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**Robust R-S-T Digital Control  
and  
Open Loop System Identification  
*A Brief Review***

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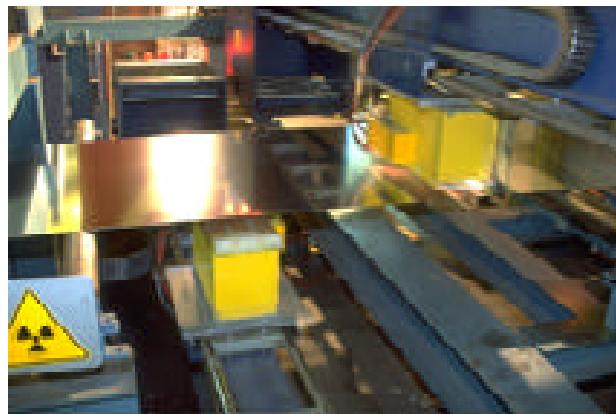


Peugeot (PSA)

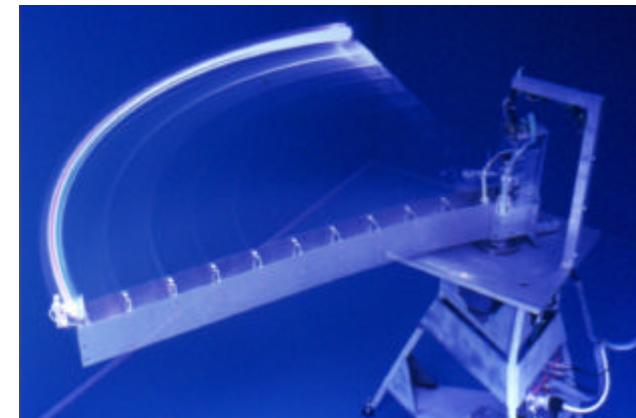


Double Twist Machine  
(Pourtier)

## Applications of R-S-T Controllers



Sollac (Florange)  
Hot Dip Galvanizing



360° Flexible Arm (LAG)

## Implementation of R-S-T Digital Controllers



PLC Leroy implements  
R-S-T digital controllers and  
Data acquisition modules

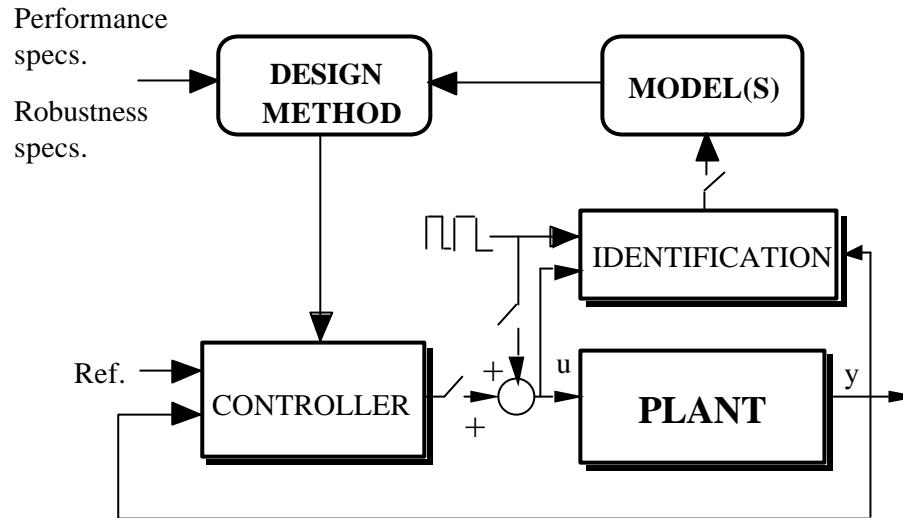


**ALSPA 320**



ALSPA 320 implements  
R-S-T digital controllers and  
Data acquisition modules

# Controller Design and Validation



- 1) Identification of the dynamic model
- 2) Performance and robustness specifications
- 3) Compatible controller design method
- 4) Controller implementation
- 5) Real-time controller validation  
(and on site re-tuning)
- 6) Controller maintenance (same as 5)

## **Outline**

### **Robust digital control**

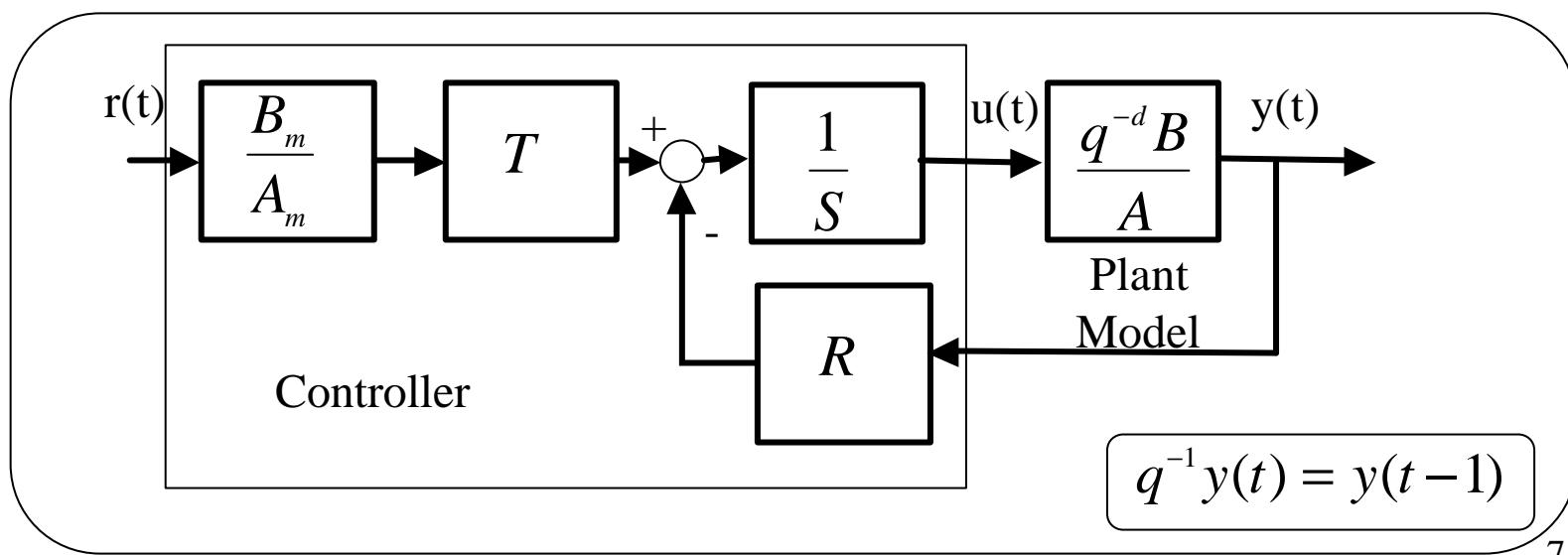
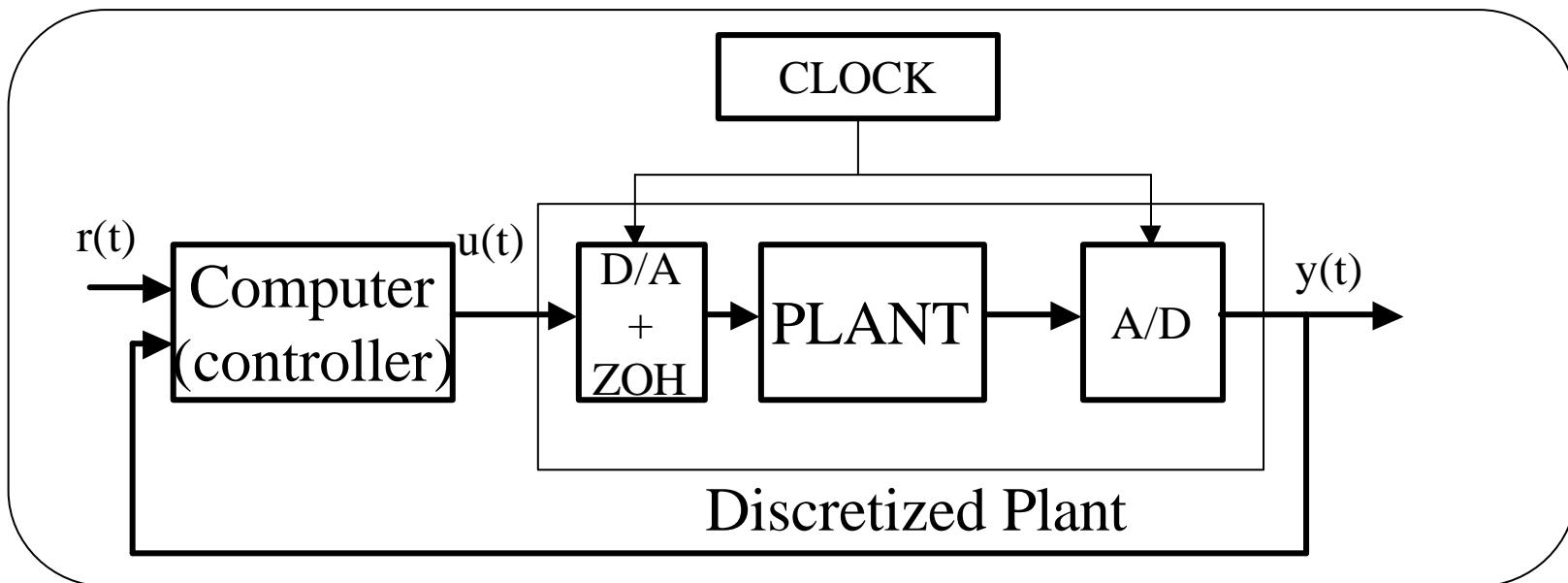
- The R-S-T digital controller
- Basic design
- Robustness issues
- An example

### **Open loop system identification**

- Data acquisition
- Model complexity
- Parameter estimation
- Validation

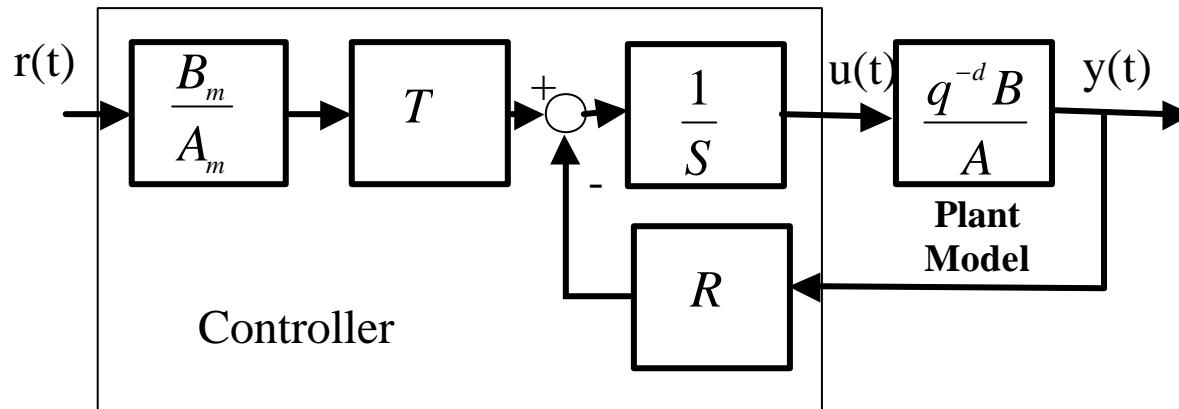
## **Robust Digital Control**

# The R-S-T Digital Controller



$$q^{-1}y(t) = y(t-1)$$

## The R-S-T Digital Controller



*Plant Model:*

$$G(q^{-1}) = \frac{q^{-d} B(q^{-1})}{A(q^{-1})} = \frac{q^{-d-1} B^*(q^{-1})}{A(q^{-1})}$$

$$A(q^{-1}) = 1 + a_1 q^{-1} + \dots + a_{n_A} q^{-n_A} \quad B(q^{-1}) = b_1 q^{-1} + \dots + b_{n_B} q^{-n_B} = q^{-1} B^*(q^{-1})$$

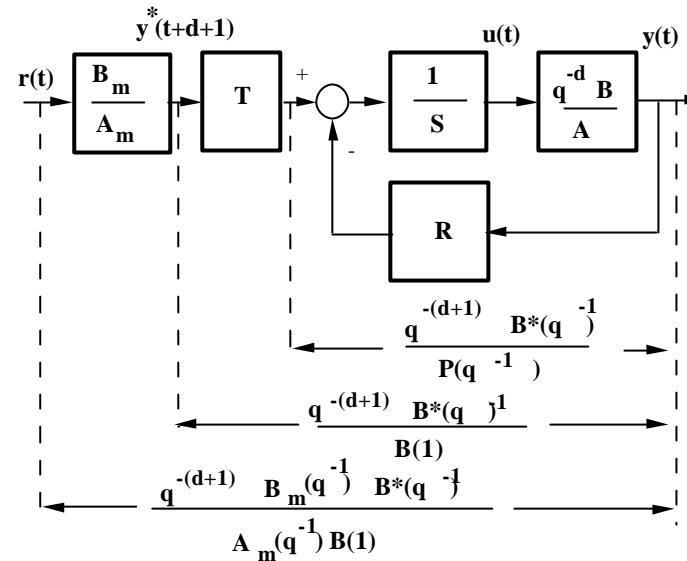
*R-S-T Controller:*

$$S(q^{-1})u(t) = T(q^{-1})y^*(t+d+1) - R(q^{-1})y(t)$$

*Characteristic polynomial (closed loop poles):*

$$P(q^{-1}) = A(q^{-1})S(q^{-1}) + q^{-d} B(q^{-1})R(q^{-1})$$

# Pole Placement with R-S-T Controller



*Controller :*  $R = H_R R'; S = H_S S'$        $H_R, H_S$  : fixed parts

*Regulation:*  $R'$  and  $S'$  solutions of:  $AH_S S' + q^{-d} BH_R R' = P = P_D P_F$

*Tracking :*  $T = P / B(1)$

dominant poles      auxiliary poles

*Reference trajectory:*  $y^*$  computer file

$y^* = (B_m / A_m)r$

## Connections with other Control Strategies

- Digital PID :  $n_R = n_S = 2; H_s = 1 - q^{-1}$
- Tracking and regulation with independent objectives(MRC):

$$P = B^* P_D P_F$$

(Hyp.:  $B^*$  has stable damped zeros)

- Minimum variance tracking and regulation (MVC):

$$P = B^* C$$

noise model

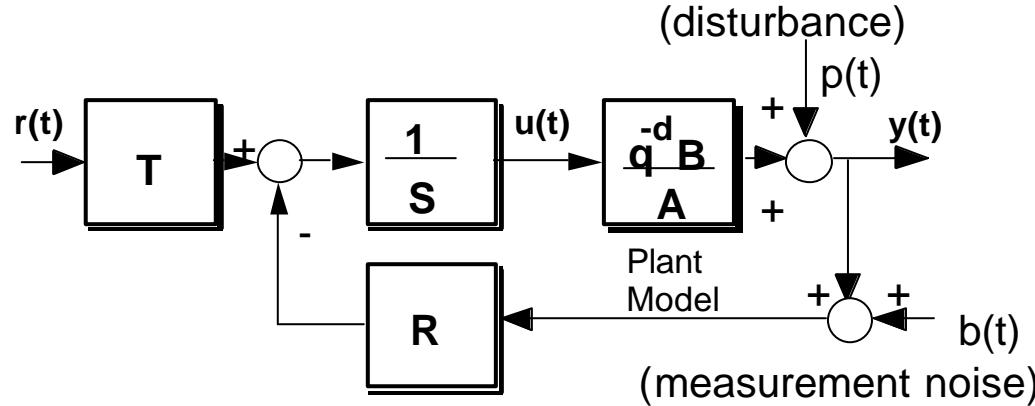
(Hyp.:  $B^*$  has stable damped zeros)

- Internal Model Control (IMC):

$$P = A P_F$$

(Hyp.:  $A$  has stable damped poles)

## The Sensitivity Functions



**Output sensitivity function ( $p \rightarrow y$ )**

$$S_{yp}(q^{-1}) = \frac{AS}{AS + q^{-d}BR} = \frac{AS}{P}$$

**Input sensitivity function ( $p \rightarrow u$ )**

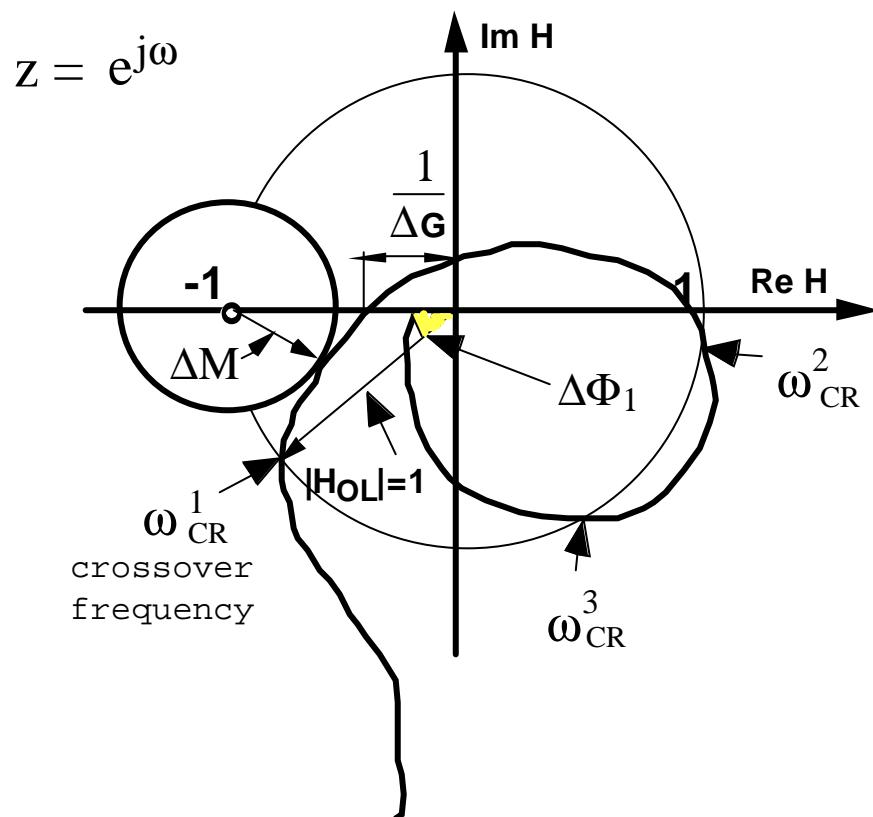
$$S_{up}(q^{-1}) = -\frac{AR}{AS + q^{-d}BR} = -\frac{AR}{P}$$

$$S_{yp} - S_{yb} = 1$$

**Noise sensitivity function ( $b \rightarrow y$ )**

$$S_{yb}(q^{-1}) = -\frac{q^{-d}BR}{AS + q^{-d}BR} = -\frac{q^{-d}BR}{P}$$

## Robustness Margins



Typical values:

$$\Delta M \geq 0.5 \text{ (-6dB)}, \quad \Delta \tau > T_s$$

$$\Delta M \geq 0.5 \Rightarrow \Delta G \geq 2 ; \Delta \Phi > 29^\circ$$

*The inverse is not necessarily true!*

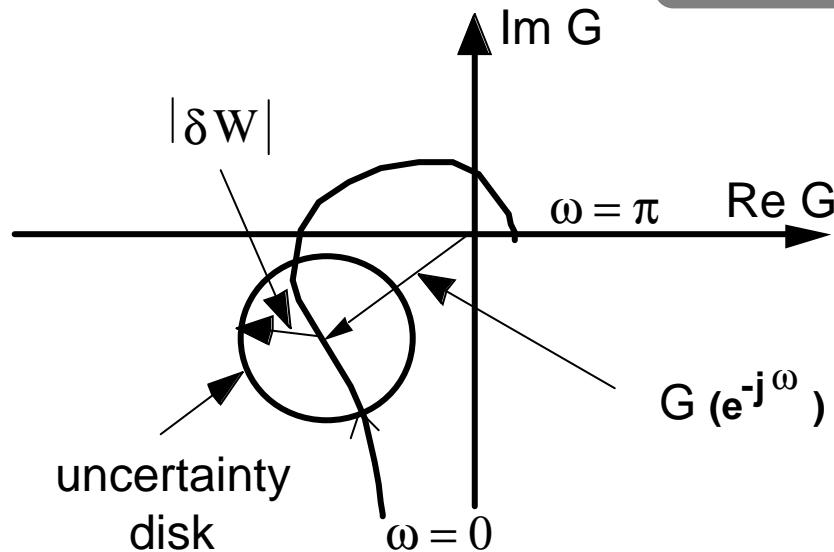
Modulus Margin:

$$\Delta M = |1 + H_{OL}(z^{-1})|_{\min} = (|S_{yp}(z^{-1})|_{\max})^{-1} = (\|S_{yp}\|_{\infty})^{-1}$$

Delay Margin:

$$\Delta \tau = \min_i \frac{\Delta \Phi_i}{\omega_{\text{CR}}}$$

## Robust Stability



Family of plant models:

$$G' \in F(G, d, W_{xy})$$

$G$  – nominal model;  $\|d(z^{-1})\|_\infty \leq 1$

$W_{xy}(z^{-1})$  - size of uncertainty

*Robust stability condition:*

a related sensitivity function

a type of uncertainty

$$\|S_{xy}W_{xy}\|_\infty < 1$$

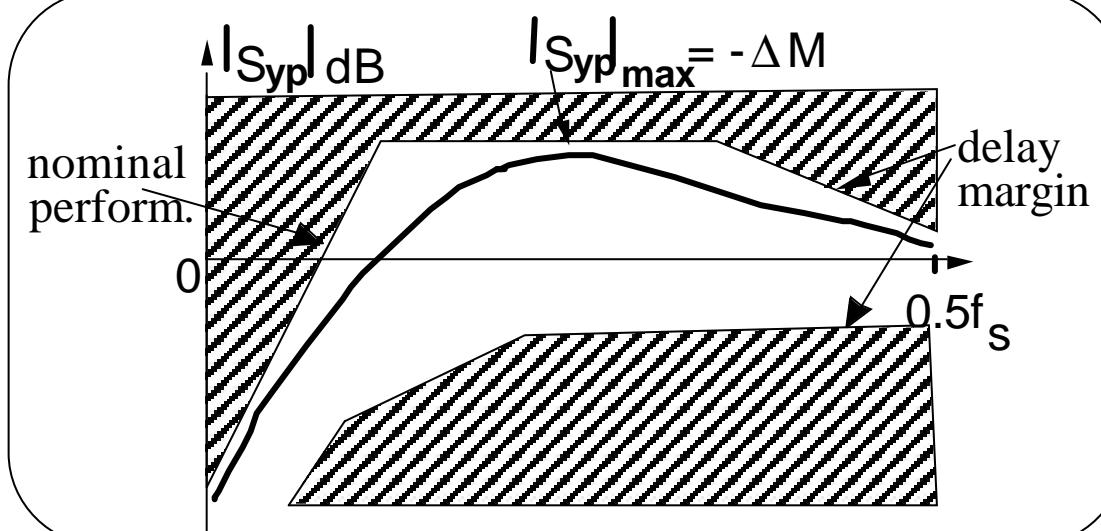
defines the size of the tolerated uncertainty

defines an upper template for the modulus of the sensitivity function

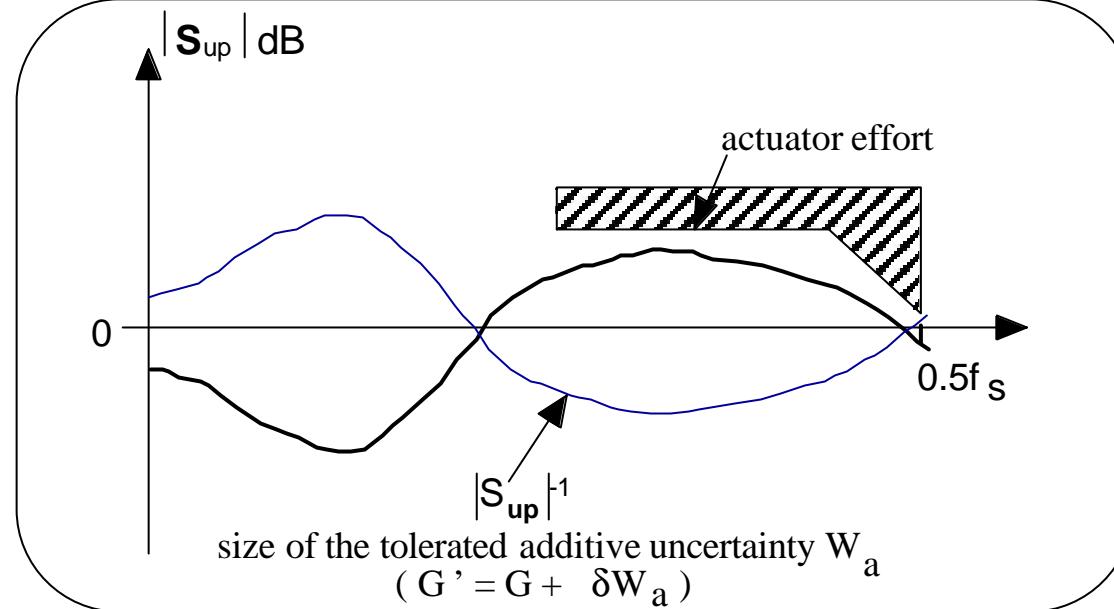
$$|S_{xy}| < |W_{xy}|^{-1}$$

There also lower templates (because of the relationship between various sensitivity fct.)

## Templates for the Sensitivity Functions



Output Sensitivity  
Function



Input Sensitivity  
Function

## Robust Controller Design

### Pole placement with sensitivity functions shaping

Nominal performance:  $P_D$  and part of  $H_R$  and  $H_S$

$$\begin{aligned} P &= P_D \circledcirc P_F \\ R &= R' \circledcirc H_R \\ S &= S' \circledcirc H_S \end{aligned}$$

*Allow to shape the sensitivity functions*

Several approaches to design :

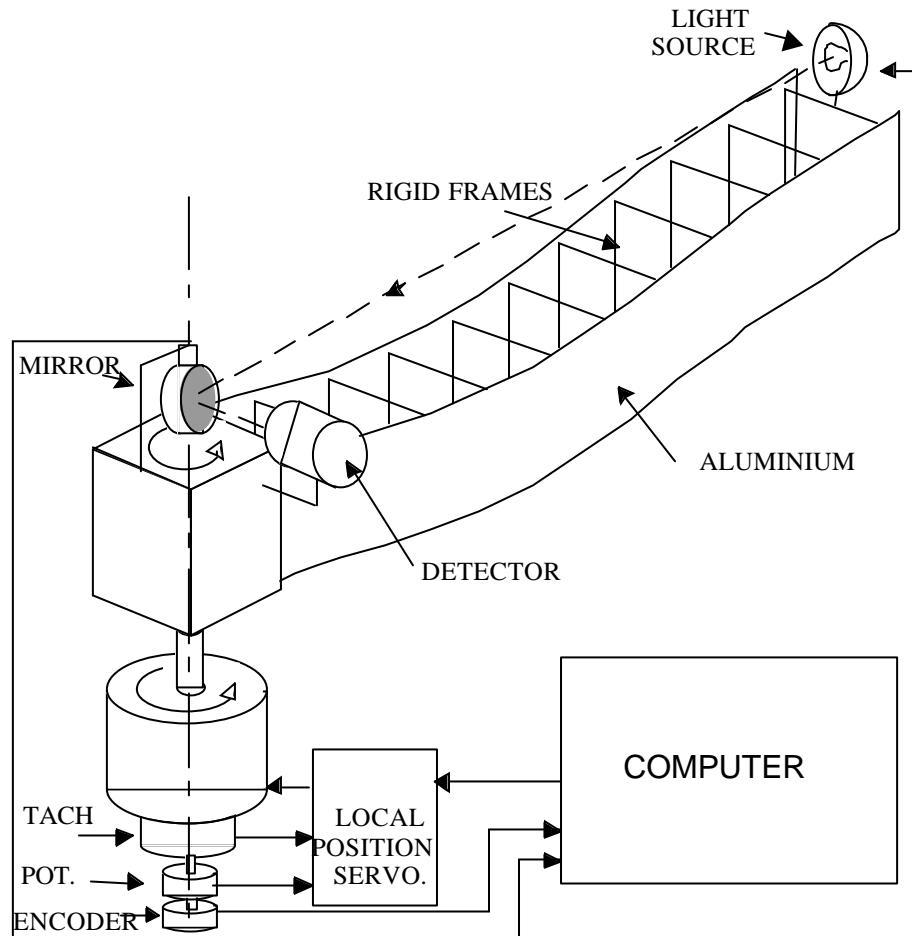
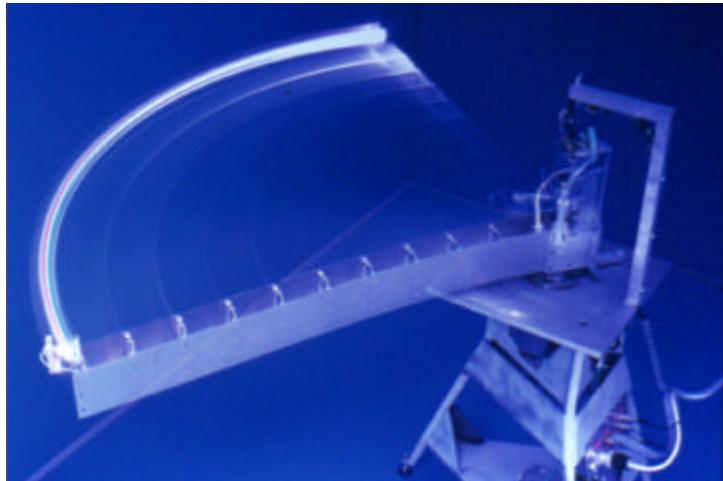
-Iterative

*Choosing  $P_F$  and using band stop filters  $H_{Ri} / P_{Fi}$ ,  $H_{Sj} / P_{Fj}$*

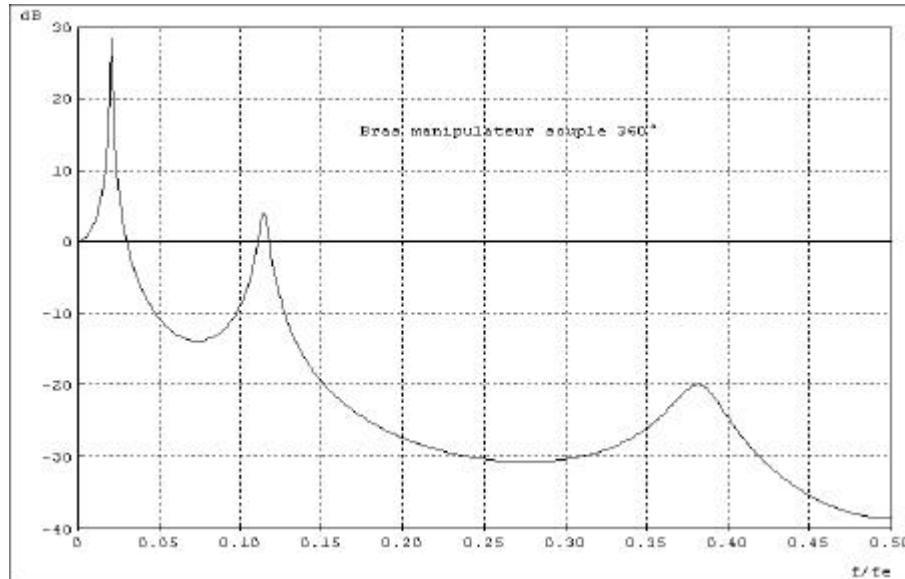
-Convex optimization

(see Langer, Landau, Automatica, June99, *Optreg* (Adaptech) )

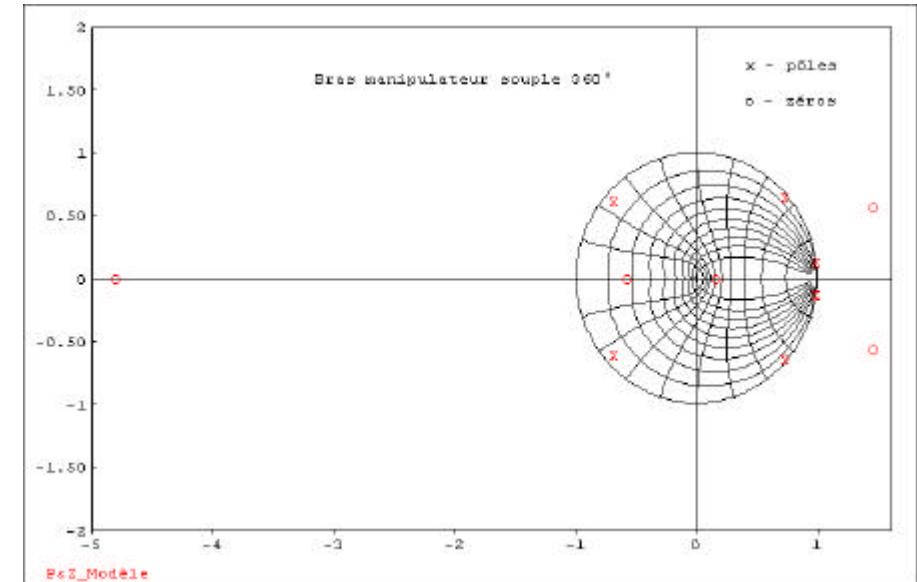
## 360° Flexible Arm



# 360° Flexible Arm



Frequency characteristics

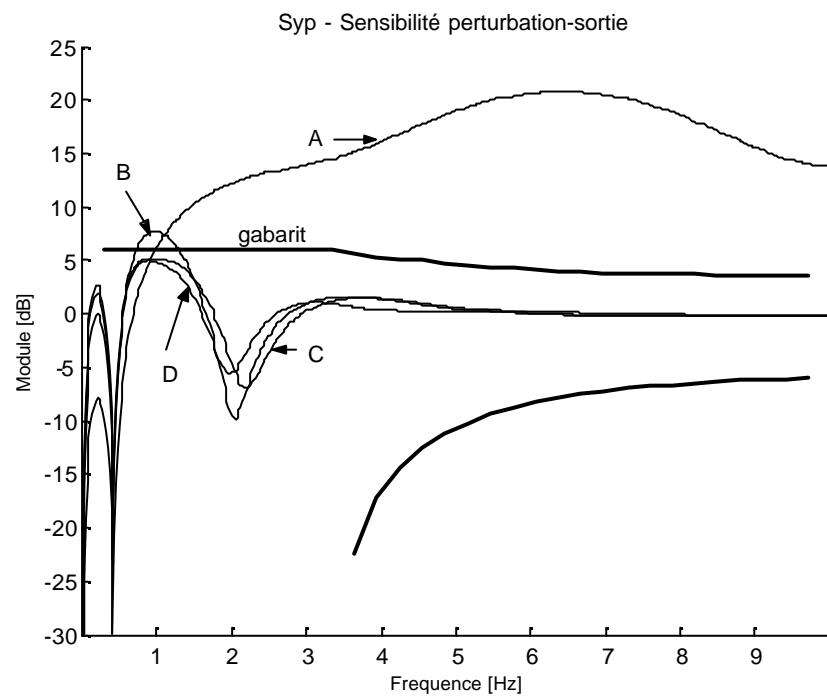


Poles-Zeros

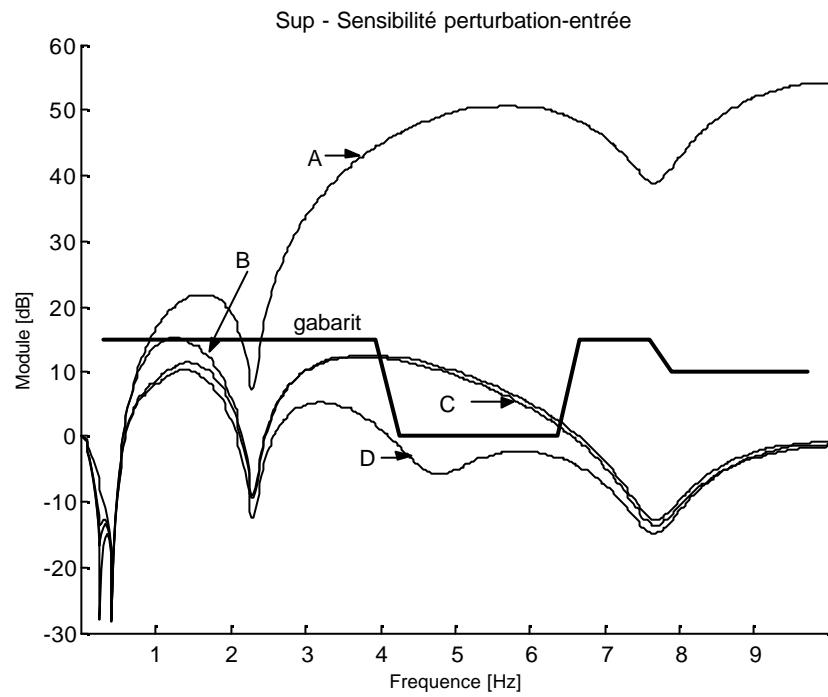
*(Identified Model)*

# Shaping the Sensitivity Functions

Output Sensitivity Function -  $S_{yp}$



Input Sensitivity Function -  $S_{up}$



A- without auxiliary poles

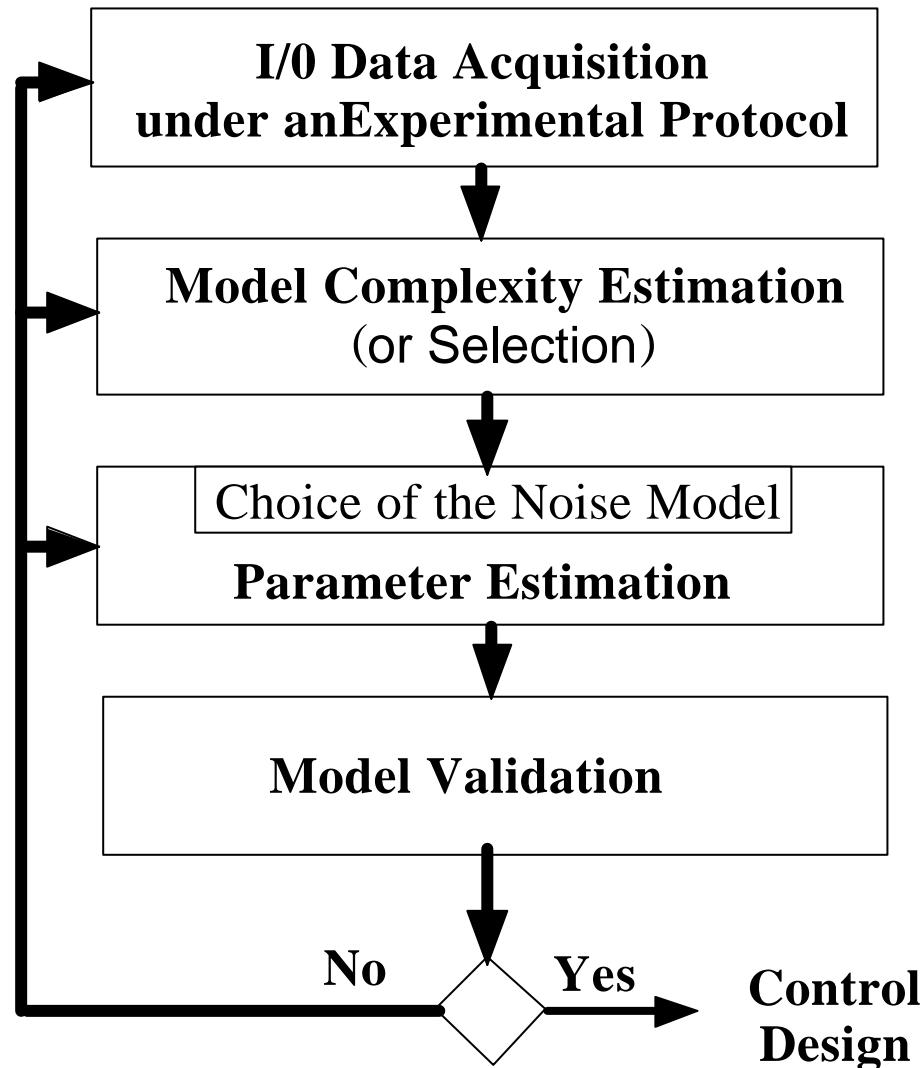
B- with auxiliary poles

C- with stop band filter  $H_{S1} / P_{F1}$

D- with stop band filter  $H_{R2} / P_{F2}$

## **Open Loop System Identification**

# System Identification Methodology



## I/O Data Acquisition

Signal : a P.R.B.S sequence

Magnitude : few % of the input operating point

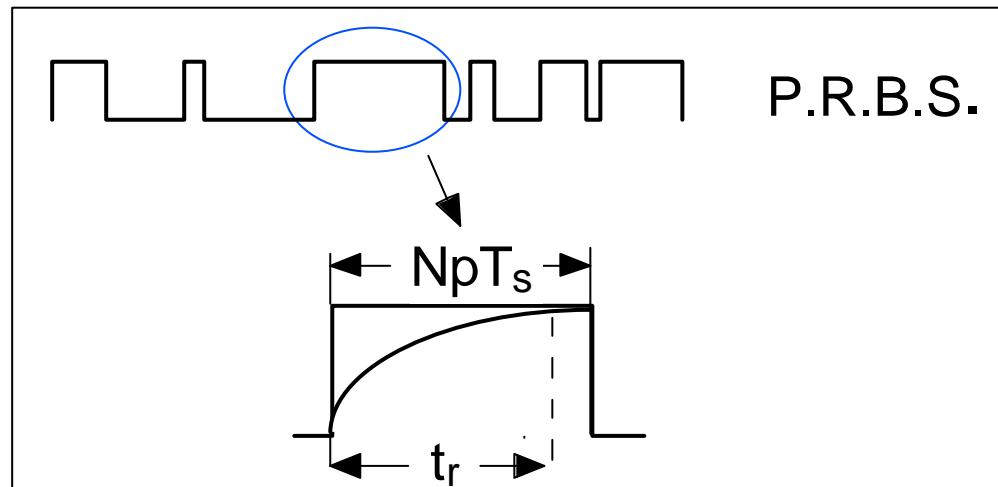
Clock frequency :  $f_{clock} = (1/p)f_s$ ;  $p = 1, 2, 3$  ( $f_s$  = sampling frequency)

Length :  $(2^{N-1} - 1)pT_s$ ;  $N$  = number of cells,  $T_s = 1/f_s$

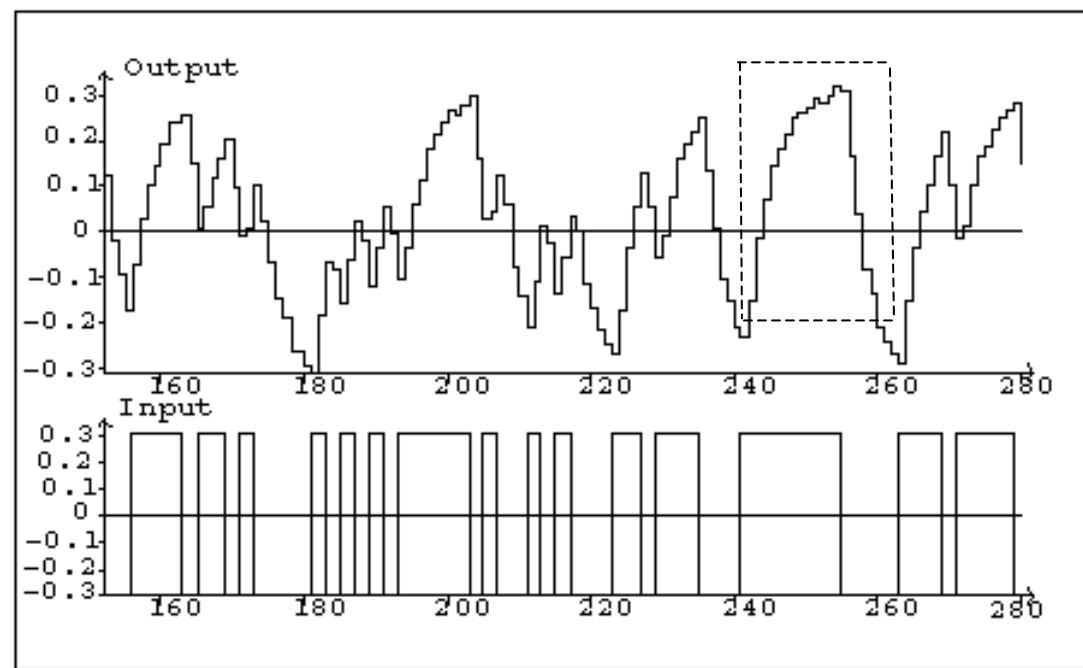
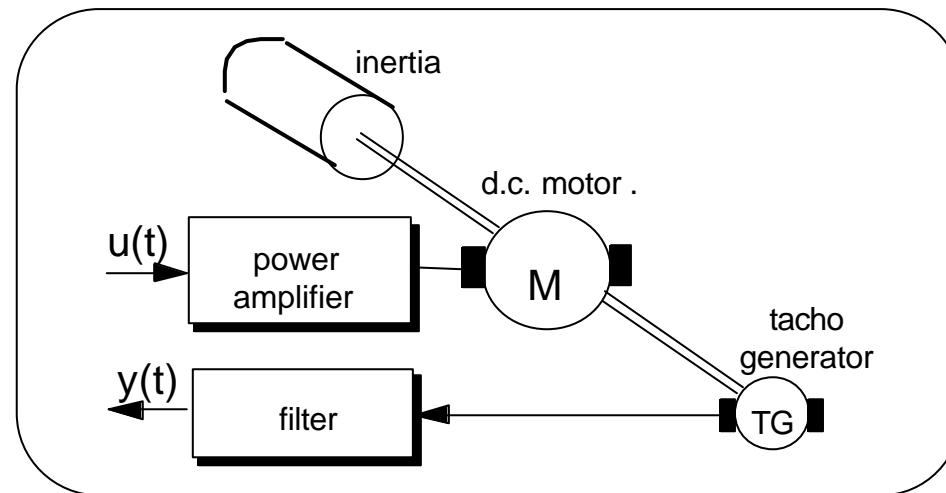
Largest pulse :  $NpT_s$

**Length** : < allowed duration of the experiment

**Largest pulse** :  $\geq t_r$  (rise time)



## An I/O File



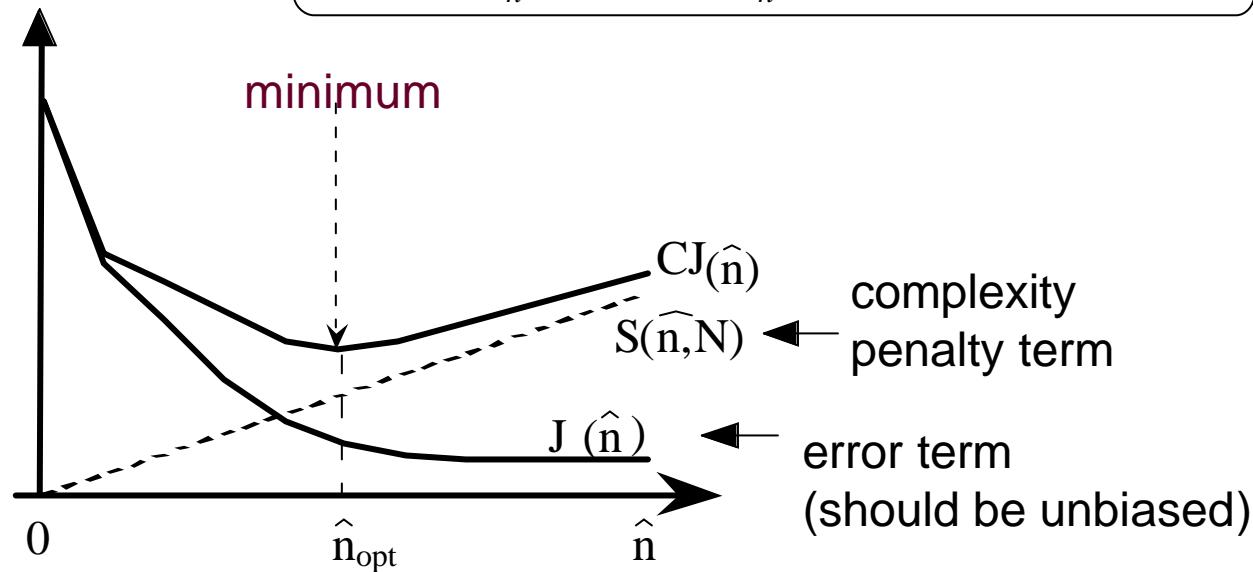
## Complexity Estimation from I/O Data

### Objective :

To get a good estimation of the model complexity ( $n_A, n_B, d$ ) directly from noisy data

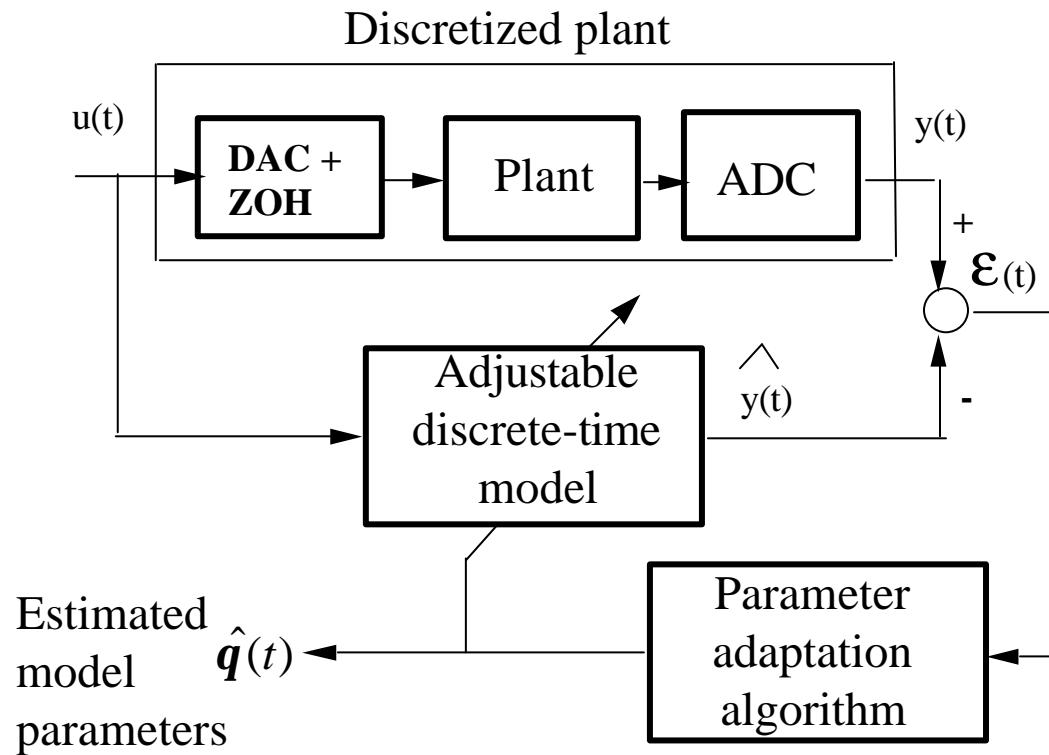
$$n = \max(n_A, n_B + d)$$

$$\hat{n}_{opt} = \min_{\hat{n}} CJ = \min_{\hat{n}} [J(\hat{n}) + S(\hat{n}, N)]$$



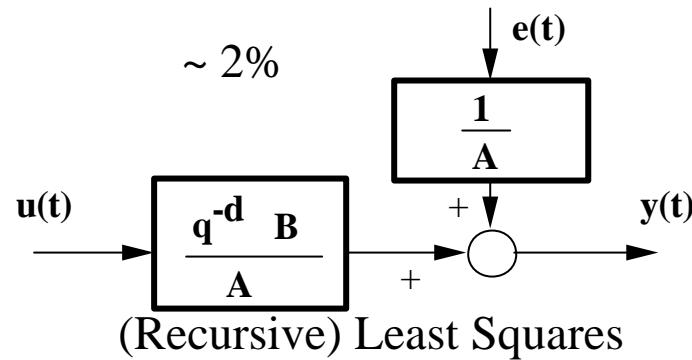
*To get a good order estimation,  $J$  should tend to the value for noisy free data when  $N \rightarrow \infty$  (use of instrumental variables)*

## Parameter Estimation

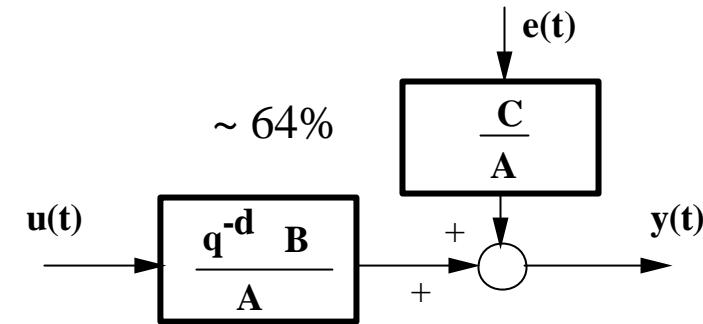


## «Plant + Noise » Models

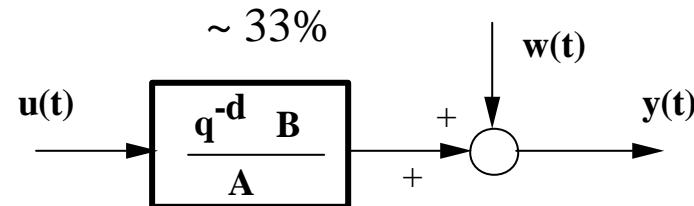
$$S1: A(q^{-1})y(t) = q^{-d}B(q^{-1})u(t) + e(t)$$



$$S3: A(q^{-1})y(t) = q^{-d}B(q^{-1})u(t) + C(q^{-1})e(t)$$

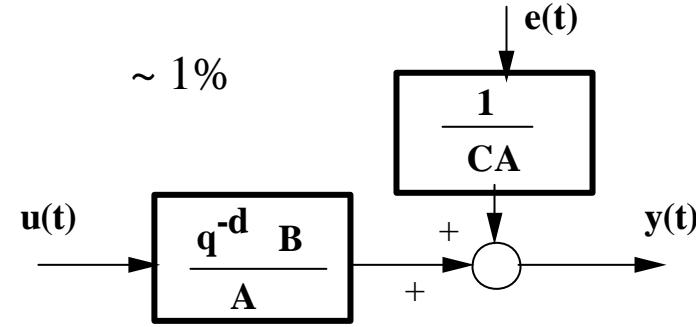


$$S2: A(q^{-1})y(t) = q^{-d}B(q^{-1})u(t) + A(q^{-1})w(t)$$



Output Error(O.E.)  
Instrumental Variable...

$$S4: A(q^{-1})y(t) = q^{-d}B(q^{-1})u(t) + [1/C(q^{-1})]e(t)$$



Generalized Least Squares

## Parameter Estimation Methods

Plant Model

$$y(t+1) = -A * (q^{-1})y(t) + B * (q^{-1})u(t-d) = \mathbf{q}^T \mathbf{y}(t)$$

$\mathbf{q}$  – parameter vector;  $\mathbf{y}$  – measurement vector

Estimated model

$$\hat{y}^o(t+1) = \hat{\mathbf{q}}^T(t) \mathbf{f}(t)$$

$\hat{\mathbf{q}}$  – estimated parameter vector;  $\mathbf{f}$  – observation vector

Prediction error

$$\mathbf{e}^0(t+1) = y(t+1) - \hat{\mathbf{q}}^T(t) \Phi(t) = y(t+1) - \hat{y}^o(t+1)$$

Parameter adaptation algorithm

$$\hat{\mathbf{q}}(t+1) = \hat{\mathbf{q}}(t) + F(t+1) \Phi(t) \mathbf{e}^0(t+1)$$

$$F^{-1}(t+1) = \mathbf{I}_1(t) F^{-1}(t) + \mathbf{I}_2(t) \Phi(t) \Phi^T(t)$$

$$0 < \mathbf{I}_1(t) \leq 1; 0 \leq \mathbf{I}_2(t) < 2$$

$$\Phi(t) = f[\mathbf{f}(t)]$$

## Parameter Estimation Methods

I- *Based on the asymptotic whitening of the prediction error*

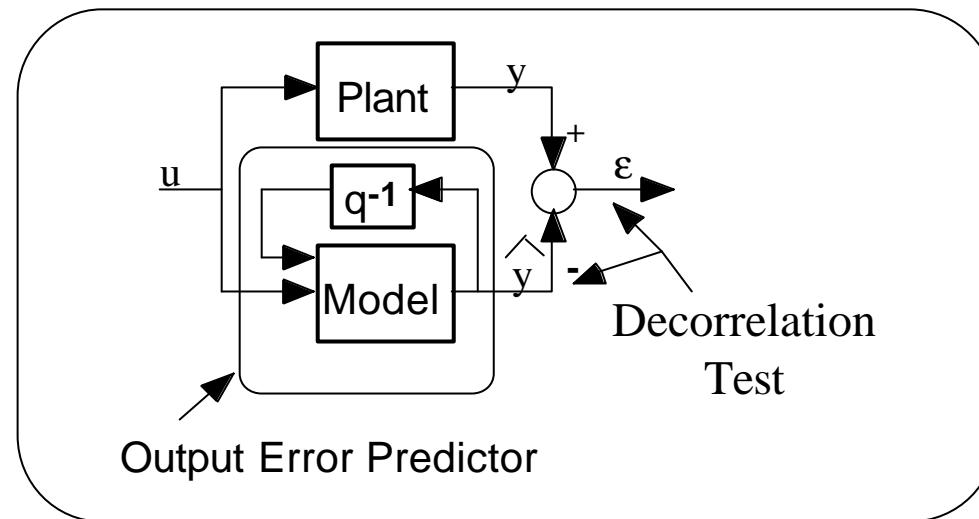
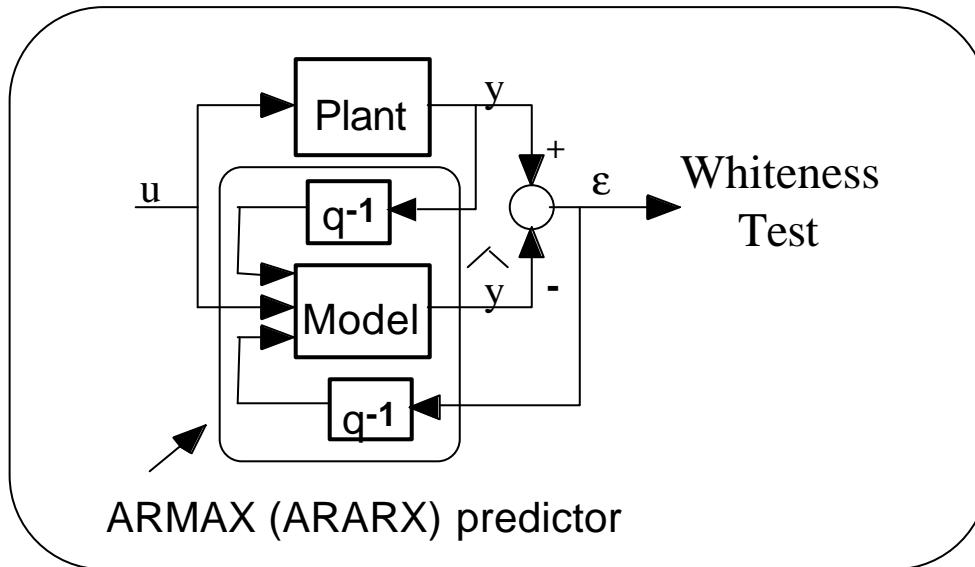
(Recursive Least Squares, Extended Least Squares, Recursive Max. Likelihood, O.E. with Extended Prediction Model )

II- *Based on the asymptotic decorrelation between the prediction error and the observation vector*

(Output Error, Instrumental Variable)

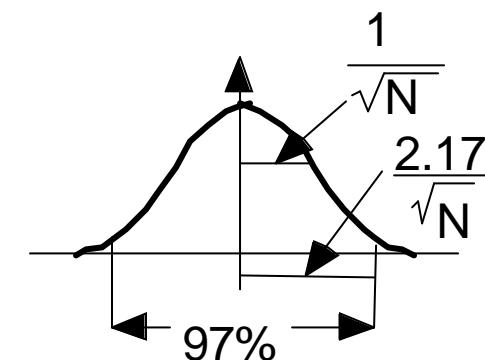
# Validation of Identified Models

## Statistical Validation



$$|RN(i)| \leq \frac{2.17}{\sqrt{N}}; i \geq 1$$

↑  
normalized crosscorelation      ↓  
number of data



$$N = 256 \rightarrow |RN(i)| \leq 0.136$$

practical value :  $|RN(i)| \leq 0.15$

## **Software Tools for Implementing the Methodology**

### System Identification

- Winpim (Adaptech)  
identification in open loop and closed loop operation
- CLID (Adaptech)  
identification in closed loop (Matlab Toolbox)

### Controller Design

- Winreg (Adaptech)  
design and optimisation of R-S-T digital controllers
- Optreg (Adaptech)  
automated design of robust digital controllers (under Matlab)

### Real-time implementation

- Wintrac (Adaptech): cascade digital control

## « Personal » References

Landau.I.D.,(1990) *System Identification and Control Design*, Prentice Hall, N.J.,USA

Landau I.D., (1995) : « Robust digital control of systems with time delay  
(the Smith predictor revisited) », *Int. J. of Control*, vol. 62, pp. 325-347.

Landau I.D., Lozano R., M'Saad M., (1997) : *Adaptive Control*, Springer, London,U.K.

Landau I.D., (1993) *Identification et Commande des Systèmes*, 2nd edition,  
Hermes, Paris (June)

Landau I.D., (2002) *Commande des Systèmes – Conception, identification  
et mise en œuvre*, Hermes, Paris (June)