User's Guide for Pole Placement Master MATLAB®

graphical toolbox for robust digital controller design

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Chapter 1

About Pole Placement Master

The Pole Placement Master (briefly PPMaster) is a toolbox programmed in MATLAB® 5.3 environment[1] (it works also under MATLAB 6.0 and higher). It is user friendly application with graphical user interface (GUI) for design of digital SISO (single-input, single-output) robust controller. The implemented controller design procedure is combined pole placement with sensitivity shaping design technique presented in [2] and improved in [6].

The toolbox was developed in Laboratoire d'Automatique de Grenoble as a part of doctoral thesis "H. Prochazka, Synthèse de régulateurs multivariables en utilisant le placement de pôles avec calibrage des fonctions de sensibilité par optimisation convexe, Institut National Polytechnique de Grenoble". The principal reference for the toolbox is [5].

[1]MATLAB is a registrated trademark of The MathWorks, Inc.

Chapter 2

Basic theory

2.1 Notation

A standard digital pole placement configuration using a polynomial controller (denoted R-S) is shown in Fig.2.1. The plant model $G(z^{-1})$ is of the form:

$$G(z^{-1}) = \frac{B(z^{-1})}{A(z^{-1})} = \frac{z^{-d}(b_1 z^{-1} + ... + b_{n_B} z^{-n_B})}{1 + a_1 z^{-1} + ... + a_{n_A} z^{-n_A}} \tag{2.1}$$

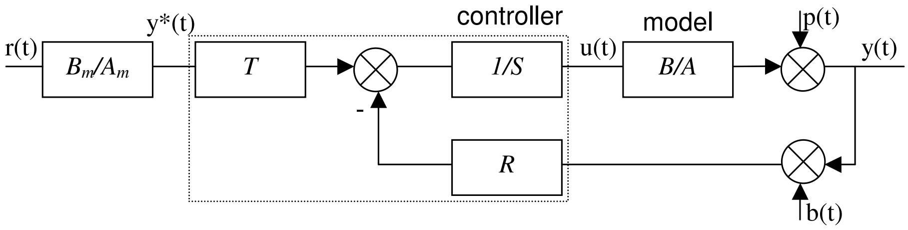

Figure 2.1: Considered closed loop configuration

The R-S part of the controller has the transfer function:

$$\frac{R(z^{-1})}{S(z^{-1})} = \frac{R_0(z^{-1})H_R(z^{-1})}{S_0(z^{-1})H_S(z^{-1})} \tag{2.2}$$

where $H_R(z^{-1})$ and $H_S(z^{-1})$ denote the fixed parts of the controller (either imposed by the design or introduced in order to shape the sensitivity functions). $R_0(z^{-1})$ and $S_0(z^{-1})$ are solutions of the Bezout equation (poles of the closed loop):

$$A(z^{-1})S_0(z^{-1})H_S(z^{-1}) + B(z^{-1})R_0(z^{-1})H_R(z^{-1}) = \underbrace{P_D(z^{-1})P_F(z^{-1})}_{=P(z^{-1})} \tag{2.3}$$

where $P(z^{-1})$ represents the desired closed loop poles, $P_D(z^{-1})$ defines the dominant poles (specified) and $P_F(z^{-1})$ defines the auxiliary poles (which in part can be specified by design specifications and the remaining part is introduced in order to shape the sensitivity function).

The tracking part $T(z^{-1})$ of the controller is used to compensate the closed loop dynamic in such way that the entire system transfer function (from $r(t)$ to $y(t)$) has the dynamic of the reference model $\frac{B_m}{A_m}$. The polynomial $T(z^{-1})$ is considered to have three basic forms:

- T contains all closed-loop poles given by the polynomial $P = AS + BR$ and its static gain is adjusted so the static gain of the transfer function from $y*(t)$ to $y(t)$ is 1. Hence,

$$T(z^{-1}) = \frac{P(z^{-1})}{B(1)} \tag{2.4}$$

- T contains dominant closed-loop poles given by the polynomial P_D and its static gain is adjusted so the static gain of the transfer function from $y*(t)$ to $y(t)$ is 1. Hence,

$$T(z^{-1}) = \frac{P_D(z^{-1})P_F(1)}{B(1)} \tag{2.5}$$

- T is a gain with the value $T = \frac{P(1)}{B(1)}$.

The reference model $\frac{B_m}{A_m}$ is considered to be either 2nd order transfer function with dynamics defined by natural frequency and damping, or two same 2nd order transfer functions connected in cascade.

Sensitivity function shaping is one of the way how to assure controller and closed-loop performances, since the sensitivity functions are one of the crucial indicator of these performances. The considered sensitivity functions are:

The output sensitivity function:

$$S_{yp}(z^{-1}) = \frac{A(z^{-1})S_0(z^{-1})H_S(z^{-1})}{P(z^{-1})} \tag{2.6}$$

The input sensitivity function:

$$S_{up}(z^{-1}) = \frac{-A(z^{-1})R_0(z^{-1})H_R(z^{-1})}{P(z^{-1})} \tag{2.7}$$

The complementary sensitivity function:

$$S_{yb}(z^{-1}) = \frac{B(z^{-1})R_0(z^{-1})H_R(z^{-1})}{P(z^{-1})} \tag{2.8}$$

where S_{yp} is shaped to obtain a sufficient closed-loop robust stability, the shaping of S_{up} allows to limit controller gain and hence actuator effort and S_{yb} shaping help to limit noise sensitivity of the closed loop and it serves to fix a desired closed-loop tracking performance. More details can be found in [1, 2].

We can now introduce the following parameterization:

$$\begin{aligned} H_R(z^{-1}) &= H'_R(z^{-1})\gamma_R(z^{-1}) \\ H_S(z^{-1}) &= H'_S(z^{-1})\gamma_S(z^{-1}) \\ P_F(z^{-1}) &= P'_F(z^{-1})\delta_R(z^{-1}) \\ P_F(z^{-1}) &= P''_F(z^{-1})\delta_S(z^{-1}) \end{aligned}$$

With these notations we get:

$$|S_{yp}(z^{-1})| = \left|\frac{A(z^{-1})S_0(z^{-1})H'_S(z^{-1})}{P_D P''_F(z^{-1})}\right| \left|\frac{\gamma_S(z^{-1})}{\delta_S(z^{-1})}\right| \tag{2.9}$$

$$|S_{up}(z^{-1})| = \left|\frac{A(z^{-1})R_0(z^{-1})H'_R(z^{-1})}{P_D P'_F(z^{-1})}\right| \left|\frac{\gamma_R(z^{-1})}{\delta_R(z^{-1})}\right| \tag{2.10}$$

where the filters $F_{yp}(z^{-1}) = \frac{\gamma_S(z^{-1})}{\delta_S(z^{-1})}$ and $F_{up}(z^{-1}) = \frac{\gamma_R(z^{-1})}{\delta_R(z^{-1})}$ consist of several 2nd order notch filters (2zeros/2poles band-stop filters with limited attenuation) simultaneously tuned. The tuning means in fact searching for appropriate frequency characteristics of $|F_{yp}(z^{-1})|$ and $|F_{up}(z^{-1})|$. Specifically in our case, we are interested in frequency band-stop with limited attenuation characteristics and thus the tuning concerns the frequency of band-stop, its bandwidth and the maximum attenuation in the band-stop frequency.

Polynomial $T(z^{-1})$...

2.2 Controller design procedure

Suppose to dispose with a digital model G of the plant to be controlled. The controller design consists of the following steps:

1. *Design specifications* - Determine desired closed loop and tracking performances. The closed loop performances, such as robust stability, disturbance rejection, etc., has to be expressed by some templates imposed on sensitivity functions. The tracking properties include rise time, maximal overshoot, or settling time.

2. *Closed-loop part R-S design* - The sensitivity functions are shaped to satisfy design specifications (to enter the frequency responses to imposed templates). As we can see from the previous section, one disposes with the following design parameters:

   - P_D polynomial of desired dominant (the slowest) closed loop poles
   - P'_F/P''_F polynomial of desired auxiliary closed loop poles
   - H_R fixed part of the controller numerator
   - H_S fixed part of controller denominator
   - F_{yp} 2nd order digital notch filters on S_{yp}
   - F_{up} 2nd order digital notch filters on S_{up}

   which allow us to shape appropriately the sensitivity functions S_{yp}, S_{up}, S_{yb}.

3. *Tracking part design* - If the tracking properties are not satisfied by closed loop controller part R-S, the tracking part has to be designed. One has to choose an appropriate structure of T and to design the reference model $\frac{B_m}{A_m}$ corresponding to the desired tracking performances. For reference model adjusting the natural frequency and damping of the reference model denominator is modified.

Chapter 3

How to use the application

3.1 Software requirements

The PPMaster is MATLAB toolbox, and so it can works only in the MATLAB environment. Specifically, the toolbox was programmed for MATLAB version 5.3, but it works also under version 6.0 or version 6.5.

Moreover, the PPMaster requires the standard MATLAB toolboxes and Signal Processing toolbox (it uses only the function freqz for discrete-time frequency responses).

3.2 Getting started

1. To run Pole-Placement Master (ppmaster):
   - Run MATLAB version 5.3 or higher.
   - Change the MATLAB current directory on the deirectory with m-files / p-files of ppmaster (using command cd or Path Browser) or add this directory to the PATH variable (for example using path-browser).
   - Type "ppmaster" to MATLAB command line
2. To load a model and start design you go to menu bar:
   Model→Load model→Matlab format / WinPim format
3. To load a whole design workspace (two examples are in the directory "Examples") go to menu bar:
   Model→Current Parameter Set →Load Parameters and model
4. To save a whole design workspace go to menu bar:
   Model→Current Parameter Set→Save Parameters and model

For any problems contact: landau@lag.ensieg.inpg.fr.

3.3 Description of the toolbox

3.3.1 Principal closed-loop controller design window

Launching the command "ppmaster", the principle window for closed-loop controller (R/S) design will be opened, see Fig.3.1.

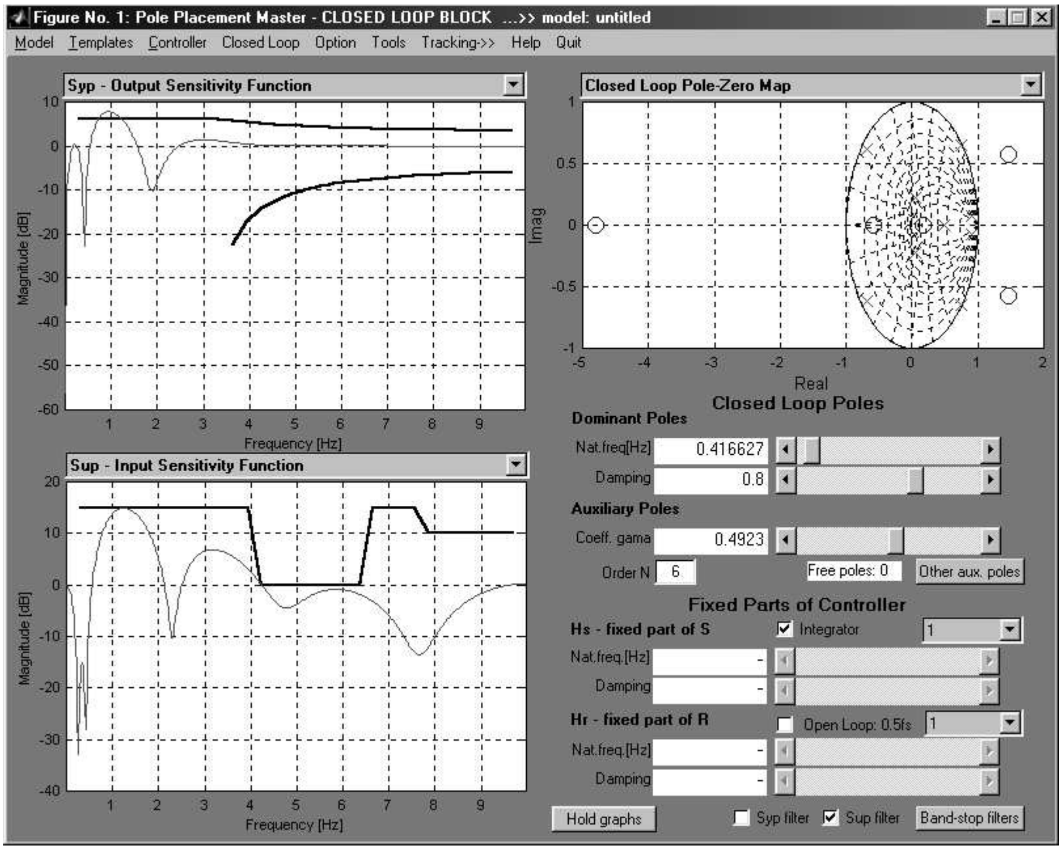

Figure 3.1: Principal window for closed-loop design

Menu bar

The menu bar of the principal window has following options (see Fig.3.1 at the top):

- *Model* - allows to load, save, edit and make analysis of the current digital plant model. The sub-menu contains:
  - Load model - for loading the digital plant model saved either in WinPim (extension .mod), or in MATLAB format .mat (variable A is a vector of denominator coeffeicients, B is a vector of numerator coefficients, Ts is sampling time).

Example: Consider to have a digital plant model as follows:

$$G(z^{-1}) = \frac{z^{-1}(b_1 z^{-1} + b_2 z^{-2})}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3}} = \frac{B(z^{-1})}{A(z^{-1})} \qquad (3.1)$$

with a sampling time $T_s = 0.05$s. The corresponding MATLAB variables will have the following format: $A = [1\ a_1\ a_2\ a_3]; B = [0\ 0\ b_1\ b_2]; Ts = 0.05;$.

- Save model - allows to save actual model either in WinPim, or in MATLAB format.
- Edit model - opens a small dialog for editing the coefficients of the model, the pure delay and the sampling time.
- Sampling time - allows to modify actual sampling time.
- Analysis of model - open sub-window for transfer-function analysis. For details on this window see bellow.
- Current Parameter Set - allows to save and load whole actual workspace of PPMaster. The loading is possible, either with model, or without model. **This is the most efficient way how to store actual design stage!**

- *Templates* - allows to load, save and edit templates for S_{yp} and S_{up} sensitivity functions. The option "Edit templates" opens a sub-window where some basic shapes can be created. For loading/saving the extension .tpl is considered, but the format of the file is standard MATLAB .mat format. The file with templates contains vectors: Wsyp - vector of S_{yp} upper template magnitude points in linear scale, Wsyp2 - vector of S_{yp} lower template magnitude points in linear scale, Wsup - vector of S_{up} upper template magnitude points in linear scale, fn - vector of normalized frequency points for all templates (the values are from 0 to 0.5).

- *Controller* - This option has following sub-options:
  - Save controller - for saving the final controller either in WinReg format or in MATLABfig:ppmwin version 5/version 4 .mat format. The saved variables are vectors R, S, T, Am, Bm of corresponding polynomial coefficients in increasing power of z^{-k} (see Load model example).
  - Display controller - opens transfer-function analysis sub-window for analysis of controller transfer function $R(z^{-1})/S(z^{-1})$. For details on this sub-window see bellow.
  - Stability Window - opens small sub-window displaying controller pole locations and indicating the controller stability. This sub-window is refreshed after every design parameter modification and it can be opened throughout entire controller design.

- *Closed loop* - opens transfer-function analysis sub-window for analysis of closed-loop transfer function $B(z^{-1})R(z^{-1})/P(z^{-1})$. For details on this sub-window see bellow.

- *Option* - Two sub-options allows to modify application colors and axes scales.

- *Tools* - contains some useful tools, these are:
  - Point Cursor - activating this tool (it is activated by default), when the left mouse-button is pressed and the mouse is positioned on some graph, the mouse actual position (X-Y scale values) on the graph is displayed.
  - Zoom - activates zoom capability of the mouse.
  - Figure - opens a dialog allowing to select which graphs will be plotted, when the selection is done, an independent figure is plotted with selected graphs.
- *Tracking*– >> - closes the closed-loop design principal window and opens the tracking part design principal window. In this window the tracking part T of the controller can be designed. For details see below.
- *Help* - The sub-option "Theory" opens a sub-window with theoretical background concerning the controller design. The sub-option "About application" gives some general information about the toolbox.
- *Quit* - close the application.

Graphs

The principal window contains three graphs (in left and right upper corners and in left lower corner) allowing to display the principal closed-loop properties, these are:

- $|S_{yp}(z^{-1})|$ output sensitivity modulus with corresponding imposed upper and lower template.
- $|S_{up}(z^{-1})|$ input sensitivity modulus with corresponding imposed upper template.
- $|S_{yb}(z^{-1})|$ complementary sensitivity modulus with corresponding imposed upper template.
- $P(z^{-1})$ closed-loop poles.
- $S(z^{-1})$ controller poles.
- step disturbance rejection (output signal evolution, when a step is added to $p(t)$).

The property, which is displayed on a graph, can be chosen using popup selector situated above the graph.

Design parameter adjusting tools

Adjusting slider/button object - The right lower part of the window contains several sliders and buttons making accessible all the design parameters. Every numerical parameter is adjustable either by slider, or by a button displaying current parameter value. Pressing the button, a small edit-window allows to modify from the keyboard the actual value.

Description of tools - The right lower part consists of several sections, which have following significance:

- Section "Dominant Poles" - contains two sliders/buttons to modify natural frequency (Nat.freq[Hz]) and damping (Damping) of the dominant closed-loop pole pair P_D.
- Section "Auxiliary Poles" - contains one slider/button allowing to set desired value of one real multiple pole. The pole's multiplicity is displayed bellow and it can be modified using '+/-' buttons. Aside, the value denoted "Free poles" indicates how many poles can be assigned yet. Finally, the button "Other aux. poles" opens sub-window which allows to assign more complicated auxiliary pole structures (complex pairs, other real poles). For sub-window description see below.
- Section "Hs - fixed part of S" - contains one check-box allowing to introduce one integrator as a part of the fixed part H_S. Next, in pop-up selection object, one can choose the form of the rest of the fixed part. The possibilities include:
  - 1 - no fixed parts except eventual integrator.
  - 1st order - one real pole. Its position on the real axe is adjustable form the lower slider/button denoted for this case "Coeff".
  - 2nd order - complex pair of poles with adjustable natural frequency and damping using two slider/buttons denoted "Nat.freq.[Hz]" and "Damping" respectively.
- Section "Hr - fixed part of R" - contains one check-box allowing to introduce one zero at -1 (opening of the loop at $1/2T_s$ Hz) as a part of the fixed part H_R. Next, in pop-up selection object, one can choose the form of the rest of the fixed part. The possibilities include:
  - 1 - no fixed parts except eventual loop opening.
  - 1st order - one real zero. Its position on the real axe is adjustable form the lower slider/button denoted for this case "Coeff".
  - 2nd order - complex pair of poles with adjustable natural frequency and damping using two slider/buttons denoted "Nat.freq.[Hz]" and "Damping" respectively.
- Section at the bottom - contains two buttons: "Hold graphs" and "Band-stop filters" and two check-boxes: "Syp filter" and "Sup filter".
  - "Hold graph" button allows to freeze actual cures, they are displayed in grey and dashed. The freezed curves are constantly displayed together with new actual curves. Once the "Hold graphs" button pressed it changed to "Release graphs" button allowing to erase the freezed curves.
  - "Band-stop filters" button opens a new sub-window for design of 2nd order digital notch filters F_{yp}, F_{up}. For detailed description of this sub-window see bellow.
  - The check-boxes "Syp filter" and "Sup filter" signal, if the designed sets of 2nd order notch filters (band-stop filters) F_{yp}, F_{up} will be used for controller design or not.

Transfer-function analysis sub-window

The transfer-function analysis sub-window (see Fig.3.2) is opened when plant model, controller or closed-loop transfer-function analysis is demanded (accessible from menu bar). The following graphical objects are displayed in this window:

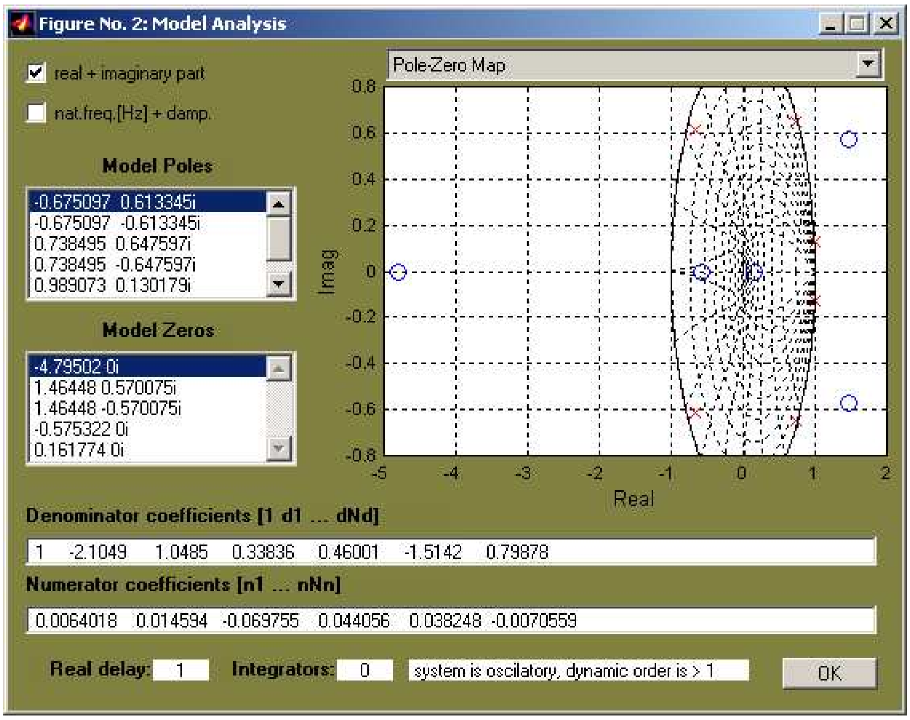

Figure 3.2: Transfer function analysis window

- *Graph* - allows to display pole-zero map, frequency response amplitude and phase. The choice of what is displayed is made by popup selector placed above the graph.
- *Check-boxes* - "real + imaginary part" and "nat.freq.[Hz] + damp." switch the display format for poles and zeros listed bellow these checkboxes. Either the real and the imaginary parts of the poles/zeros, or the natural frequencies and dampings are displayed.
- *Lists* - as mentioned, they list the transfer function poles and zeros in one of the possible format.
- *Edit lines* - contain transfer function denominator and numerator coefficients.
- *Real delay / Integrators* - give real delay of the transfer function (the delay in number of samples including zero-order holder delay) and the number of integrators. aside these numbers, some comment on the system dynamic is displayed.

Auxiliary closed-loop poles sub-window

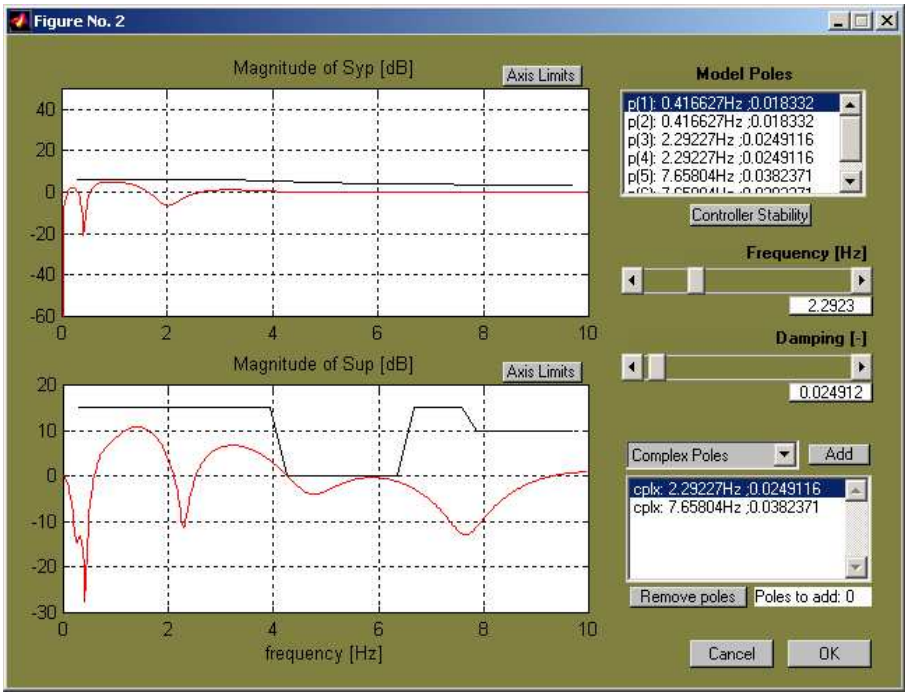

Figure 3.3: Auxiliary closed-loop poles selection window

This window is used to set any desired auxiliary closed-loop pole's structure, thus it modifies the design parameter P_F'/P_F''. The window contains following graphical objects:

- *Graphs* - two graphs displays S_{yp} and S_{up} sensitivity function characteristics for observing the effects of the placed auxiliary poles onto the closed loop. The buttons "Axis Limits" above every graph serve for adjusting the scales limits. They open small dialog displaying the actual limits. To modify the limits, one has to change the displayed values and to close the dialog.

- *Model Poles List* - the listing object in the upper right corner list all model's poles in the format of natural frequency and damping. The cursor in this list marks one pole. When a new pole is added to auxiliary poles in this window, it has by default the same position as the pole marked in model poles, but only if the same kind of poles (complex or real pole) are marked and added.

- *List of auxiliary poles* - The list of all desired auxiliary poles (except the real multiple pole adjustable from the principal window) is at the bottom on the right. On the contrary to the model pole list, a complex pair of poles is denoted only by one item named "cplx:" with the corresponding natural frequency and damping. Also a multiple real pole is denoted only by one item named "real:" and with pole's position followed with pole's multiplicity.

- *Add button and popup* - To add a pole one has to use "Add" button. The popup selector on the left of this button allows to choose what type of pole will be added (real multiple pole or complex pair).
- *Remove poles button* - This button remove auxiliary pole/s marked by the cursor in the auxiliary pole list.
- *Poles to add value* - this value signify how many poles can be assigned yet (number of free non-assigned poles).
- *Controller Stability button* - the same as in the bar menu option - "Controller→Stability Window".
- *Slider/buttons* - The sliders and buttons with the actual values allow us to modify the properties of the auxiliary pole/s marked by cursor in auxiliary pole list. If a complex pair of poles is marked, one can change natural frequency and damping, if a real multiple pole is marked, the pole's position on real axe and its multiplicity can be modified.

Band-stop filter design sub-window

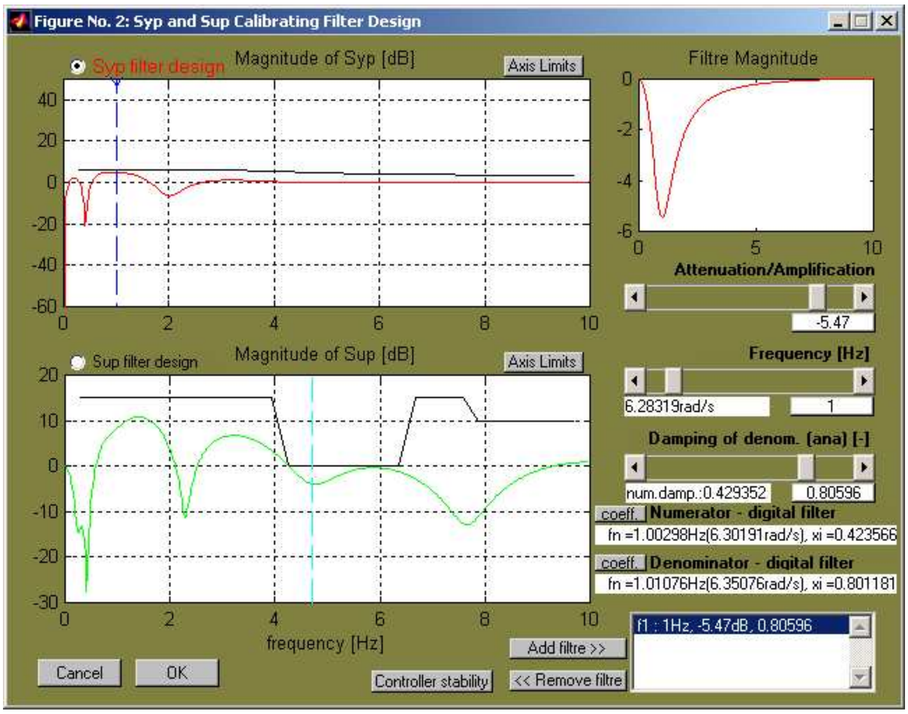

Figure 3.4: Band-stop filter design window

The band-stop filter window serve for design of 2nd order notch filters F_{yp} and F_{up} of pole-placement shaping. The number of filter is not limited, by one filter rise the controller complexity by 2. The graphical objects in the window has following utility:

- *Graphs* - two large graphs on the left displays S_{yp} and S_{up} sensitivity function characteristics for observing the effects of the filters onto the closed loop. Any added filter is marked by one dashed line describing his actual position. The buttons "Axis Limits" above every graph serve for adjusting the scales limits. They open small dialog displaying the actual limits. To modify the limits, one has to change the displayed values and to close the dialog.

  One small graph on the right display frequency response of actually modified 2nd order notch (band-stop) filter.

- *Radio buttons "Syp filter design", "Sup filter desig"* - they determine what filters are designed, if F_{yp} for S_{yp} sensitivity function or F_{up} for S_{up} sensitivity function.

- *Slider/buttons* - The sliders and the buttons displaying actual value of each slider allow to modify the three design properties of actual notch filter. The properties are :
  - M_t - band-stop attenuation/amplification.
  - f_t - band-stop frequency.
  - ζ_d - analog equivalent denominator damping (adjust band-stop width).

- *Numerator and Denominator - digital filter* - The white text lines entitled "Numerator - digital filter" and "Denominator - digital filter" display respectively the natural frequency (in Hertz and in rad/s) and damping of digital filter numerator and denominator.

- *"coeff" button* - It opens a small dialog with one edit line containing the coefficients of the actual digital filter numerator/denominator.

- *List of filters* - The listing object at the bottom lists all applied filters F_{yp} / F_{up}. The cursor marks the actually modified filter. This filter is also marged in the corresponding graph by two triangles.

- *Add/Remove filter button* - The button "Add filter" adds one filter F_{yp} or F_{up} to the list. The button "Remove filter" removes the filter marked by the cursor in the filter list.

- *Controller Stability button* - the same as in the bar menu option - "Controller→Stability Window".

3.3.2 Tracking part design window

The tracking part design window enable to design the tracking part T of the controller and the tracking model B_m/A_m. These graphical objects are used:

- *Graphs* - The three graphs can display tracking (time domain) properties of the closed loop system. These properties are accessible:
  - System output $y(t)$ step response.

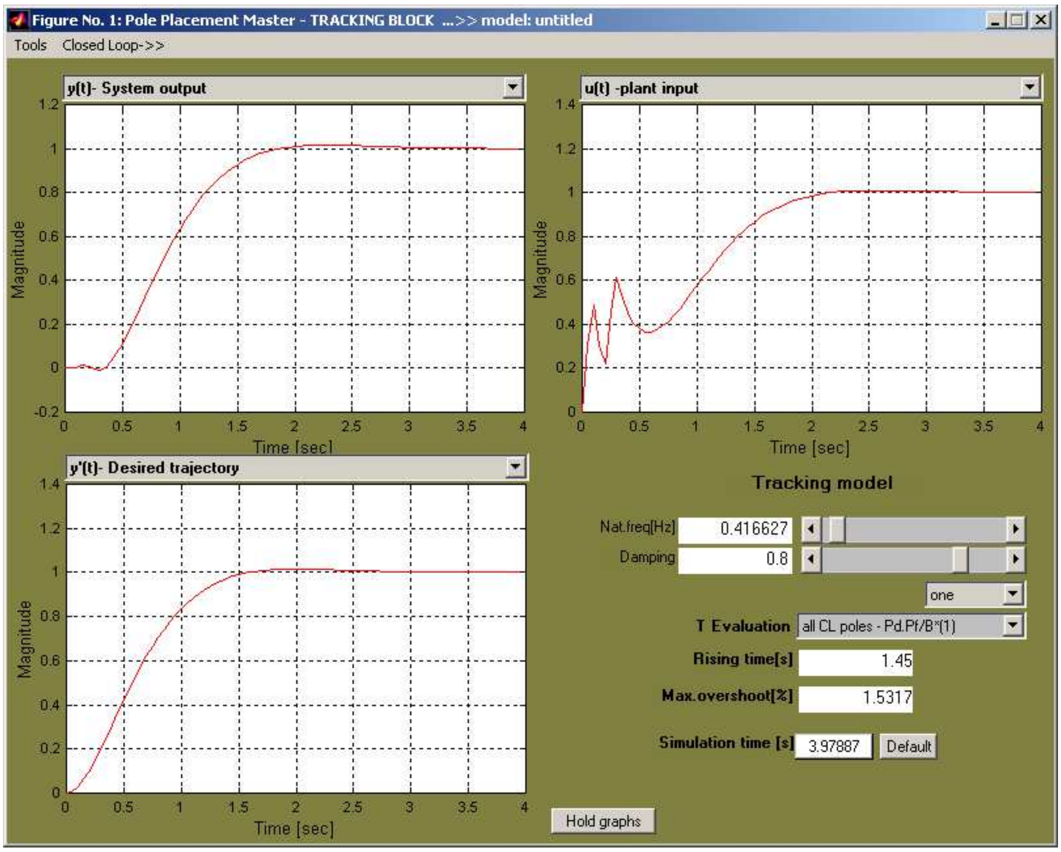

Figure 3.5: Tracking part design window

- Desired trajectory $y*(t)$ step response.
- Command $u(t)$ step response.

- *Upper popup selector* - It is used for choosing either one or two tracking models connected in cascade.
- *Lower popup selector* - This popup determine the form of the tracking part T. There are three possibilities:
  - "All CL poles - Pd.Pf/B*(1)" - $T(z^{-1})$ is a polynomial with adjusted static gain containing all closed loop poles.
  - "dominant CL poles - Pd.Pf(1)/B*(1) - $T(z^{-1})$ is a polynomial with adjusted static gain containing one complex pair of roots corresponding to closed loop dominant poles defined by P_D.
  - "Gain - P(1)/B(1)" - T is a constant with an appropriate value setting static gain of the whole system equal to one.
- *Sliders* - the two sliders modify actual tracking model properties, specifically the natural frequency and the damping. The current values are displayed on the left.

- *Rising time value* - It gives the system rising time (step response time to reach 90% of the reference final value) in seconds.
- *Max.overshoot value* - It represents the overshoot over the final reference value of step response in %.
- *Simulation time button* - On this button, the actual simulation time of displayed responses is showed. Pressing the button, the simulation time can be modified. The "Default" button compute a default sampling time according to system dynamics.

Chapter 4

Bibliographical indications

An integral description of the theory related to the pole placement with sensitivity shaping can be found in [1]. The method was introduced in [2] and improved in [6]. The software toolbox ”PPMaster” was presented in [5]. Some practical examples of robust controller design can be found in [3, 6, 4, 1].

Bibliography

[1] I.D. Landau. *Commande des systèmes.* Hermes,Paris, 2002.

[2] I.D. Landau and A. Karimi. Robust digital control using pole placement with sensitivity function shaping method. *Int. J. Robust and Nonlin. Cont.*, 8:191–210, 1998.

[3] I.D. Landau, J. Langer, D. Rey, and J. Barnier. Robust control of 360 flexible arm using the combined pole placement/sensitivity function shaping method. *IEEE Trans. Control Systems Tchnol.*, pages 369–383, 1996.

[4] H. Prochazka. Comments on the paper "controller order reduction by identification in closed-loop applied to a benchmark". *accepted to European Journal of Control*, 2003.

[5] H. Prochazka and I.D. Landau. Logiciel pour l'enseignement et le calcul du placement de pôles robuste. *Conf. Int. Francophone d'Automatique Nante*, pages 694–698, 7 2002.

[6] H. Prochazka and I.D. Landau. Pole placement with sensitivity function shaping using 2nd order digital notch filters. *Automatica*, 39(6):1103–1107, 2003.