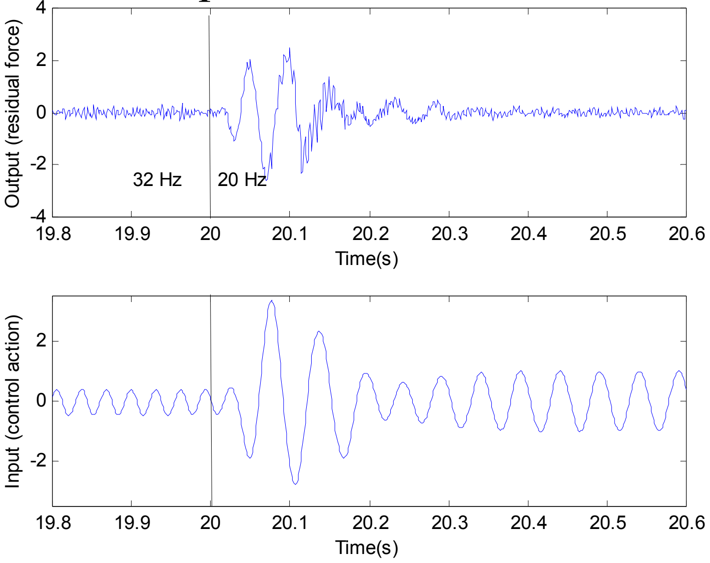


Output

Initialization of the
adaptive controller

Input

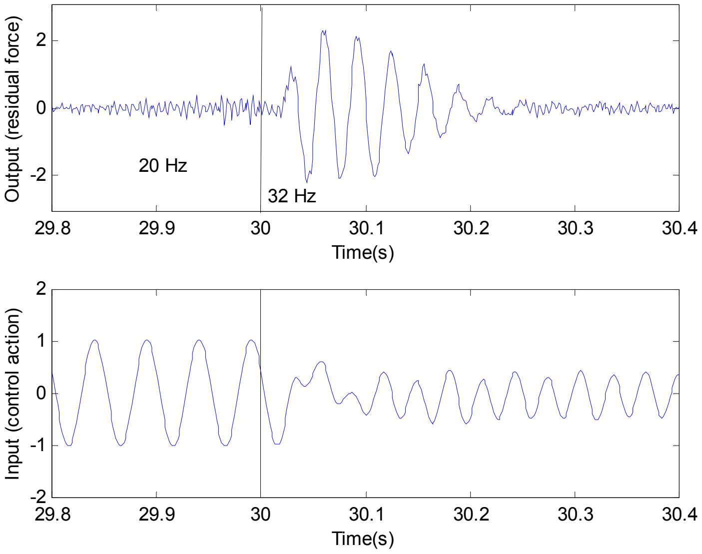
Adaptation transient



output

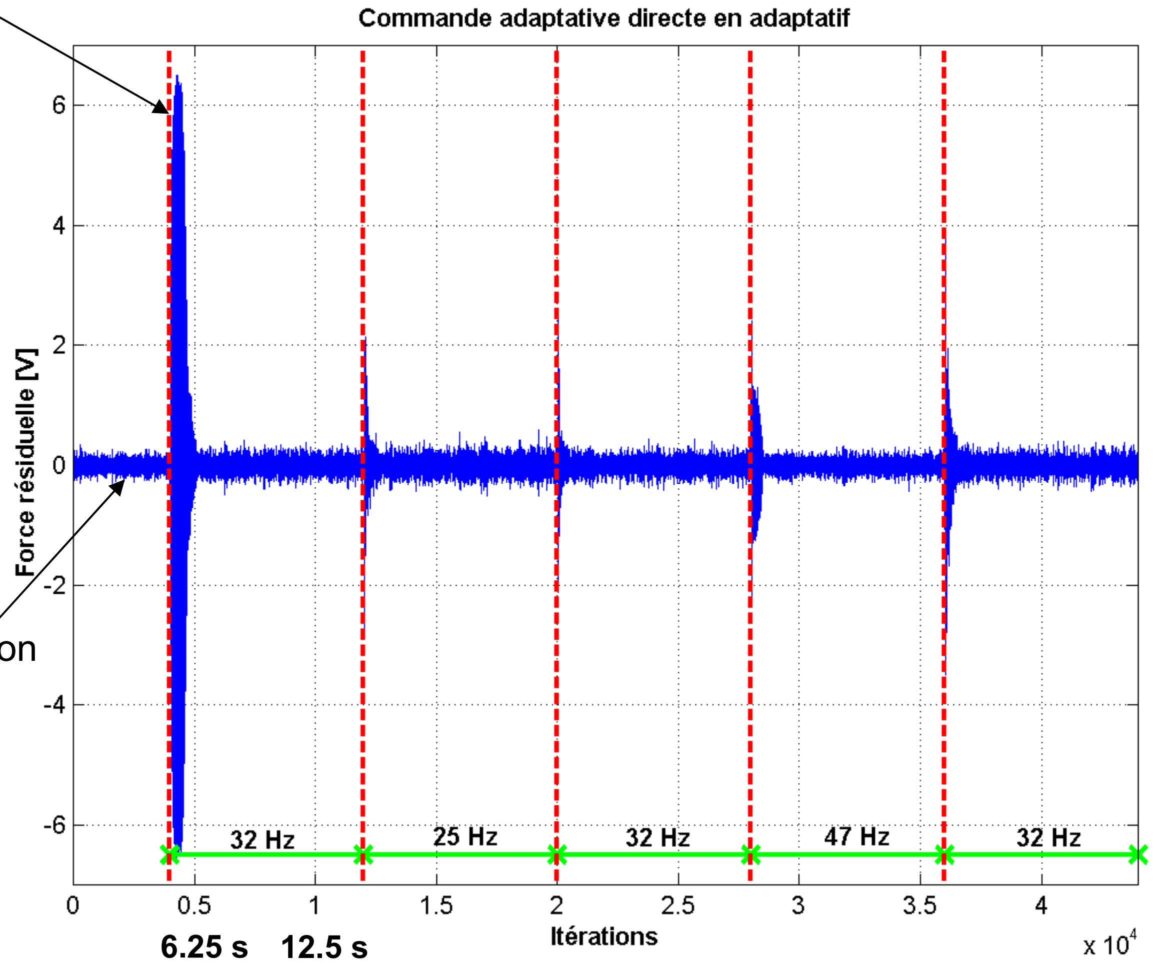
input

Adaptation transient



Direct Adaptive Control

Worst case (stupid) initialization



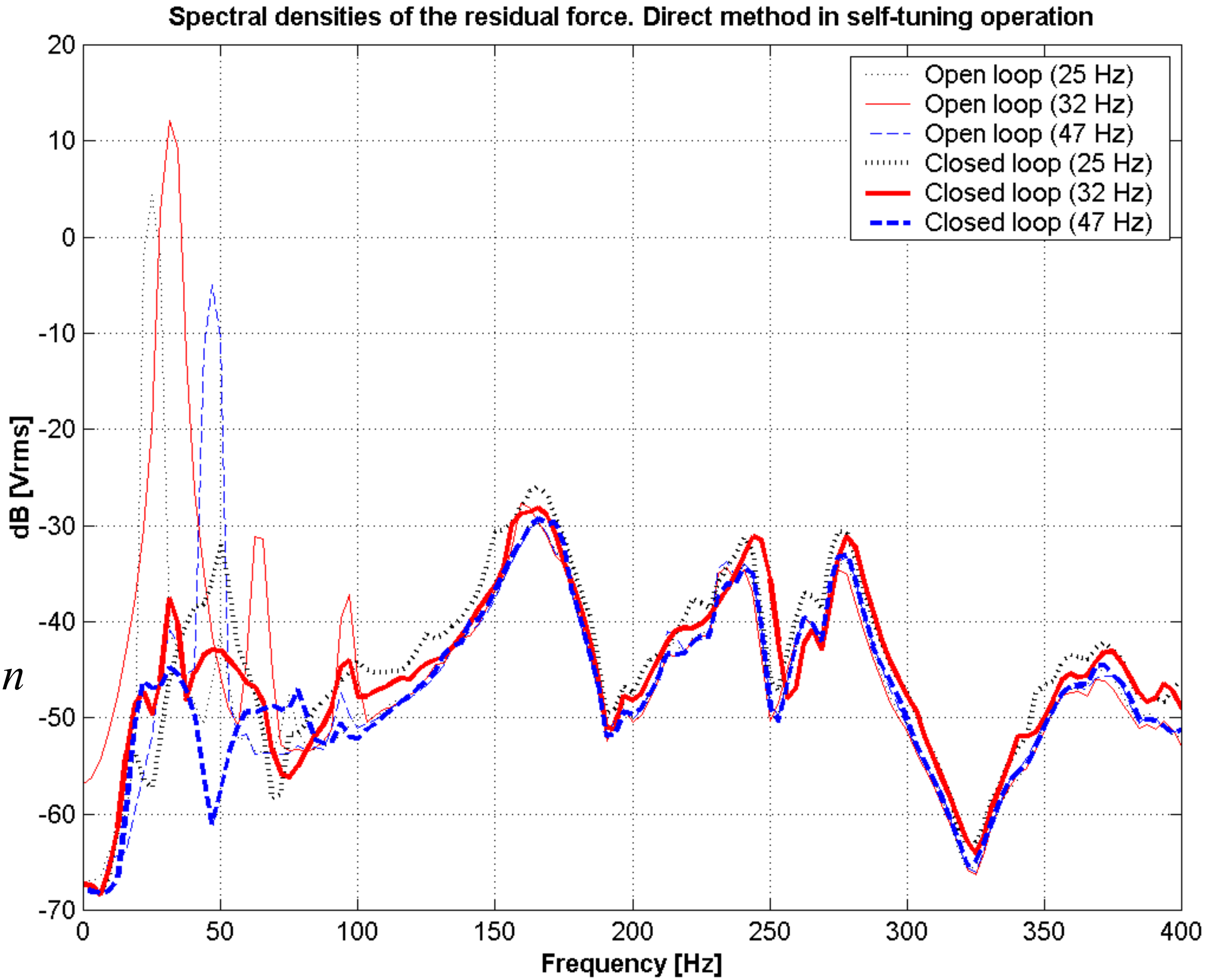
Simultaneous disturbance application and controller initialization

Step frequency changes

Open loop operation

6.25 s 12.5 s

Frequency domain results – direct adaptive method



Attenuation
> 50 dB

Self-tuning operation = time decreasing adaptation gain

I.D.Landau, A. Constantinescu, M. Alma « Adaptive rejection of unknown disturbances »

Rejection of unknown multiple narrow band disturbances

- **Assumption:** Plant model almost constant and known (*obtained by system identification*)
- **Problem:** Attenuation of unknown and/or variable « stationary » disturbances without using an additional measurement
- **Solution:** ***Adaptive feedback control***

A {
- Estimate the model of the disturbance (*indirect adaptive control*)
- Use the internal model principle (IMP)

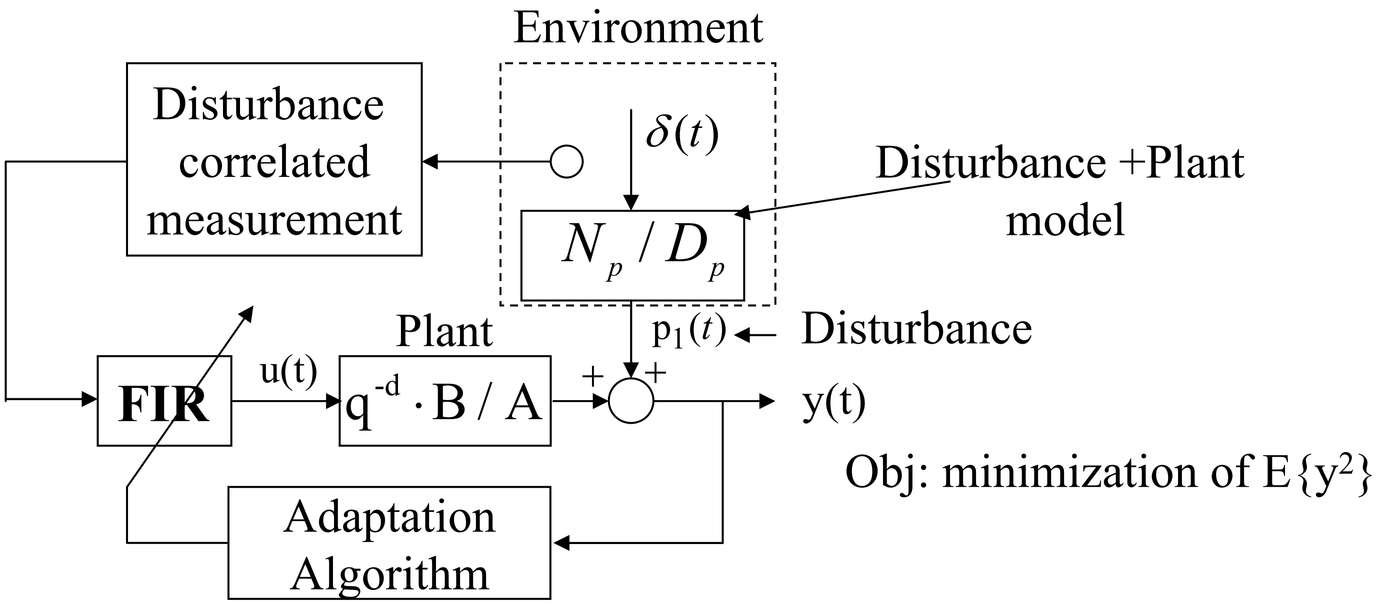
B
- Use of the Youla parametrization + IMP (*direct adaptive control*)

A robust control design should be considered assuming that the model of the disturbance is known

A class of applications: *suppression of unknown vibrations (active vibration control)*

Attention: *The area was "dominated " by adaptive signal processing solutions (Widrow's adaptive noise cancellation) which require an additional transducer*

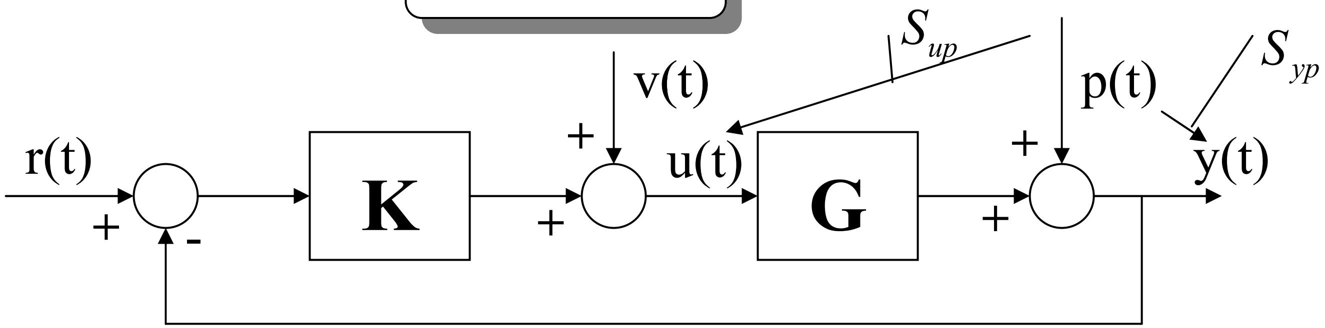
Unknown disturbance rejection – classical solution



Disadvantages:

- requires the use of an additional transducer
- difficult choice of the location of the transducer
- adaptation of many parameters

Notations



$$G(q^{-1})=\frac{q^{-d}B(q^{-1})}{A(q^{-1})}$$

$$K(q^{-1})=\frac{R(q^{-1})}{S(q^{-1})}=\frac{R'(q^{-1})H_R(q^{-1})}{S'(q^{-1})H_S(q^{-1})}$$

H_R and H_S are pre-specified

Output Sensitivity function :

$$S_{yp}(z^{-1})=\frac{A(z^{-1})S'(z^{-1})H_S(z^{-1})}{P(z^{-1})}$$

Input Sensitivity function :

$$S_{up}(z^{-1})=-\frac{A(z^{-1})R'(z^{-1})H_R(z^{-1})}{P(z^{-1})}$$

Closed loop poles :

$$P(z^{-1})=A(z^{-1})S(z^{-1})+z^{-d}B(z^{-1})R(z^{-1})$$

Perfect attenuation of a disturbance at the frequency ω :

$$S_{yp}(e^{-j\omega})=0\,; \quad \left|S_{up}(e^{-j\omega})\right|=\left|\frac{A(e^{-j\omega})}{B(e^{-j\omega})}\right|$$

Disturbance model

Deterministic framework

$$p(t) = \frac{N_p(q^{-1})}{D_p(q^{-1})} \cdot \delta(t): \text{ deterministic disturbance}$$

$D_p \rightarrow$ may have poles on the unit circle; $\delta(t) =$ Dirac

(Sinusoid: $D_P = 1 + \alpha q^{-1} + q^{-2}; \ \alpha = -2\cos(2\pi f / f_s)$)

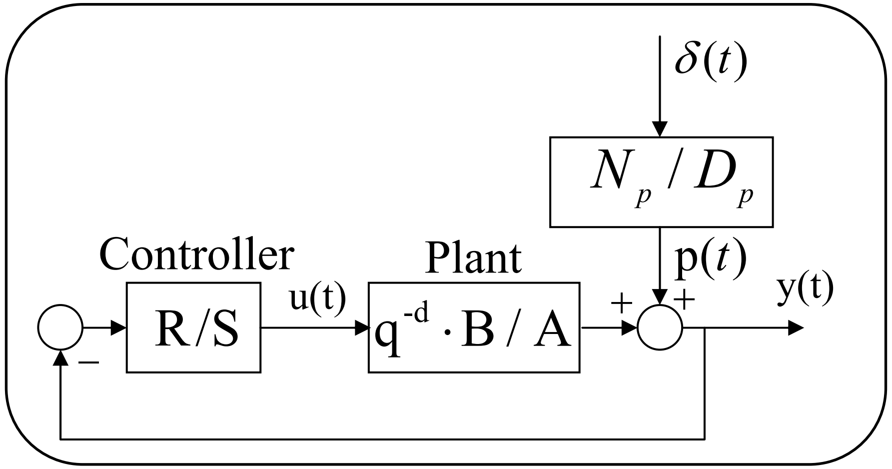
Stochastic framework

$$p(t) = \frac{N_p(q^{-1})}{D_p(q^{-1})} \cdot e(t): \text{stochastic disturbance}$$

$e(t) =$ Gaussian white noise sequence $(0, \sigma)$

$D_p \rightarrow$ may have poles on the unit circle

Closed loop system. Notations



$$p(t) = \frac{N_p(q^{-1})}{D_p(q^{-1})} \cdot \delta(t) : \text{deterministic disturbance}$$

$D_p \rightarrow$ poles on the unit circle; $\delta(t) = \text{Dirac}$

Controller :

$$R(q^{-1}) = R'(q^{-1}) \cdot H_R(q^{-1});$$

$$S(q^{-1}) = S'(q^{-1}) \cdot H_S(q^{-1}).$$

Internal model principle: $H_S(q^{-1})=D_p(q^{-1})$

$$\text{Output} : y(t) = \frac{A(q^{-1})S(q^{-1})}{P(q^{-1})} \cdot p(t) = S_{yp}(q^{-1}) \cdot p(t) \longrightarrow \quad y(t) = \frac{A(q^{-1})\boxed{H_S(q^{-1})}S'(q^{-1})}{P(q^{-1})} \cdot \frac{N_p(q^{-1})}{\boxed{D_p(q^{-1})}} \cdot \delta(t)$$

$$\text{CL poles} : P(q^{-1}) = A(q^{-1})S(q^{-1}) + z^{-d}B(q^{-1})R(q^{-1})$$

$$\Downarrow$$

$$\lim_{t\rightarrow\infty} y(t) = 0$$

Robustness design considerations

- Assuming that the model of the disturbance is known
- That one uses "internal model principle"

For *all possible* parameters values of the disturbance model, the resulting controller should be *robust.*
(acceptable: modulus margin, delay margin, maximum value of $|S_{up}|$)

At the frequencies where perfect rejection occurs ($S_{yp}=0$) one has:

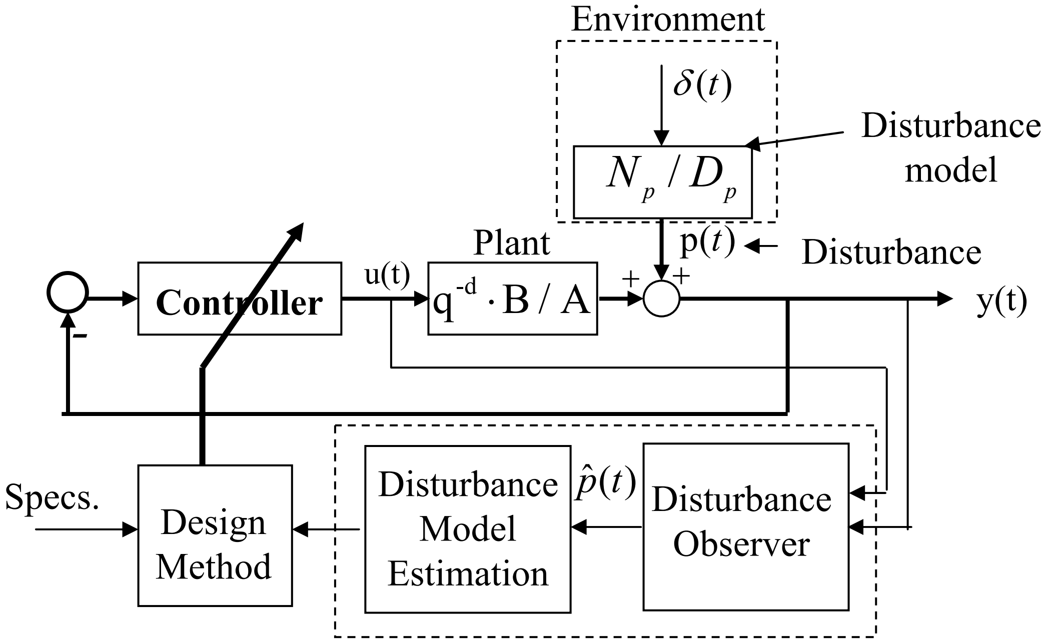
$$\left|S_{up}(e^{-j\omega})\right| = \left|\frac{A(e^{-j\omega})}{B(e^{-j\omega})}\right|$$

Consequence:
Perfect rejection of disturbances can be done only in the frequency range where the plant (secondary path) has enough gain.

Indirect adaptive regulation

Two-steps methodology:

1. Estimation of the disturbance model, $D_p(q^{-1})$
2. Computation of the controller, imposing $H_S(q^{-1}) = \hat{D}_p(q^{-1})$



It can be time consuming (if the plant model B/A is of large dimension)

Indirect adaptive regulation

Step I : Estimation of the disturbance model
(ARMA identification - Recursive Extended Least Squares)

Step II: Computation of the controller

Poles placement - *Solving Bezout equation (for S' and R)*

$$H_S = \hat{D}_p$$

?

$$A\hat{D}_p S' + q^{-d} BR = P$$

$$S = \hat{D}_p S'$$

Remarks :

- The computation method should assure also reasonable sensitivity functions (by selecting appropriate fixed parts in the controller and closed loop poles)
- Other controller design techniques can be used
- **It is time consuming for large dimension of the plant model**

Internal model principle and Q-parametrization)

Central contr: $[R_0(q^{-1}),S_0(q^{-1})]$.

CL poles: $P(q^{-1})=A(q^{-1})S_0(q^{-1})+q^{-d}B(q^{-1})R_0(q^{-1})$.

Control: $S_0(q^{-1})\ u(t) = -R_0\,(q^{-1})\ y(t)$

Q-parameterization (Y-K):

$\boldsymbol{R(z^{1})=R_0(q^{-1})+A(q^{-1})Q(q^{-1});}$

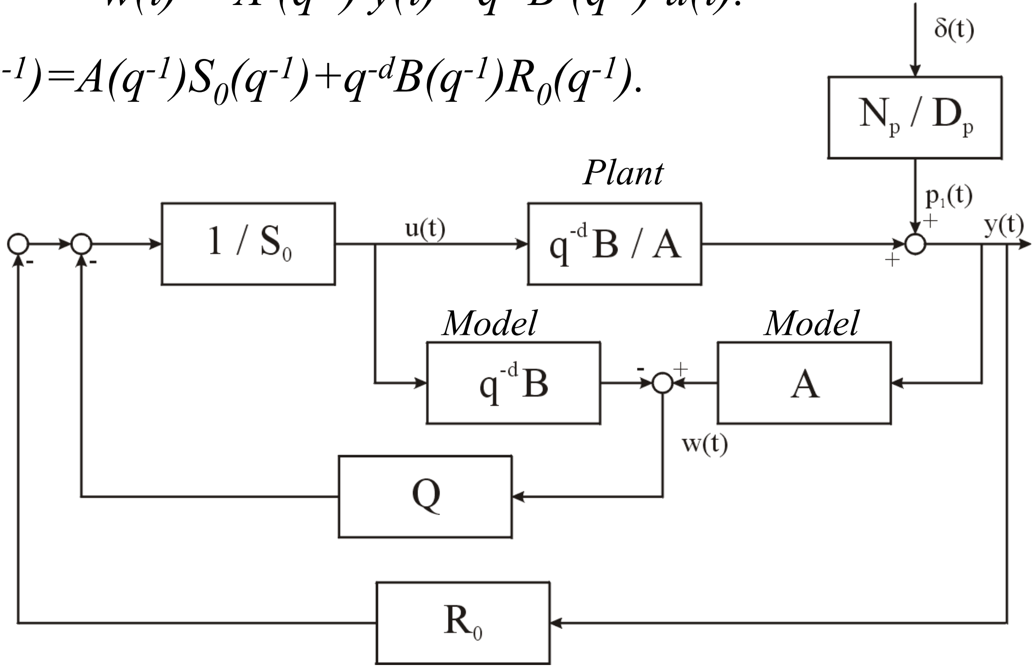
$\boldsymbol{S(q^{-1})=S_0(z^{-1})-q^{-d}B(q^{-1})Q(q^{-1}).}$

$$Q(q^{-1}) = q_0 + q_1 q^{-1} + ...q_{n_Q} q^{-n_Q}$$

Control: $S_0(q^{-1})\ u(t) = -R_0\,(q^{-1})\ y(t) - Q\ (q^{-1})\ w(t)$,

where $w(t) = A\ (q^{-1})\ y(t) - q^{-d}B\ (q^{-1})\ u(t)$.

CL poles: $P(q^{-1})=A(q^{-1})S_0(q^{-1})+q^{-d}B(q^{-1})R_0(q^{-1})$.



Model = Plant

$w=Ap_1$ (*)

Internal model principle and Q- parameterization

Central contr: $[R_0(q^{-1}),S_0(q^{-1})]$.

CL Poles: $P(q^{-1})=A(q^{-1})S_0(q^{-1})+q^{-d}B(q^{-1})R_0(q^{-1})$.

Control: $S_0(q^{-1})\,u(t) = -R_0(q^{-1})\,y(t)$

Q-parameterization :

$\boldsymbol{R(z^{1})=R_0(q^{-1})+A(q^{-1})Q(q^{-1})}$;

$\boldsymbol{S(q^{-1})=S_0(z^{-1})-q^{-d}B(q^{-1})Q(q^{-1})}$.

Closed Loop Poles remain unchanged

Internal model assignment on Q

$Q(q^{-1})$ computed such as $[S(q^{-1})]$ contains the internal model of the disturbance

$$S=S_0-q^{-d}BQ=MD_p \quad \Longrightarrow \quad \textit{Solve:} \quad \underset{?}{M}D_p+q^{-d}B\underset{?}{Q}=S_0 \qquad (**)$$

Will lead also to an « indirect adaptive control solution »

BUT:

*Q can be used to “**directly**”tune the internal model without changing the closed loop poles (see next)*

Direct Adaptive Regulation (unknown D_p)

(Based on an idea of Y. Z. Tsypkin)

Hypothesis: Identified (known) plant model *(A,B,d*).

Goal: drive *y(t)* to *0*

Consider: $p_1(t) = \frac{N_p(q^{-1})}{D_p(q^{-1})} \cdot \delta(t) \quad \longrightarrow \quad w(t) = \frac{AN_p}{D_p}\delta(t) \quad (*)$

$$y(t) = \frac{A(q^{-1})\left[S_0(q^{-1}) - q^{-d}B(q^{-1})Q(q^{-1})\right]}{P(q^{-1})} \cdot \frac{N_p(q^{-1})}{D_p(q^{-1})} \cdot \delta(t) = \frac{\left[S_0(q^{-1}) - q^{-d}B(q^{-1})Q(q^{-1})\right]}{P(q^{-1})} w(t)$$

$\swarrow$

Define (construct):

$$\varepsilon(t) = \frac{S_0(q^{-1})}{P(q^{-1})} \cdot w(t) - Q(q^{-1}) \frac{q^{-d}B(q^{-1})}{P(q^{-1})} \cdot w(t).$$

Let $\hat{Q}(t,q^{-1})$ be an estimated value of $Q(q^{-1})$

We can show that (using (**))

Leads to a direct adaptive control →

$$\boxed{\varepsilon(t+1) = [Q(q^{-1}) - \hat{Q}(t+1,q^{-1})] \cdot \frac{q^{-d}B^*(q^{-1})}{P(q^{-1})} \cdot w(t) + v(t+1)}$$

($v(t+1) =$ disturbance term $\rightarrow 0$)

The Algorithm

measured

$$w(t+1) = A(q^{-1})y(t+1) - q^{-d}B^*(q^{-1})u(t);\ \ (B(q^{-1})u(t+1) = B^*(q^{-1})u(t))$$

define:

$$\boxed{\varepsilon^{\circ}(t+1) = \frac{S_0(q^{-1})}{P(q^{-1})}w(t+1) - \hat{Q}(t,q^{-1})\frac{q^{-d}B(q^{-1})}{P(q^{-1})}w(t+1).}$$

$$w_1(t+1) = \frac{S_0(q^{-1})}{P(q^{-1})}\cdot w(t+1);\quad w_2(t) = \frac{q^{-d}B^*(q^{-1})}{P(q^{-1})}\cdot w(t);$$

$$\hat{\theta}^T(\mathrm{t}) = \left[\hat{\mathrm{q}}_0(t)\quad \hat{\mathrm{q}}_1(t)\right];\quad \phi^T(t) = \left[w_2(t)\quad w_2(t-1)\right],\ \ (\text{for}\ \ n_{D_P} = 2\ \ \text{since}\ n_Q = n_{D_P}\text{-}1)$$

A priori adaptation error: $\boxed{\varepsilon^0(t+1) = w_1(t+1) - \hat{\theta}^T(t)\phi(t)}$

A posteriori adaptation error: $\varepsilon(t+1) = w_1(t+1) - \hat{\theta}^T(t+1)\phi(t)$

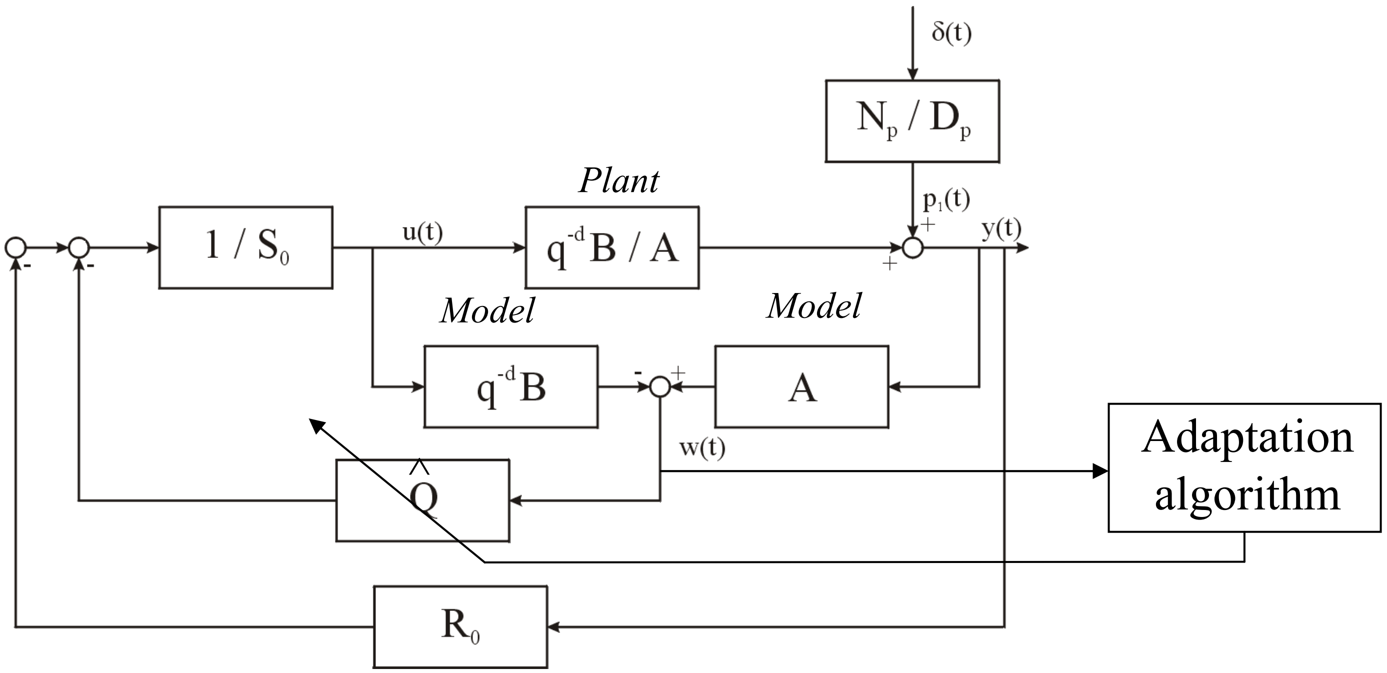
Parameter adaptation algorithm:

$$\begin{cases} \hat{\theta}(\mathrm{t}+1) = \hat{\theta}(\mathrm{t}) + \mathrm{F}(\mathrm{t}+1)\phi(\mathrm{t})\varepsilon^0(\mathrm{t}+1); \\ F^{-1}(t+1) = \lambda_1(t)F^{-1}(t) + \lambda_2(t)\phi(t)\phi^T(t). \end{cases}$$

Various choices possible for λ_1 and λ_2 *which define the adaptation gain time profile*

(For a stability proof see Automatica, 2005, no.4 pp. 563-574)

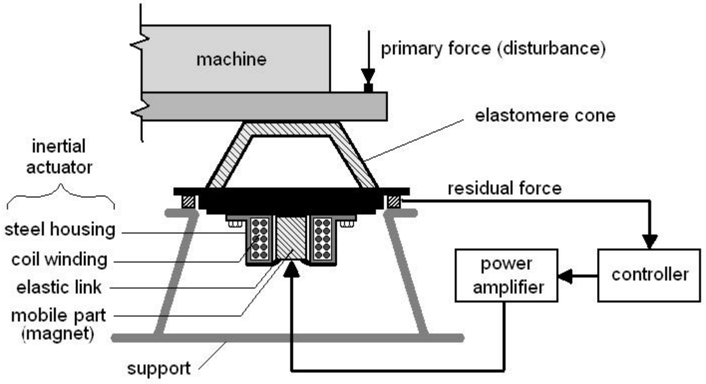
Direct adaptive rejection of unknown disturbances



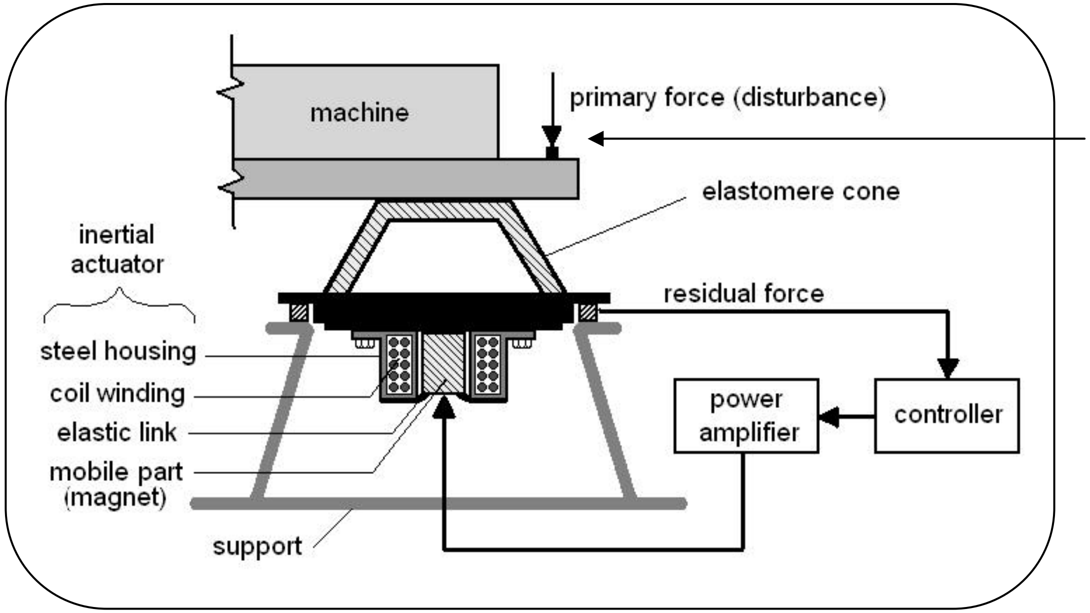
- *The order of the Q polynomial depends upon the order of the disturbance model denominator (D_P) and not upon the complexity of the plant model*
- *Less parameters to estimate then for the identification of the disturbance model*
- *Operation in "self –tuning" mode (constant unknown disturbance) or "adaptive" mode (time varying unknown disturbance)*
- *Much simpler implementation than indirect adaptive control*

Active vibration control using an inertial actuator
Real-time results

Rejection of two simultaneous sinusoidal disturbances



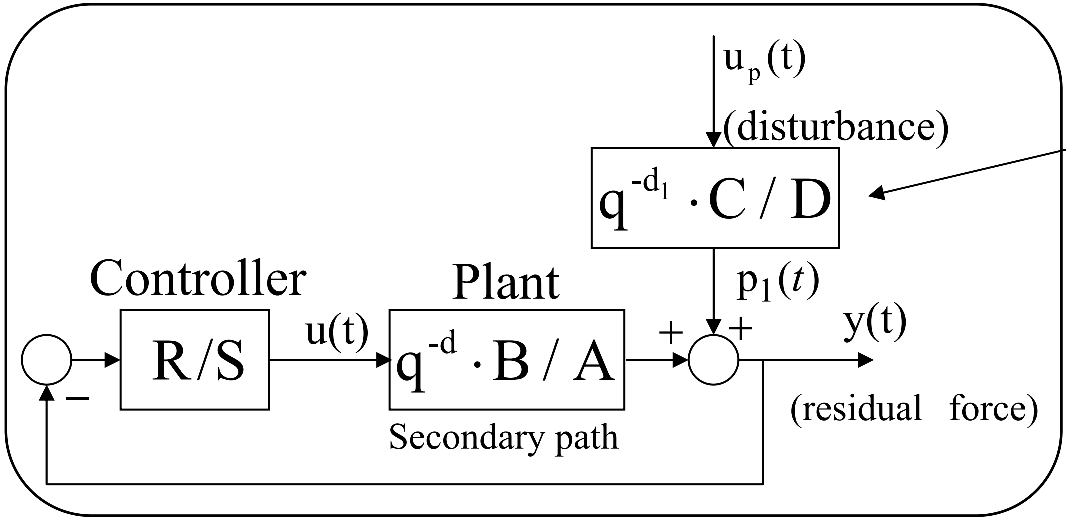
Active Vibration Control using an Inertial Actuator



Objective:

- **Reject the effect of unknown and variable narrow band disturbances**
- **Do not use an additional measurement**

Same control approach but different technology

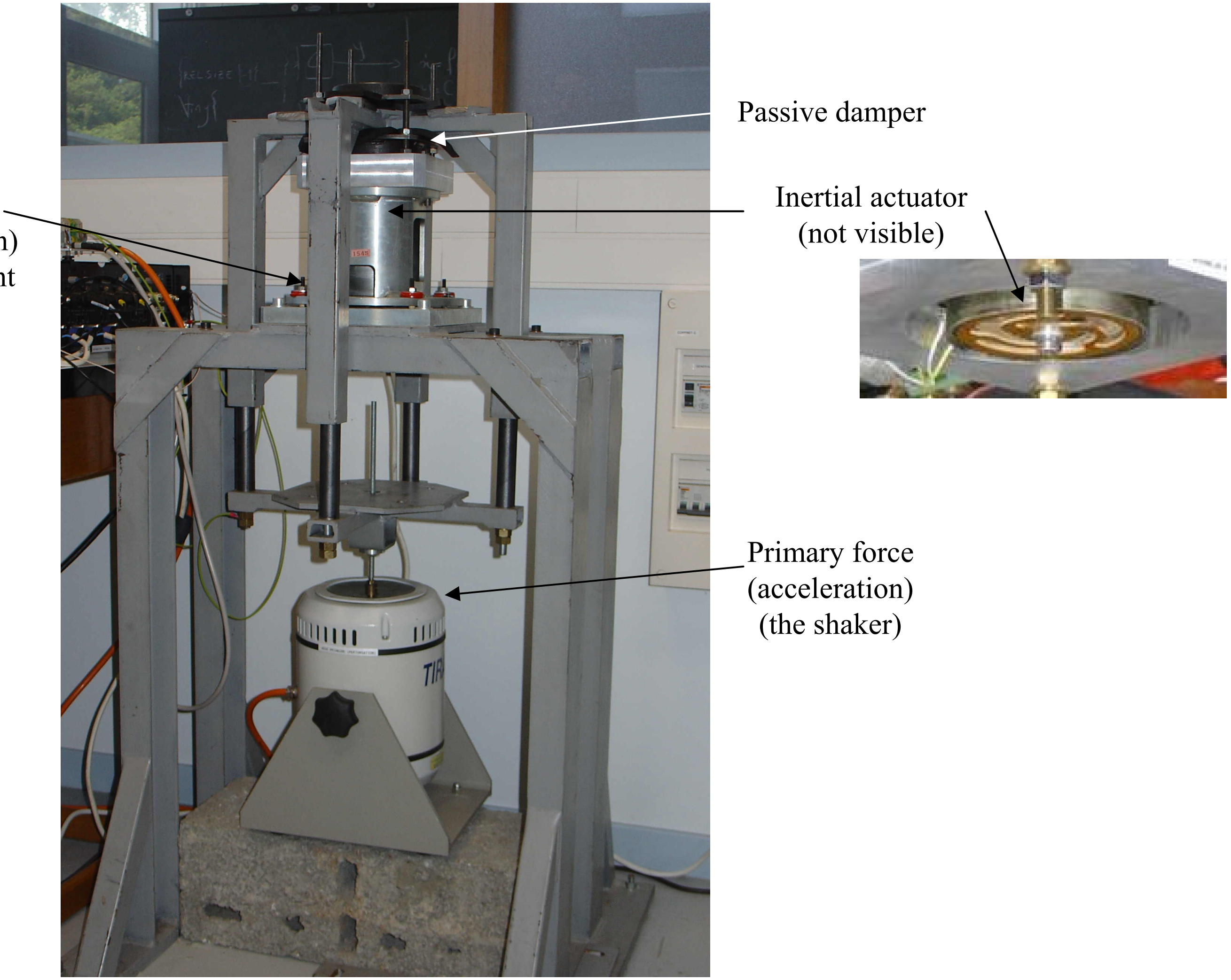


Two paths :

- Primary
- Secondary (double differentiator)

$$T_s = 1.25\,\mathrm{ms}\ (f_s = 800\,Hz)$$

View of the active vibration control using an inertial actuator



Residual
force
(acceleration)
measurement

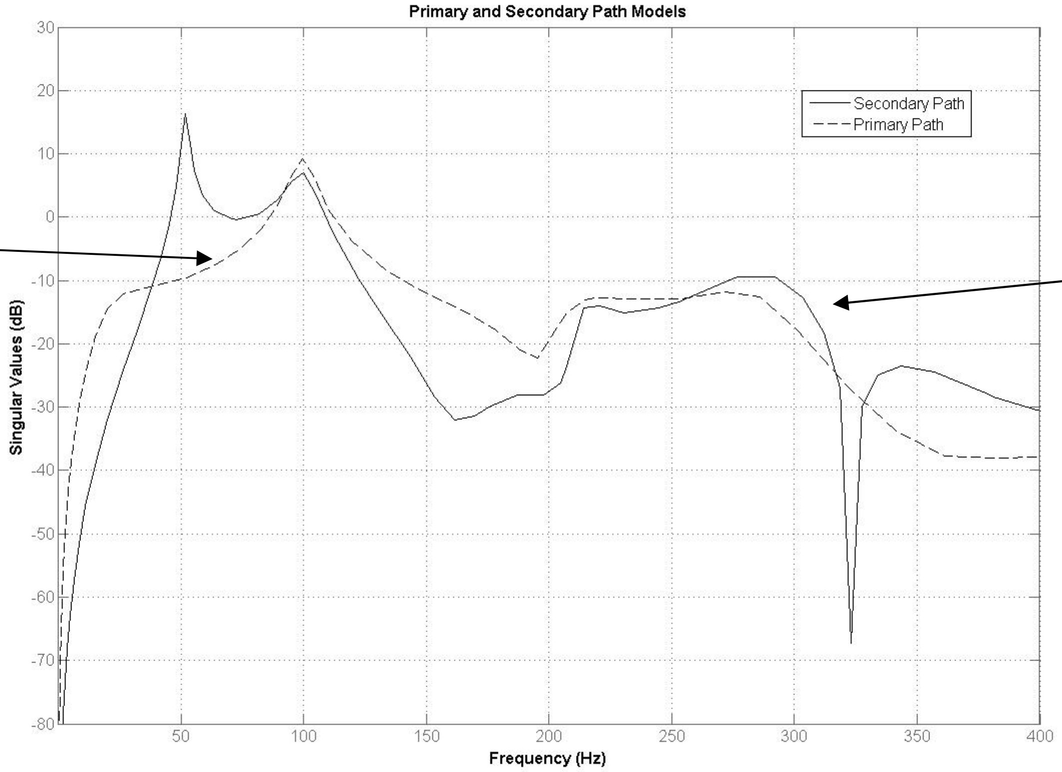
Passive damper

Inertial actuator
(not visible)

Primary force
(acceleration)
(the shaker)

Active vibration control using an inertial actuator

Frequency Characteristics of the Identified Models



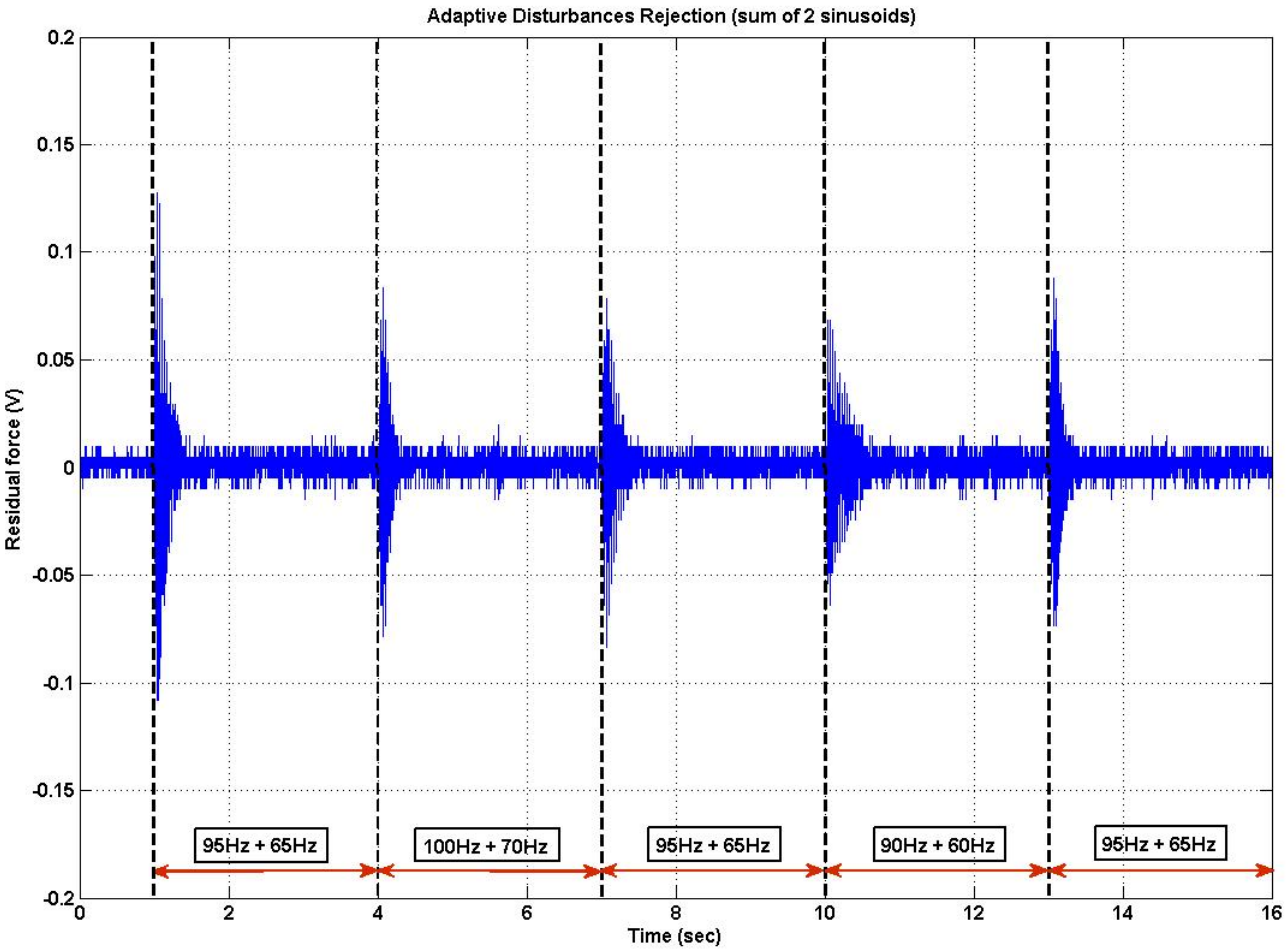
Primary path

Secondary path

Complexity of secondary path: $n_A = 10\,;\, n_B = 12\,;\, d = 0$

Time Domain Results – Direct adaptive control
Adaptive Operation

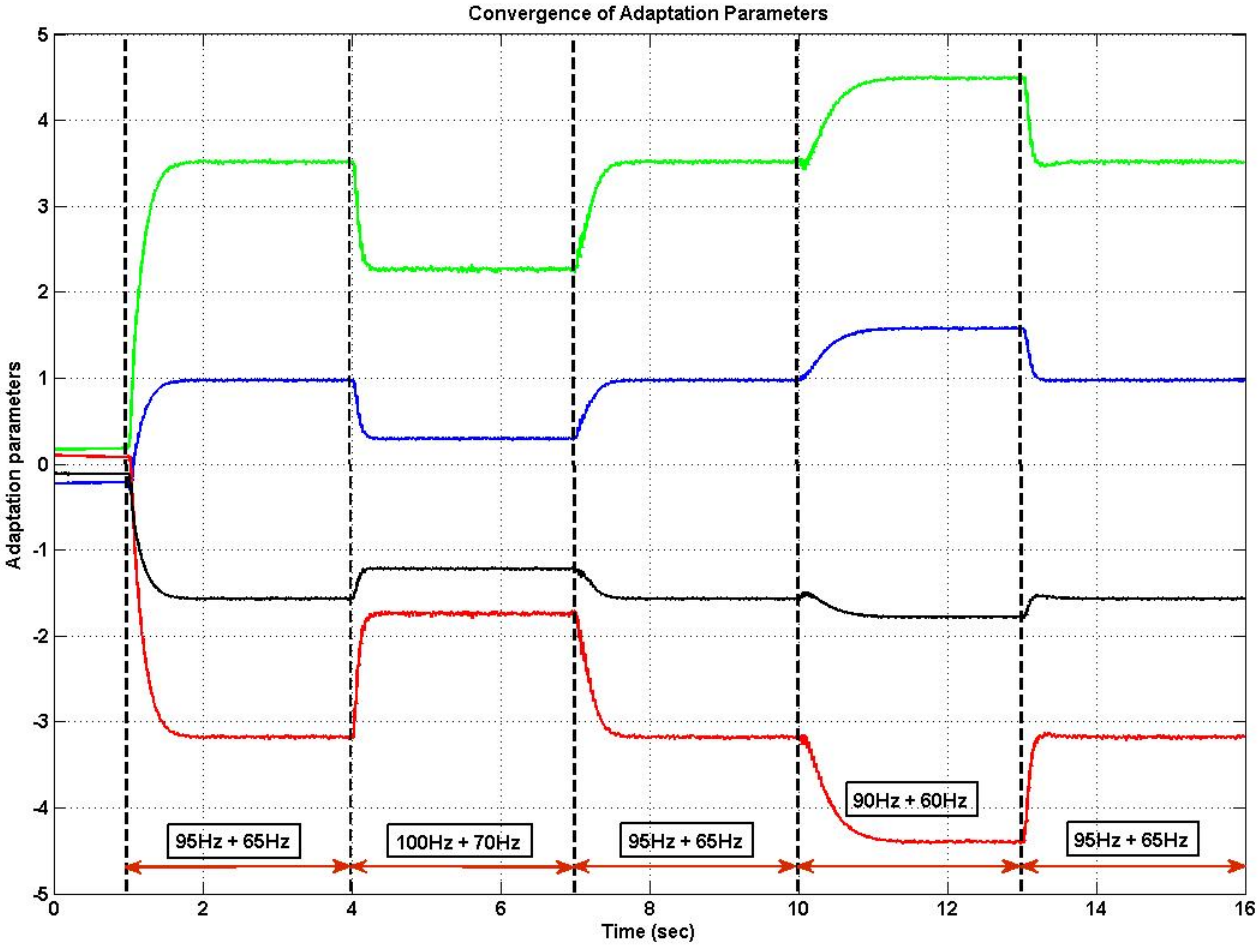
Simultaneous rejection of two time varying sinusoidal disturbances



Output

Time Domain Results – Direct adaptive control
Evolution of the Q parameters

Simultaneous rejection of two time varying sinusoidal disturbances



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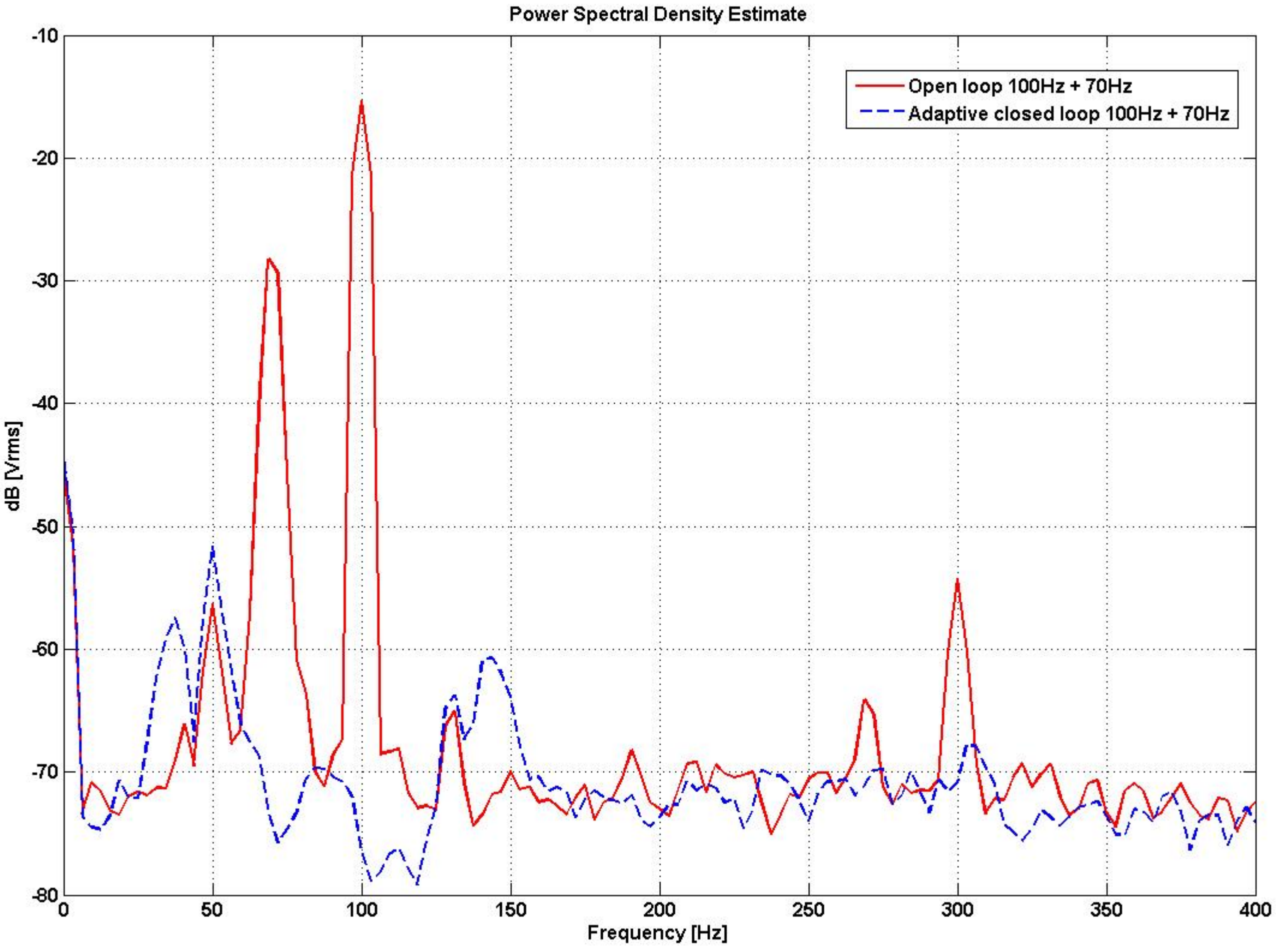
Frequency domain results – direct adaptive method

Rejection of two simultaneous sinusoidal disturbances



Attenuation > 45 dB

Conclusions

- Using internal model principle, adaptive feedback control solutions can be provided for the rejection of unknown disturbances

- Both direct and indirect solutions can be provided

- Two modes of operation can be used : self-tuning and adaptive

- **Direct adaptive control is the simplest to implement**

- Direct adaptive control offers better performance

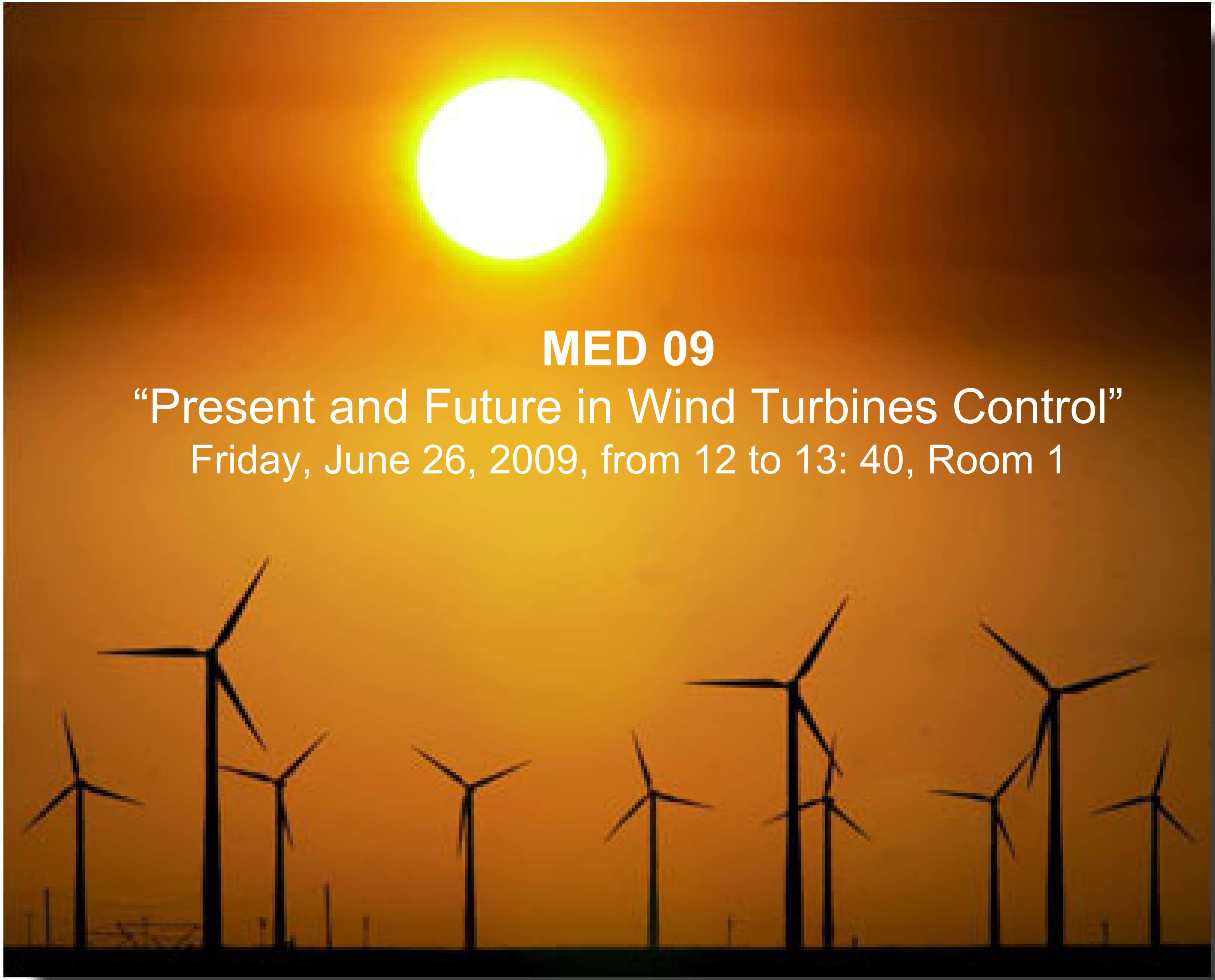
- The methodology has been extensively tested on active vibration control systems and other systems (Ben Amara, Bonvin)

New results

- Extension to the multivariable case (Ficocelli, Ben Amara)

Open problems:

- Robustness with respect to plant model error (more theory)
- Removing the hypothesis of constant plant model



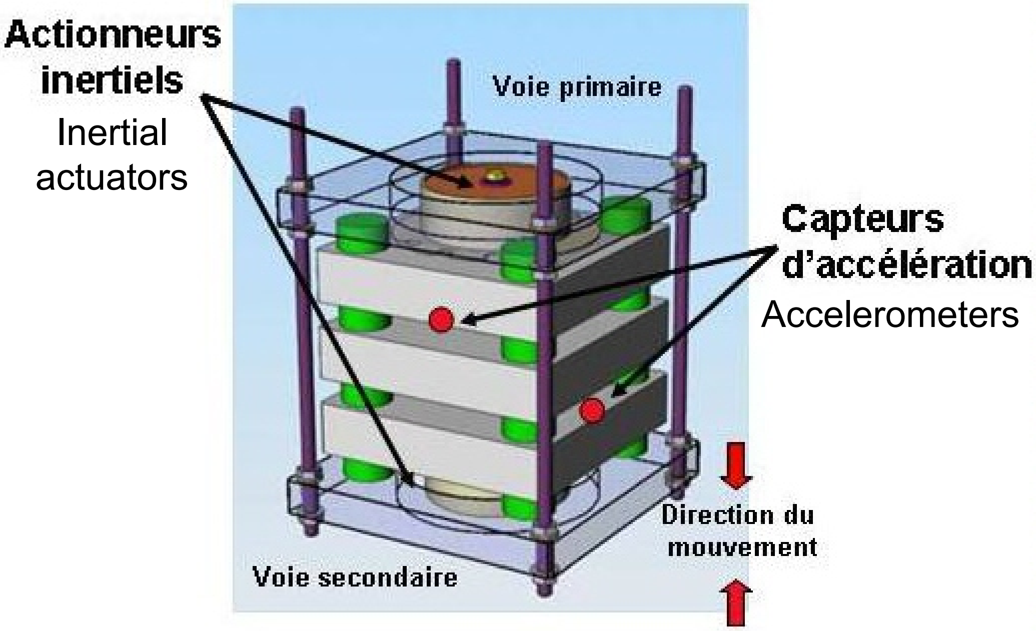
MED 09

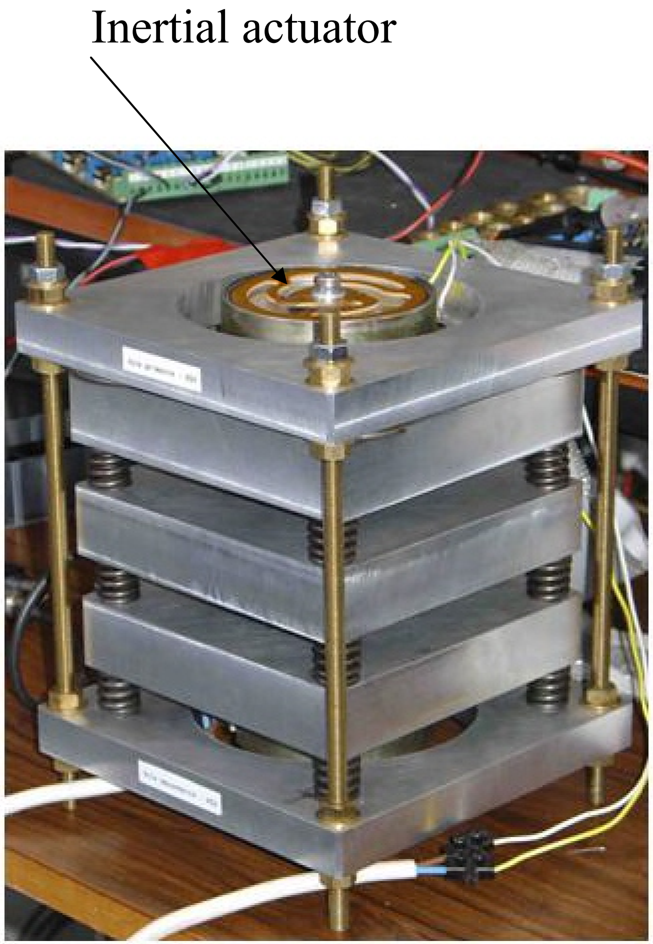
“Present and Future in Wind Turbines Control”

Friday, June 26, 2009, from 12 to 13: 40, Room 1

Additional Slides

View of a flexible controlled structure using inertial actuators





Adaptive Control (Regulation) – Regimes of operation

Adaptive

- The adaptation algorithm is continuously operating
- The controller is updated at each sample
- Use *non vanishing adaptation gain*

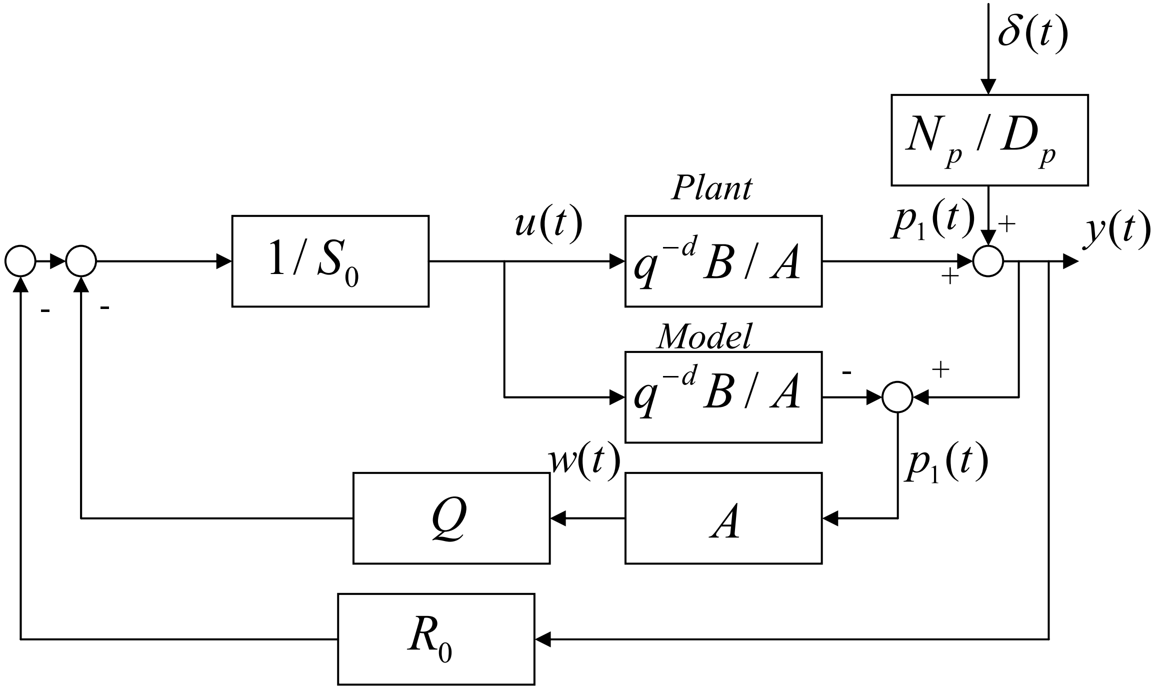
Self -tuning

- Parameter estimation and/or controller updating is done with a *time deceasing adaptation gain.*
- No natural adaptation capabilities
- Restarts when the variance of the performance error is bigger than a given threshold

Auto – tuning (indirect adaptive control)

- Parameter estimation followed by controller re-computation.
- *Controller is constant during parameter estimation*
- Restarts when the variance of the performance error is bigger than a given threshold.

Q-parametrization (Yula-Kucera)
An interpretation for the case *A* asympt. stable



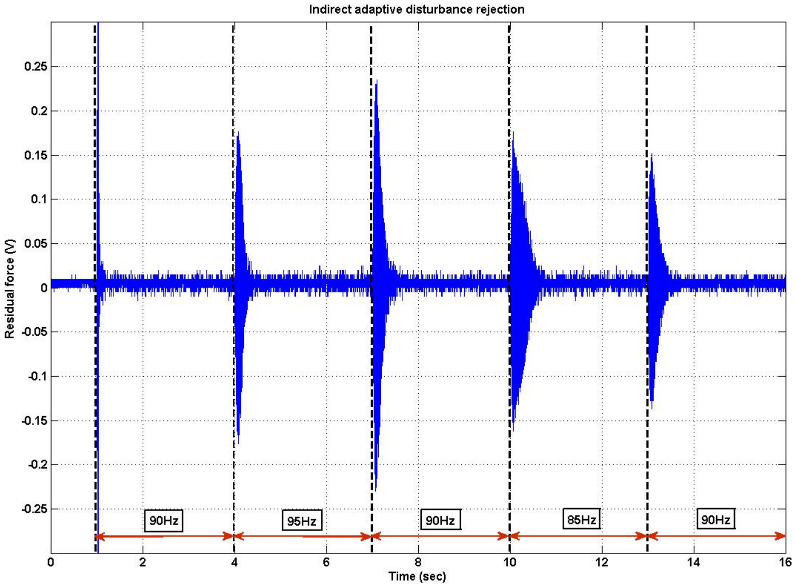
I.D.Landau, A. Constantinescu, M. Alma « Adaptive rejection of unknown disturbances »

Comparison direct/indirect adaptive regulation

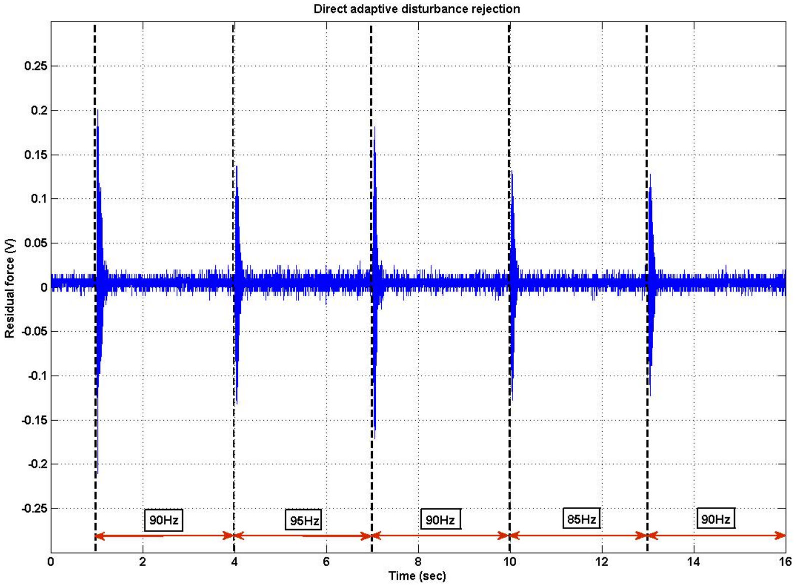
Time domain results – Adaptive regime

Active vibration control using an inertial actuator

Indirect adaptive method



Direct adaptive method



***Direct** adaptive control leads to a much simpler implementation and better performance than **Indirect** adaptive control*

Direct Adaptive Control (unknown D_p)

$$\varepsilon(t) = \frac{S_0(q^{-1})}{P(q^{-1})} \cdot w(t) - \frac{q^{-d}B(q^{-1})}{P(q^{-1})} Q(q^{-1}) \cdot w(t). \qquad (*)$$

We need to express $\varepsilon(t)$ as:

$$\varepsilon(t+1) = \left[Q(q^{-1}) - \hat{Q}(t+1, q^{-1})\right]\Psi(t)$$

Using: $MD_p + q^{-d}BQ = S_0$, (*) becomes

Vanishing term

$$\varepsilon(t+1) = [Q(q^{-1}) - \hat{Q}(t+1, q^{-1})] \cdot \frac{q^{-d}B^*(q^{-1})}{P(q^{-1})} \cdot w(t) + \frac{M(q^{-1})D_p(q^{-1})}{P(q^{-1})} p(t+1)$$

Instead of solving $MD_p + q^{-d}BQ = S_0$ *search recursively for :*

$$\hat{Q}(t, q^{-1})^* = \arg\min_{\hat{Q}} \sum_{i=0}^{t} \varepsilon^2\left[i, \hat{Q}\right]$$

Details:

$$\frac{M(q^{-1})D_p(q^{-1})}{P(q^{-1})} p(t+1) = \frac{M(q^{-1})N_p(q^{-1})}{P(q^{-1})} \delta(t+1)$$

$$\frac{q^{-d}B^*(q^{-1})}{P(q^{-1})} \cdot w(t) = \frac{q^{-d}B(q^{-1})}{P(q^{-1})} \cdot w(t+1)$$

Time Domain Results – Direct adaptive control
Evolution of the control input

Simultaneous rejection of two time varying sinusoidal disturbances