

N. Marchand

Introduction

Modeling

Cartesian coordinates

Frames

Newton

X4

Kinematics

Arm robots Inner-loop Geometrical mo Kinematic mod

Dynamical mode

Conclusion

Path planning

Workspace and obstacles

path planning

Mobile robotics

Visual servoing

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Robotics

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INTRODUCTION

- Historical perspective
 - A great interest in "automatic/robotic" systems even in mythology:
 - Talos, created by Hephaistos and offered to the king Minos
 - In Iliad, Hephaistosis referred as creator of technical artificial creatures
 - 384-322 BJC, Aristote is speaking about machines doing human work



- First realizations
 - In Egypt: moving jaw of Anubis mask, moving arm of Amon's statue to designate the new pharaon
 - In Alexandria: fountains with moving birds (hydraulic systems)
 - Early mechanical automatons in the 9th/10th century (mainly in the Arabic world)
 - Clocks and automatons in the 13th/14th century (Europe)
 - Industrial automatons: 18th/19th century (e.g. Vaucanson, Jacquard)



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INTRODUCTION

- Historical perspective
 - First use of the word Robot (means forced labor or serf in Czech) in the play R.U.R. (Rossum's Universal Robots) by Karel Capek (1890-1938) in January 1921.

In R.U.R., Capek poses a paradise, where the machines initially bring so many benefits but in the end bring an equal amount of blight in the form of unemployment and social unrest



RUR

- Science fiction
 - Often a bad image: men against robots, dystopic society, etc. More and more a good image.

Formal definition (Robot Institute of America)

A reprogrammable, multifunctional manipulator designed to move material, parts, tools, or specialized devices through various programmed motions for the performance of a variety of tasks



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• Robots have a bad image (1930-1960)

- Robots take human works
- Robots are dangerous since potentially independent and more intelligent than we are

ROBOTS AND THEIR IMAGE

- Robots have a better image (1960-today)
 - Robots can make things that human can not do (space, etc.)
 - Human can do things that robots can not do (we still are clever)
 - Robots can be games
 - Robots can be good or bad





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ROBOTICS INDUSTRY (1/MANY)

• Where are the robots ?

France:

- 61% in automotive industry
- 14% in chemical industry
- ...
- 4% in electricity industry
- 3% in food industry

• What kind of robots ?



- Industry: ground fixed robots: manipulators, arm robots, ...
- Private individuals: mobile robots: service, games, ...
- Future of robots:
 - Industrial mobile robotics
 - Medical robotics
 - Service robots (growing field)



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Robotics industry... today (2/MANY)

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Vacuum cleaner



Micromanipulator



Surgical robot



Forest robot



Kuka robot for automotive industry



Hollywood robots

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... AND IN THE FUTURE ? (2 BIS/MANY)

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Nanorobotics (FEMTO-ST, Fr)



Military robots (Black Hornet, FLIR Systems, No) 2015 : Autonomous weapons: an open letter from AI & Robotics researchers [link]



Bio-inspired robots Spot from Boston Dynamics





Microrobotics (Harvard, USA)



Exoskeleton

Modular robots

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Robotics industry (3/many)

Robotics

Past

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Total Industrial and Non-Industrial Robotics Revenue, World Markets: 2015-2020



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Robotics industry (3/many)

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Total Industrial and Non-Industrial Robotics Revenue, World Markets: 2015-2020



• Enova Robotics, Sousse, Tunisia

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A D > A B > A B > A B >

Estimated worldwide annual supply of industrial robots





ROBOTICS INDUSTRY: UAVS (4/MANY)

UAV's Manufacturer

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2016 2015 2014 2013 2012 Année 2011 2010 2009 2008 Nb. RPAS 2007 2006 Nb. fabricants Nb. pays producteurs 2005 500 1000 1500 2000 2500 0

• UAVs by countries



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ROBOTICS INDUSTRY: UAVS (5/MANY)

• Development of the drone's industry:

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publications par

Introduction

Χ4

6000 coaxial UAV 2016 coaxial (helicopter OR UAV) 5000 2015 helicopter UAV 2014 quadcopter OR quadrotor 2013 4000 -fixed wing UAV 2012 flapping wing UAV Année 2011 3000 2010 P 2000 1000 1000 2009 2008 2007 Nb. RPAS 2006 Nb. fabricants Nb. pays producteurs 2005 0 2000 2008 2012 2016 1996 2004 500 1000 1500 0 2000 2500 Année Foundation of DJI: 2006 53% D.II 25%

- Very competitive market with a high technological level of intergration
- Commercial margin of 10% to 15% (more than 50% on iPhone) •



ROBOTICS INDUSTRY: UAVS (6/MANY)

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250 200 150 100 50 36% 103% 30% **59**% 81% Agriculture Building Construction Mining Transportation Inspection 126% 102% 97% 16% Delivery & Oil & Gas Power Lines Renewable Energy Media & Logistics Entertainment 58% 54% 12% **11()**% 8%

Environmental

Monitoring

Disaster

Response

Public Land Management

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Police & Fire

Robotics

Traffic Monitoring

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ROBOLUTION (1/MANY)

• Number of robots for every 10 000 workers:

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- 70% of robots in companies with more than 1000 employees
- 17% of robots in companies with less than 300 employees
- In 2002, 95% of robots > 30k€ and 32% of robots > 60k€
- $\bullet~79\%$ of decrease of the mean price between 1990 and 2002
- Big robots manufacturers: ABB (S), KUKA (G), Fanuc (JP), etc.



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Robolution (2/MANY)

- Robotics enables 90% of cost reduction (60% for delocation)
- Each new robot destroys 6.2 jobs [MIT/Boston 1990-2007, 2017]
- 47% of jobs in the US, 50% of jobs in Europe have a high risk of being replaced by robots in the next 20 years [Oxford, 2013] ... but only 9% according [OCDE, 2016]
- Poor countries are more vulnerable, especially world factories (85% of the jobs in Ethiopia, 77% in Chine [World Bank])
- Sectors with high impact: Administration et Production
- Winner sectors: Finance, Maths/Sciences, Education
- No link between unemployment and robots
- Helps to relocate jobs in countries where the consumers are
- Very few studies on created jobs (compared to destroyed jobs)
- 800 000 direct jobs in robotics in 2020 and more than 2 millions in connected domains (electronic, energy, agriculture, etc.)



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Robolution (3/MANY)

- What about the previous industrial revolution ?
 - Machines have created more jobs than they have replaced in the last 140 years
 - Working is getting less and less exhausting
 - Increase of new jobs (+580% éducation)
 - But we had fears, as in any big change periods:
 - 1675 : Destruction of machines by weavers (England), 1788 : 2000 workers break weaving machines (France), 1811-1812 : Luddism (Angleterre)
 - 1858 : Karl Marx is prophesies the replacement of the humans by machines

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• 1930 : John Maynard Keynes invents the term "technological unemployment"

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Outline

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• Basic mechanics for robotics

- Space representation
 - frames, coordinate transformation, etc.
- Force and torques
- Modelisation
- Control for robots
 - All potential problems:
 - Oscillations, dry friction, saturations, etc.
 - Linear approaches
 - Nonlinear approaches



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Position and speed

• The **position** of some point *P* in the **fixed** frame $\mathcal{F}(o, \vec{e_x}, \vec{e_y}, \vec{e_z})$ is the vector $\vec{p} = (x, y, z)^T$



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Position and speed

• The **position** of some point *P* in the **fixed** frame $\mathcal{F}(o, \vec{e}_x, \vec{e}_y, \vec{e}_z)$ is the vector $\vec{p} = (x, y, z)^T$

• The **speed** of *P* in \mathcal{F} is the vector $\vec{s} = \dot{\vec{p}} = (\dot{x}, \dot{y}, \dot{z})^T$



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Robotics	\bullet A rotation is represented by a 3 \times 3 ma	etrix R such that $R^T=R^{-1}$ and det $R=1$
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A rotation is represented by a 3 × 3 matrix R such that R^T = R⁻¹ and det R = 1
A rotation of angle θ around:

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Robotics • A rotation is represented by a 3×3 matrix R such that $R^T = R^{-1}$ and det R = 1• A rotation of angle θ around:

• axis \vec{e}_x is given by:

$$R_{x} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$$

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ROTATIONS

• A rotation is represented by a 3×3 matrix R such that $R^T = R^{-1}$ and det R = 1

N. Marchand	 A rotation of angle θ around: axis e_x is given by: 	(1 0		
Introduction		$R_{\rm x} = \begin{pmatrix} 1 & 0 \\ 0 & \cos\theta \end{pmatrix}$	$-\sin\theta$	
Modeling Cartesian coordinates Orientation Frames Newton X4	• axis $\vec{e_y}$ is given by:	$\begin{pmatrix} 0 & \sin \theta \\ \\ R_y = \begin{pmatrix} \cos \theta & 0 \\ 0 & 1 \\ -\sin \theta & 0 \end{pmatrix}$	$ \begin{array}{c} \cos\theta \\ \sin\theta \\ 0 \\ \cos\theta \end{array} $	
Kinematics Arm robots Inner-loop Geometrical model Kinematic model Dynamical model Conclusion				
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Robotics	• A rotation is represented by a 3 \times 3 matrix R such that $R^T = R^{-1}$ and det $R = 1$				
N. Marchand	 A rotation of angle θ aroun axis e_x is given by: 	ıd:			
Introduction		$R_{\rm x} = \begin{pmatrix} 1 & 0 \\ 0 & \cos \theta \end{pmatrix}$	$\begin{pmatrix} 0\\ -\sin\theta \end{pmatrix}$		
Modeling		$\begin{pmatrix} 0 & \sin \theta \end{pmatrix}$	$\cos\theta$		
Cartesian coordinates Orientation Frames Newton	• axis $\vec{e_y}$ is given by:	$R_{y} = \begin{pmatrix} \cos\theta & 0\\ 0 & 1\\ \sin\theta & 0 \end{pmatrix}$	$\begin{pmatrix} & \sin \theta \\ & 0 \\ & \cos \theta \end{pmatrix}$		
X4					
Kinematics Am robots Inner-loop Geometrical model Kinematic model Dynamical model Conclusion	 axis e _z is given by: 	$R_z = \begin{pmatrix} \cos\theta & -s\\ \sin\theta & \cos\theta\\ 0 & 0 \end{pmatrix}$	$ \begin{array}{ccc} \sin\theta & 0\\ \cos\theta & 0\\ 0 & 1 \end{array} \right) $		
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• A rotation is represented by a 3 \times 3 matrix R such that $R^T = R^{-1}$ and det R = 1Robotics • A rotation of angle θ around: N. Marchand • axis \vec{e}_x is given by: $R_{\rm x} = \begin{pmatrix} 1 & 0 & 0\\ 0 & \cos\theta & -\sin\theta\\ 0 & \sin\theta & \cos\theta \end{pmatrix}$ • axis \vec{e}_v is given by: Orientation $R_{y} = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$ • axis \vec{e}_{τ} is given by: $R_z = \begin{pmatrix} \cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1 \end{pmatrix}$ • a unit vector $\vec{u} = (u_x, u_y, u_z)^T$: $\begin{pmatrix} u_x^2 + (1 - u_x^2)c_\theta & u_x u_y(1 - c_\theta) - u_z s_\theta & u_x u_z(1 - c_\theta) + u_y s_\theta \\ u_x u_y(1 - c_\theta) + u_z s_\theta & u_y^2 + (1 - u_y^2)c_\theta & u_y u_z(1 - c_\theta) - u_x s_\theta \\ u_y u_y(1 - c_\theta) - u_y s_\theta & u_y u_y(1 - c_\theta) + u_y s_\theta & u_y^2 + (1 - u_y^2)c_\theta \end{pmatrix}$ with $c_{i} = \cos(\cdot)$ and $s_{i} = \sin(\cdot)$ (and later on $t_{i} = \tan(\cdot)$)

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• The scalar product $\langle v_1, v_2 \rangle$ is defined by: $\langle v_1, v_2 \rangle := v_1^T v_2 \in \mathbb{R}$ Robotics N. Marchand Orientation Newton obstacles servoing

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- Skew-symmetric matrices and cross product:

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• Skew-symmetric matrices and rotations

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• Skew-symmetric matrices and rotations

$$u^{\times} \sin \theta + (I - uu^{T}) \cos \theta + uu^{T} = \exp((u\theta)^{\times})$$

is the rotation of angle θ leaving axis u fixed

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- Attitude:
 - equivalent of position for angles: what is the orientation of an object w.r.t. the ground ?

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ATTITUDE REPRESENTATION: ANGLES

- Attitude:
 - equivalent of position for angles: what is the orientation of an object w.r.t. the ground ?
 - gives the rotation that transforms the reference frame into the body frame

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ATTITUDE REPRESENTATION: ANGLES

- Attitude:
 - equivalent of position for angles: what is the orientation of an object w.r.t. the ground ?
 - gives the rotation that transforms the reference frame into the body frame
- Many attitude representations

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ATTITUDE REPRESENTATION: ANGLES

- Attitude:
 - equivalent of position for angles: what is the orientation of an object w.r.t. the ground ?
 - gives the rotation that transforms the reference frame into the body frame
- Many attitude representations
 - Euler angles



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- Attitude:
 - equivalent of position for angles: what is the orientation of an object w.r.t. the ground ?
 - gives the rotation that transforms the reference frame into the body frame
- Many attitude representations
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- Euler angles: 3 angles, 27 possible rotations



L. Euler (1707-1783)

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W.R. Hamilton (1805-1865)

Quaternions



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W.R. Hamilton (1805-1865)

• Quaternions



- u fixed by rotation of angle θ
- the quaternion is:

$$q = \begin{pmatrix} u_x \sin \theta/2 \\ u_y \sin \theta/2 \\ u_z \sin \theta/2 \\ \cos \theta/2 \end{pmatrix} = \begin{pmatrix} \vec{q} \\ q_0 \end{pmatrix}$$

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ATTITUDE REPRESENTATION : ANGULAR VELOCITIES

• The angular velocity $\omega = (\omega_1, \omega_2, \omega_3)^T$ represents the rotation speed w.r.t. each axis of the body frame



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• **Caution:** Angular velocities <u>are not</u> the time derivatives of Euler angles

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Angular velocities are given by:

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 - Rotation matrix:

$$\dot{R} = R\omega^{\times}$$

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 Angular velocities are given by:
 - Rotation matrix:

$$\dot{R} = R\omega^{\times}$$

• Quaternions :

$$\dot{\vec{q}} = \frac{1}{2} \Omega(\vec{\omega}) q \qquad \text{with} \begin{cases} \Omega(\vec{\omega}) = \begin{pmatrix} 0 & -\vec{\omega}^T \\ \vec{\omega} & -\vec{\omega}^\times \end{pmatrix} \\ \exists (q) = \begin{pmatrix} -\vec{q}^T \\ b_{3\times 3} q_0 + \vec{q}^\times \end{pmatrix} \end{cases}$$

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MOVING FRAMES





P. Varignon (1654-1722)

Varignon's formula

 $=rac{dec{U}}{dt}^{\mathcal{F}}+\Omega^{\mathcal{F}/\mathcal{M}} imesec{U}^{\mathcal{F}}$ $d\vec{U}^{\mathcal{M}}$ dt

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MOVING FRAMES

• $\mathcal{F} := (O, \vec{e_x}, \vec{e_y}, \vec{e_z})$ fixed inertial frame

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MOVING FRAMES

• $\mathcal{F} := (O, \vec{e_x}, \vec{e_y}, \vec{e_z})$ fixed inertial frame • $\mathcal{M} := (M, \vec{t_1}, \vec{t_2}, \vec{t_3})$: mobile frame



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- R: rotation matrix s.t. $\mathcal{M} = R\mathcal{F}$



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- $\Omega^{\mathcal{M}/\mathcal{F}}$: angular velocity matrix of \mathcal{M} w.r.t.



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$MOVING \ FRAMES$ • $\mathcal{F} := (\textit{O}, \vec{e_x}, \vec{e_y}, \vec{e_z})$ fixed frame

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- $\mathcal{M} := (M, \vec{t_1}, \vec{t_2}, \vec{t_3})$: mobile frame
- R: rotation matrix s.t. $\mathcal{M} = R\mathcal{F}$
- $\Omega^{\mathcal{M}/\mathcal{F}}$: angular velocity matrix of \mathcal{M} w.r.t. \mathcal{F}
- Acceleration

$$\ddot{P}^{\mathcal{F}} := \left(\frac{d\dot{P}^{\mathcal{F}}}{dt}\right)^{\mathcal{F}} = \frac{d\dot{P}^{\mathcal{M}}}{dt}^{\mathcal{F}} + \frac{d\Omega^{\mathcal{M}/\mathcal{F}} \times P^{\mathcal{F}}}{dt}$$



$$\begin{aligned} \frac{d\dot{P}^{\mathcal{M}}}{dt}^{\mathcal{F}} &= \ddot{P}^{\mathcal{M}} + \Omega^{\mathcal{M}/\mathcal{F}} \times \dot{P}^{\mathcal{M}} \text{ (Varignon's formula)} \\ \frac{d\Omega^{\mathcal{M}/\mathcal{F}} \times P^{\mathcal{F}}}{dt} &= \dot{\Omega}^{\mathcal{M}/\mathcal{F}} \times P^{\mathcal{F}} + \Omega^{\mathcal{M}/\mathcal{F}} \times \dot{P}^{\mathcal{F}} \\ &= \dot{\Omega}^{\mathcal{M}/\mathcal{F}} \times P^{\mathcal{F}} + \Omega^{\mathcal{M}/\mathcal{F}} \times \dot{P}^{\mathcal{M}} + \Omega^{\mathcal{M}/\mathcal{F}} \times (\Omega^{\mathcal{M}/\mathcal{F}} \times P^{\mathcal{F}}) \end{aligned}$$

all together:





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a body of mass m := ∑_i m_i composed of elements located in p_i with speed v_i in F
or a body of mass m := ∫_{body} dm composed of elementary part located in p_{dm} with speed v_{dm} in F
p := ∑_i m_ip_i/m defines the position of its center of mass G in F
or p := ∫_{body} dmp_{dm}/m defines the position of its center of mass G in F
v := p defines speed of the center of mass

•
$$\vec{r}_i := (\vec{p}_i - \vec{p}) (\text{resp. } \vec{r}_{dm} := (\vec{p}_{dm} - \vec{p}))$$

Linear Momentum

Consider[.]

• an inertial frame \mathcal{F}

$$egin{array}{rcl} ec{P} & := & \sum_i m_i ec{v}_i = m ec{v} \in \mathbb{R}^3 \ ec{P} & := & \int_{ ext{body}} ec{v}_{dm} dm \in \mathbb{R}^3 \end{array}$$

Angular Momentum $\vec{L} := \sum_{i} m_{i}(\vec{p}_{i} - \vec{p}) \times \vec{v}_{i}$ $\vec{L} := \int_{body} (\vec{p}_{dm} - \vec{p}) \times \vec{v}_{dm} dm$ $= \underbrace{\int_{body} ||\vec{r}_{dm}||^{2} dm}_{J: \text{ moment of inertia}} \vec{\omega}$

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Consider:

- a rigid body
- \bullet an inertial frame ${\cal F}$
- \bullet a moving frame ${\cal M}$ centered in the center of mass and aligned with the main axis of the rigid body

NEWTON'S LAWS

• Let $\vec{F_i}$'s be forces applying on the body with moment arm $\vec{a_i}$



Conservation of the angular momentum $\sum \vec{\tau} = \frac{d\vec{L}}{dt}^{\mathcal{F}}$

• In a moving frame (Varignon's formula):

$$\frac{d\vec{L}^{\mathcal{F}}}{dt} = \frac{d\vec{L}^{\mathcal{M}}}{dt} + \Omega \times \vec{L}$$



I. Newton (1643-1727) J. L. Lagrange (1736-1813)

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• 4 fixed rotors with controlled rotation speed *s_i*



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N. Marchand (gipsa-lab)

- 4 fixed rotors with controlled rotation speed *s_i*
- 4 generated forces F_i



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- 4 counter-rotating torques Γ_i
- Roll movement



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- Roll movement generated with a dissymmetry between left and right forces:

$$\Gamma_r = I(F_4 - F_2)$$





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HOW IT WORKS

- 4 generated forces F_i
- 4 counter-rotating torques Γ_i
- Roll movement generated with a dissymmetry between left and right forces:

 $\Gamma_r = l(F_4 - F_2)$

Pitch movement





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- 4 counter-rotating torques Γ_i
- Roll movement generated with a dissymmetry between left and right forces:

 $\Gamma_r = I(F_4 - F_2)$

 Pitch movement generated with a dissymmetry between front and rear forces:

$$\Gamma_p = I(F_1 - F_3)$$





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- 4 counter-rotating torques Γ_i
- Roll movement generated with a dissymmetry between left and right forces:

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 Pitch movement generated with a dissymmetry between front and rear forces:

$$\Gamma_p = I(F_1 - F_3)$$

Yaw movement



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HOW IT WORKS

- 4 generated forces F_i
- 4 counter-rotating torques Γ_i
- Roll movement generated with a dissymmetry between left and right forces:

 $\Gamma_r = I(F_4 - F_2)$

 Pitch movement generated with a dissymmetry between front and rear forces:

 $\Gamma_p = l(F_1 - F_3)$

• Yaw movement generated with a dissymmetry between front/rear and left/right torques:

$$\Gamma_y = \Gamma_1 + \Gamma_3 - \Gamma_2 - \Gamma_4$$



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• Electrical motor: A 2nd order system with friction and saturation

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• Electrical motor: A 2nd order system with friction and saturation usually *approximated* by a 1^{rst} order system:

$$\dot{s}_i = -\frac{k_m^2}{J_r R} s_i - \frac{1}{J_r} \eta_{\text{oad}} + \frac{k_m}{J_r R} \operatorname{sat}_{\bar{U}_i}(U_i) \quad i \in \{1, 2, 3, 4\}$$

- s_i: rotation speed
- U_i : voltage applied to the motor; real control variable

 τ_{load} : motor load: $\tau_{\text{load}} = k_{gearbox} c_D |s_i| s_i$ with c_D drag coefficient



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- τ_{load} : motor load: $\tau_{\text{load}} = k_{gearbox} c_D |s_i| s_i$ with c_D drag coefficient

• Aerodynamical forces and torques: Very complex models exist





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• Electrical motor: A 2nd order system with friction and saturation

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- s_i: rotation speed
- U_i : voltage applied to the motor; real control variable
- τ_{load} : motor load: $\tau_{\text{load}} = k_{gearbox} c_D |s_i| s_i$ with c_D drag coefficient
- Aerodynamical forces and torques: Very complex models exist but overcomplicated for control, better use the *simplified* model:

$$\begin{array}{rcl} F_i &=& c_T s_i^2 \\ \Gamma_r &=& l c_T (s_4^2 - s_2^2) \\ \Gamma_p &=& l c_T (s_1^2 - s_3^2) \\ \Gamma_y &=& l c_D (s_1^2 + s_3^2 - s_2^2 - s_4^2) \end{array} \qquad i \in \{1,2,3,4\}$$

 c_T : thrust coefficient, c_D : drag coefficient

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• Two frames

- a fixed frame $\mathcal{E}(\vec{e}_1, \vec{e}_2, \vec{e}_3)$
- a frame attached to the X4 $\mathcal{T}(\vec{t_1}, \vec{t_2}, \vec{t_3})$



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- Frame change
 - a rotation matrix R from $\mathcal T$ to $\mathcal E$



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State variables:



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• State variables:

- Cartesian coordinates (in \mathcal{E})
 - position \vec{p}
 - velocity \vec{v}

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- State variables:
 - Cartesian coordinates (in \mathcal{E})
 - position \vec{p}
 - velocity \vec{v}
 - Attitude coordinates:
 - \bullet angular velocity $\vec{\omega}$ in the moving frame $\mathcal T$
 - either: Euler angles three successive rotations about \vec{t}_3 , \vec{t}_1 and \vec{t}_3 of angles angles ϕ , θ and ψ giving R
 - or: Quaternion representation $(q_0, \vec{q}) = (\cos \beta/2, \vec{u} \sin \beta/2)$ represent a rotation of angle β about \vec{u}

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$$\vec{p} = \vec{v}$$

$$\vec{mv} = -mg\vec{e}_3 + R \underbrace{\sum_i F_i(s_i)\vec{t}_3}_{i} + \vec{F}_{ext}$$

$$\vec{T} : \text{ control thrust}$$

Robotics

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Robotics

Cartesian coordinates:

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N. Marchand

Newton

• Attitude:

$$\begin{cases} p = v \\ m\vec{v} = -mg\vec{e}_3 + R \\ \vec{\tau} : \text{ control thrust} \end{cases} + \vec{F}_{ext}$$

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Visual servoing

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$\begin{cases} \dot{\vec{p}} = \vec{v} \\ m \dot{\vec{v}} = -mg \vec{e}_3 + R \underbrace{\sum_{i} F_i(s_i) \vec{t}_3}_{\vec{T} : \text{ control thrust}} + \vec{F}_{ext} \end{cases}$

Attitude:

• Rotation matrix formalism:

$$\left\{ \begin{array}{rrr} \dot{R} & = & R\vec{\omega}^{\times} \\ J\vec{\omega} & = & -\vec{\omega}^{\times}J\vec{\omega} + \vec{\Gamma}_c + \vec{\Gamma}_{ext} \end{array} \right. \qquad \text{with } \vec{\omega}^{\times} = \left(\begin{matrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{matrix} \right)$$

 $\vec{\omega}^{\,\times}$ is the skew symmetric tensor associated to $\vec{\omega}$



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$$\vec{\vec{p}} = \vec{v} \vec{m\vec{v}} = -mg\vec{e}_3 + R \underbrace{\sum_i F_i(s_i)\vec{t}_3}_{i} + \vec{F}_{ext} \vec{\vec{T}} : \text{ control thrust}$$

• Attitude:

• Rotation matrix formalism:

$$\begin{cases} \dot{R} = R\vec{\omega}^{\times} \\ J\dot{\vec{\omega}} = -\vec{\omega}^{\times}J\vec{\omega} + \vec{\Gamma}_{c} + \vec{\Gamma}_{ext} \end{cases} \quad \text{with } \vec{\omega}^{\times} = \begin{pmatrix} 0 & -\omega_{3} & \omega_{2} \\ \omega_{3} & 0 & -\omega_{1} \\ -\omega_{2} & \omega_{1} & 0 \end{pmatrix}$$

 $\vec{\omega}^{\times}$ is the skew symmetric tensor associated to $\vec{\omega}$ • Quaternion formalism:

$$\begin{cases} \dot{\vec{q}} &= \frac{1}{2}\Omega(\vec{\omega})\vec{q} \\ &= \frac{1}{2}\Xi(q)\vec{\omega} \\ J\dot{\vec{\omega}} &= -\vec{\omega}^{\times}J\vec{\omega} + \vec{\Gamma}_{c} + \vec{\Gamma}_{ext} \end{cases} \text{ with } \begin{cases} \Omega(\vec{\omega}) = \begin{pmatrix} 0 & -\vec{\omega}^{T} \\ (\vec{\omega} & -\vec{\omega}^{\times}) \\ \Xi(q) = \begin{pmatrix} 0 & -\vec{\omega}^{T} \\ -\vec{q}^{T} \\ l_{3\times 3}q_{0} + \vec{q}^{\times} \end{pmatrix}$$

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$\begin{cases} \vec{p} = \vec{v} \\ m\vec{v} = -mg\vec{e}_3 + R \underbrace{\sum_{i} F_i(s_i)\vec{t}_3}_{i} + \vec{F}_{ext} \\ \vec{T} : \text{ control thrust} \end{cases}$

Attitude:

• Rotation matrix formalism:

$$\begin{cases} \dot{R} = R\vec{\omega}^{\times} \\ \dot{J\vec{\omega}} = -\vec{\omega}^{\times}J\vec{\omega} + \vec{\Gamma}_{c} + \vec{\Gamma}_{ext} \end{cases} \quad \text{with } \vec{\omega}^{\times} = \begin{pmatrix} 0 & -\omega_{3} & \omega_{2} \\ \omega_{3} & 0 & -\omega_{1} \\ -\omega_{2} & \omega_{1} & 0 \end{pmatrix}$$

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where
$$\vec{\Gamma}_{c} = \begin{pmatrix} \Gamma_{r}(s_{2}, s_{4}) \\ \Gamma_{p}(s_{1}, s_{3}) \\ \Gamma_{y}(s_{1}, s_{2}, s_{3}, s_{4}) \end{pmatrix}$$
 are the control torques

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The wronskian matrix

- Consider the 1-2-3 Euler angles ($\phi, \theta, \psi)$
- The rotation matrix is given by:

$$R = R_z R_y R_x = \begin{pmatrix} c_\theta c_\phi & s_\phi s_\theta c_\psi - c_\phi s_\psi & c_\phi s_\theta c_\psi + s_\phi s_\psi \\ c_\theta s_\phi & s_\phi s_\theta s_\psi + c_\phi c_\psi & c_\phi s_\theta s_\psi - s_\phi c_\psi \\ -s_\theta & s_\phi c_\theta & c_\phi c_\theta \end{pmatrix}$$

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• The relation between the time derivative of the Euler angles and the angular velocity is:

$$\vec{\omega} = \begin{pmatrix} \dot{\phi} \\ 0 \\ 0 \end{pmatrix} + R_z \begin{pmatrix} 0 \\ \dot{\theta} \\ 0 \end{pmatrix} + R_z R_y \begin{pmatrix} 0 \\ 0 \\ \dot{\psi} \end{pmatrix} = W^{-1} \begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix}$$



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N. Marchand (gipsa-lab)

The wronskian matrix

- Consider the 1-2-3 Euler angles ($\phi, \theta, \psi)$
- The rotation matrix is given by:

$$R = R_z R_y R_x = \begin{pmatrix} c_\theta c_\phi & s_\phi s_\theta c_\psi - c_\phi s_\psi & c_\phi s_\theta c_\psi + s_\phi s_\psi \\ c_\theta s_\phi & s_\phi s_\theta s_\psi + c_\phi c_\psi & c_\phi s_\theta s_\psi - s_\phi c_\psi \\ -s_\theta & s_\phi c_\theta & c_\phi c_\theta \end{pmatrix}$$

• The relation between the time derivative of the Euler angles and the angular velocity is: $(\dot{\phi})$ (0) ($\dot{\phi}$) ($\dot{\phi}$)

$$\vec{\omega} = \begin{pmatrix} \phi \\ 0 \\ 0 \end{pmatrix} + R_z \begin{pmatrix} 0 \\ \dot{\theta} \\ 0 \end{pmatrix} + R_z R_y \begin{pmatrix} 0 \\ 0 \\ \dot{\psi} \end{pmatrix} = W^{-1} \begin{pmatrix} \phi \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix}$$

• W is called the wronskian matrix given by (for 1-2-3 Euler angles):

$$\mathcal{N} = egin{pmatrix} 0 & rac{s_\phi}{c_ heta} & rac{c_\phi}{c_ heta} \ 0 & c_\phi & -s_\phi \ 1 & s_\phi t_ heta & c_\phi t_ heta \end{pmatrix}$$

• This matrix is singular for
$$\theta = \pi/2 + k\pi$$

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A FIRST MODEL: REVIEW OF NONLINEARITIES

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$$\begin{aligned} \dot{s}_{i} &= -\frac{k_{m}^{2}}{J_{r}R}s_{i} - \frac{k_{gearbox}c_{D}}{J_{r}}|s_{i}|s_{i} + \frac{k_{m}}{J_{r}R}\operatorname{sat}_{\bar{U}_{i}}(U_{i})\\ \dot{\vec{p}} &= \vec{v}\\ m\dot{\vec{v}} &= -mg\vec{e}_{3} + R\begin{pmatrix}0\\\\0\\\sum_{i}F_{i}(s_{i})\end{pmatrix}\\ \dot{\vec{k}} &= R\vec{\omega}^{\times}\\ J\dot{\vec{\omega}} &= -\vec{\omega}^{\times}J\vec{\omega} + \begin{pmatrix}\Gamma_{r}(s_{2},s_{4})\\\Gamma_{p}(s_{1},s_{3})\\\Gamma_{y}(s_{1},s_{2},s_{3},s_{4})\end{pmatrix} \end{aligned}$$

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 $\dot{s}_{i} = -\frac{k_{m}^{2}}{J_{r}R}s_{i} - \frac{k_{gearbox}c_{D}}{J_{r}}|s_{i}|s_{i} + \frac{k_{m}}{J_{r}R}\operatorname{sat}_{\bar{U}_{i}}(U_{i})$ $\dot{\vec{p}} = \vec{v}$ $-mg\vec{e}_3 + R\left(\begin{array}{c} 0\\ \sum F_i(s_i) \end{array} \right)$ mΫ $\dot{R} = R\vec{\omega}^{\times}$ $J\dot{\vec{\omega}} = -\vec{\omega}^{\times}J\vec{\omega} + \begin{pmatrix} \Gamma_r(s_2, s_4) \\ \Gamma_p(s_1, s_3) \\ \Gamma_y(s_1, s_2, s_3, s_4) \end{pmatrix}$

In red: nonlinearities In blue: where the control variables act

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PARAMETER IDENTIFICATION

• Electrical motor:

- For small input steps, the system behaves very close to a **linear** first order system
- Hence, use linear identification tools
- \bar{U}_i is found on the data-sheet of the motor (damage avoidance)



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- \bar{U}_i is found on the data-sheet of the motor (damage avoidance)

• Aerodynamical parameters: b and c_D

b and *c*_D measured with specific test beds, depends upon temperature, distance from ground, etc.





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• Mechanical parameters:

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• Aerodynamical parameters: b and cD

b and c_D measured with specific test beds, depends upon temperature, distance from ground, etc.

• Mechanical parameters:

I length of an arm of the helicopter, easy to measure



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PARAMETER IDENTIFICATION

• Electrical motor:

- For small input steps, the system behaves very close to a **linear** first order system
- Hence, use linear identification tools
- \bar{U}_i is found on the data-sheet of the motor (damage avoidance)

• Aerodynamical parameters: b and cD

b and c_D measured with specific test beds, depends upon temperature, distance from ground, etc.

• Mechanical parameters:

I length of an arm of the helicopter, easy to measure m total mass of the helicopter, easy to measure



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PARAMETER IDENTIFICATION

• Electrical motor:

- For small input steps, the system behaves very close to a **linear** first order system
- Hence, use linear identification tools
- \bar{U}_i is found on the data-sheet of the motor (damage avoidance)

• Aerodynamical parameters: b and cD

b and c_D measured with specific test beds, depends upon temperature, distance from ground, etc.

• Mechanical parameters:

- I length of an arm of the helicopter, easy to measure
- m total mass of the helicopter, easy to measure
- J body inertia, hard to have precisely



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PARAMETER IDENTIFICATION

• Electrical motor:

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- Hence, use linear identification tools
- \bar{U}_i is found on the data-sheet of the motor (damage avoidance)

• Aerodynamical parameters: b and cD

b and c_D measured with specific test beds, depends upon temperature, distance from ground, etc.

• Mechanical parameters:

- I length of an arm of the helicopter, easy to measure
- m total mass of the helicopter, easy to measure
- J body inertia, hard to have precisely
- I_r rotor inertia, hard to have precisely


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The flapping effect

- The thrust was assumed to be $\sum_{i} F_i(s_i) \vec{t}_3$, that is colinear to \vec{t}_3
- It has been proved to be false because it neglects the effect of the apparent wind speed, this is the **flapping effect**



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The flapping effect

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The flapping effect

- The thrust was assumed to be $\sum_{i} F_i(s_i) \vec{t}_3$, that is colinear to \vec{t}_3
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apparent wind



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The flapping effect

- The thrust was assumed to be $\sum_i F_i(s_i) \vec{t}_3$, that is colinear to \vec{t}_3
- It has been proved to be false because it neglects the effect of the apparent wind speed, this is the **flapping effect**
- Higher thrust on one side of the blades
- The thrust becomes $\sum_{i} R_{i}^{\text{flapping}} F_{i}(s_{i}) \vec{t}_{3}$, torques are also modified



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$\begin{array}{c} MODELING \mbox{ MORE INTO DETAILS: THE} \\ FLAPPING \mbox{ EFFECT} \\ \bullet \mbox{ The flapping matrix takes can be decomposed :} \end{array}$

$R^{\scriptscriptstyle {\mathrm{flapping}}}$	=	$R_{\!\scriptscriptstyle X}^{\scriptscriptstyle \mathrm{flapp}}$	$P^{ing} \cdot R^{fla}_y$	pping				
		(1)	0	0)		$\int c(\alpha)$	0	$s(\alpha)$
	=	0	$c(\beta)$	$-s(\beta)$	•	0	1	0
		(0	$s(\beta)$	c(β) /		$\langle -s(\alpha) \rangle$	0	$c(\alpha)/$



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MODELING MORE INTO DETAILS: THE FLAPPING EFFECT • The flapping matrix takes can be decomposed :

 $R^{\text{flapping}} = R_{\chi}^{\text{flapping}} \cdot R_{y}^{\text{flapping}}$ $= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c(\beta) & -s(\beta) \\ 0 & s(\beta) & c(\beta) \end{pmatrix} \cdot \begin{pmatrix} c(\alpha) & 0 & s(\alpha) \\ 0 & 1 & 0 \\ -s(\alpha) & 0 & c(\alpha) \end{pmatrix}$

 $\bullet \ \alpha \ {\rm and} \ \beta \ {\rm can} \ {\rm be} \ {\rm composed} \ {\rm as} \ {\rm follows}$:

$$\begin{aligned} \alpha &= \alpha_{\mathbf{v}} + \alpha_{\omega} \\ \beta &= \beta_{\mathbf{v}} + \beta_{\omega} \end{aligned}$$

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• α and β can be composed as follows :

$$\alpha = \alpha_{\mathbf{v}} + \alpha_{\omega}$$
$$\beta = \beta_{\mathbf{v}} + \beta_{\omega}$$

• α_v and β_v represent the contribution of the linear speed of the body to the flapping effect

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$R^{\scriptscriptstyle { ext{flapping}}}$	=	$R_{\!\scriptscriptstyle X}^{\scriptscriptstyle \mathrm{flapp}}$	$P^{ing} \cdot R_y^{fla}$	pping				
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	=	0	$c(\beta)$	$-s(\beta)$	•	0	1	0
		\ 0	$s(\beta)$	c(β) /		$-s(\alpha)$	0	$c(\alpha)/$

• α and β can be composed as follows :

$$\alpha = \alpha_{\mathbf{v}} + \alpha_{\omega}$$
$$\beta = \beta_{\mathbf{v}} + \beta_{\omega}$$

• α_v and β_v represent the contribution of the linear speed of the body to the flapping effect

• a_{ω} and b_{ω} represent the contribution of the rotational speed of the body to the flapping effect

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The ground effect

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• The thrust was assumed to be
$$\sum_{i} F_i(s_i) \vec{t}_3$$
, with $F_i(s_i) = c_T s_i^2$

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The ground effect

- The thrust was assumed to be $\sum_{i} F_i(s_i) \vec{t}_3$, with $F_i(s_i) = c_T s_i^2$
- Unfortunately, c_T is not constant but depends upon
 - the density of the air, therefore of the temperature
 - the ground distance : it is the ground effect, $\alpha_g(z) \geq 1$



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ROTORS EFFECTS

• Each rotor may be thought of as a rigid disc rotating around the vertical axis the body frame, with angular velocity *s_i*. The rotor's axis of rotation is itself moving with the angular velocity of the frame. This leads to the following gyroscopic torque :

$$ec{\mathsf{r}}_{\mathsf{gyro}} = \mathit{I_r}ec{\omega} imes ec{t_3} \sum_i (-1) i \ket{s_i}$$

• I_r is the inertia matrix of a rotor



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Rotors effects

• Each rotor may be thought of as a rigid disc rotating around the vertical axis the body frame, with angular velocity *s_i*. The rotor's axis of rotation is itself moving with the angular velocity of the frame. This leads to the following gyroscopic torque :

$$ec{\mathsf{r}}_{\mathsf{gyro}} = \mathit{I_r}ec{\omega} imes ec{t_3} \sum_i (-1) i \ket{s_i}$$

- *I_r* is the inertia matrix of a rotor
- Each rotor produces a counter rotating torque that can be expressed as:

$$s_{res}$$
 := $\sum_{i} (-1)^{i} |s_{i}|$
 $\vec{\Gamma}_{I} = I_{r} \dot{s}_{res} \vec{t}_{3}$

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 \vec{F} \vec{u}_3 \vec{t}_3 \vec{v}_2 \vec{u}_1 \vec{u}_2 \vec{u}_1 \vec{u}_2 \vec{u}_1 \vec{u}_2 \vec{u}_3 \vec{u}_3 \vec{u}_4 \vec{u}_3 \vec{u}_4 $\vec{u$

- Superposition of thrust center and mass center
- External forces
- Air friction: $-K_v ||\vec{v}||\vec{v}$



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THE MIXING MATRIX

• The **mixing matrix** M_x links the torques and thrust force to the rotational speed of the rotors

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THE MIXING MATRIX

- The mixing matrix M_x links the torques and thrust force to the rotational speed of the rotors
- Depends on the considered configuration (not the same for + or x configuration)











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THE MIXING MATRIX

- The mixing matrix M_x links the torques and thrust force to the rotational speed of the rotors
- Depends on the considered configuration (not the same for + or x configuration)
- For the + configuration presented before, we have:

$$\begin{pmatrix} T \\ \Gamma_r \\ \Gamma_p \\ \Gamma_y \end{pmatrix} = \underbrace{\begin{pmatrix} c_T & c_T & c_T & c_T \\ 0 & -lc_T & 0 & lc_T \\ lc_T & 0 & -lc_T & 0 \\ lc_D & -lc_D & lc_D & -lc_D \end{pmatrix}}_{M_x} \begin{pmatrix} s_1^2 \\ s_2^2 \\ s_3^2 \\ s_4^2 \end{pmatrix}$$

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• Flapping and other effect renders the relation between the rotor's speeds and control thrust and torques complex



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- Flapping and other effect renders the relation between the rotor's speeds and control thrust and torques complex
- With flapping appears coupling phenomenon: the thrust affects the yaw movement and the drag affects thrust/roll/pitch movements



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Actuation: depends upon the type of electrical drive you useBody:





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Actuation: depends upon the type of electrical drive you use
Body:

$$p = v$$

$$m\vec{v} = -mg\vec{e}_3 - K_v ||\vec{v}|| \vec{v} + R\vec{T} + \vec{F}_{ext}$$

$$\dot{R} = R\vec{\omega}^{\times}$$

$$J\vec{\omega} = -\vec{\omega}^{\times}J\vec{\omega} + I_r\dot{s}_{res}\vec{t}_3 + I_r\vec{\omega} \times \vec{t}_3 \sum_i (-1)i|s_i| + \vec{\Gamma}_c + \vec{\Gamma}_{ext}$$

• Thrust:

÷

 \rightarrow

$$\vec{\mathcal{T}} = \sum_{i} R_{i}^{\text{flapping}} \alpha_{g} c_{T} s_{i}^{2} \vec{t}_{3}$$

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Actuation: depends upon the type of electrical drive you use Body:

$$\vec{p} = \vec{v}$$

$$\vec{m}\vec{v} = -mg\vec{e}_3 - K_v ||\vec{v}|| \vec{v} + R\vec{T} + \vec{F}_{ext}$$

$$\vec{R} = R\vec{\omega}^{\times}$$

$$\vec{J}\vec{\omega} = -\vec{\omega}^{\times}J\vec{\omega} + I_r\dot{s}_{res}\vec{t}_3 + I_r\vec{\omega} \times \vec{t}_3 \sum_i (-1)i|s_i| + \vec{\Gamma}_c + \vec{\Gamma}_{ext}$$

• Thrust:

$$ec{\mathcal{T}} = \sum_{i} R_{i}^{\text{flapping}} lpha_{g} c_{T} s_{i}^{2} ec{t}_{3}$$

• Torques:

$$\vec{f}_c = \sum_i R_i^{\text{flapping}} \alpha_g c_T s_i^2 \vec{t}_3 \times p_{rotor_i}^{\mathcal{T}} + \sum_i (-1)^{i+1} c_D s_i^2 \vec{t}_3$$

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- Jointed-arm robot: A robot whose arm is constructed of rigid members connected by rotary joints
 - Two possible rotary joints:

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Jointed-arm robot: A robot whose arm is constructed of rigid members connected by rotary joints
Two possible rotary joints:

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rotary around the arm

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JOINTED-ARM ROBOTS

- Jointed-arm robot: A robot whose arm is constructed of rigid members connected by rotary joints
- Two possible rotary joints:
 - rotary around the arm

rotary perpendicular to the arm



- Cartesian coordinates
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• rotary perpendicular to the arm



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• Each possible movement is called a degree of freedom (dof)

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JOINTED-ARM ROBOTS

- Jointed-arm robot: A robot whose arm is constructed of rigid members connected by rotary joints
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• Sometimes movements are coupled (more than 1 dof/articulation)



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- Each possible movement is called a degree of freedom (dof)
- Sometimes movements are coupled (more than 1 dof/articulation)
- A "universal" robot has 12 dof:



- Jointed-arm robot: A robot whose arm is constructed of rigid members connected by rotary joints
- Two possible rotary joints:
 - rotary around the arm



rotary perpendicular to the arm



- Each possible movement is called a degree of freedom (dof)
 - Sometimes movements are coupled (more than 1 dof/articulation)
 - A "universal" robot has 12 dof:
 - 6 for spatial position (vehicle)

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- Jointed-arm robot: A robot whose arm is constructed of rigid members connected by rotary joints • Two possible rotary joints:
- rotary around the arm



rotary perpendicular to the arm



- Each possible movement is called a degree of freedom (dof)
 - Sometimes movements are coupled (more than 1 dof/articulation)
 - A "universal" robot has 12 dof:
 - 6 for spatial position (vehicle)
 - 3 for the arm

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- Jointed-arm robot: A robot whose arm is constructed of rigid members connected by rotary joints
- Two possible rotary joints:
 - rotary around the arm



rotary perpendicular to the arm



- Each possible movement is called a degree of freedom (dof)
 - Sometimes movements are coupled (more than 1 dof/articulation)
 - A "universal" robot has 12 dof:
 - 6 for spatial position (vehicle)
 - 3 for the arm
 - 3 for the terminal tool

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- Jointed-arm robot: A robot whose arm is constructed of rigid members connected by rotary joints
 Two possible rotary joints:
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• Each possible movement is called a degree of freedom (dof)

- Sometimes movements are coupled (more than 1 dof/articulation)
- A "universal" robot has 12 dof:
 - 6 for spatial position (vehicle)
 - 3 for the arm
 - 3 for the terminal tool
- In the industrial context, a polyvalent robot will have 6 dof


JOINTED-ARM ROBOTS

- Jointed-arm robot: A robot whose arm is constructed of rigid members connected by rotary joints
 Two possible rotary joints:
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- rotary around the arm



rotary perpendicular to the arm



- Sometimes movements are coupled (more than 1 dof/articulation)
- A "universal" robot has 12 dof:
 - 6 for spatial position (vehicle)
 - 3 for the arm
 - 3 for the terminal tool
- In the industrial context, a polyvalent robot will have 6 dof
- 6 dof are sufficient for any position and orientation of the terminal tool in the *reachable space*

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 Jointed-arm robot: A robot whose arm is constructed of rigid members connected by rotary joints
 Two possible rotary joints:

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- Sometimes movements are coupled (more than 1 dof/articulation)
- A "universal" robot has 12 dof:

rotary around the arm

- 6 for spatial position (vehicle)
- 3 for the arm
- 3 for the terminal tool
- In the industrial context, a polyvalent robot will have 6 dof
- 6 dof are sufficient for any position and orientation of the terminal tool in the *reachable space*
- Many tasks can be performed with less than 6 dof: "pick and place" needs only 4 dof

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• Characteristic variables:

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• Characteristic variables:

• Actuator control *u_i* of the joint *i*

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• Characteristic variables:

- Actuator control u_i of the joint i
- Actuator torques C_i of the joint i

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• Characteristic variables:

- Actuator control u_i of the joint i
- Actuator torques C_i of the joint i
- Angles θ_i of the joint



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• Characteristic variables:

- Actuator control u_i of the joint i
- Actuator torques C_i of the joint i
- Angles θ_i of the joint
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JOINTED-ARM ROBOTS

- Characteristic variables:
 - Actuator control u_i of the joint i
 - Actuator torques C_i of the joint i
 - Angles θ_i of the joint
 - Spatial position X_i of the extremity of the joint
- Controlling a robot is equivalent to mastering the relation

$$u_i \rightleftharpoons C_i \rightleftharpoons \theta_i \rightleftharpoons X_i$$



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- Characteristic variables:
 - Actuator control u_i of the joint i
 - Actuator torques C_i of the joint i
 - Angles θ_i of the joint
 - Spatial position X_i of the extremity of the joint
- Controlling a robot is equivalent to mastering the relation

• Actuator's dynamics
$$U_i \rightleftharpoons C_i \rightleftharpoons \theta_i \rightleftharpoons X_i$$

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JOINTED-ARM ROBOTS

- Characteristic variables:
 - Actuator control u_i of the joint i
 - Actuator torques C_i of the joint i
 - Angles θ_i of the joint
 - Spatial position X_i of the extremity of the joint
- Controlling a robot is equivalent to mastering the relation

$$u_i \rightleftharpoons C_i \rightleftharpoons \theta_i \rightleftharpoons X_i$$
• Actuator's dynamics
• Robot's dynamics

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• Enables to force θ to follow the reference θ_r



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- Enables to force θ to follow the reference θ_r
- The actuator is usually a first (electric) or second order system (pneumatic)



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Visual servoing $\begin{array}{c} & \text{Disturbances} \\ & \downarrow \\ & \downarrow \\ & \downarrow \\ & e \\ & e$

- Enables to force θ to follow the reference θ_r
- The actuator is usually a first (electric) or second order system (pneumatic)
- Usually controlled with a PID controller with



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Visual servoing $\begin{array}{c} \theta_{i}^{r} \\ \theta_{i}^{r} \\ \theta_{i}^{m} \end{array} \xrightarrow{e} \end{array} \xrightarrow{c} Controller} \begin{array}{c} u_{i} \\ u_{i} \\ Actuator + Robot \\ \theta_{i} \\ \end{array}$

- Enables to force θ to follow the reference θ_r
- The actuator is usually a first (electric) or second order system (pneumatic)
- Usually controlled with a PID controller with
 - filtered derivative action

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- Enables to force θ to follow the reference θ_r
- The actuator is usually a first (electric) or second order system (pneumatic)
- Usually controlled with a PID controller with
 - filtered derivative action
 - anti-windup to tackle saturations

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 $\bullet\,$ We go back to the X4 example and focus on the rotors:

INNER CONTROL LOOP Anti-windup PID

$$\left\{\dot{s}_i = -\frac{k_m^2}{J_r R} s_i - \frac{1}{J_r} \tau_{\mathsf{load}} + \frac{k_m}{J_r R} \operatorname{sat}_{\bar{U}_i}(U_i)\right\}$$

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INNER CONTROL LOOP Anti-windup PID

• We go back to the X4 example and focus on the rotors:

$$\left\{\dot{s}_i = -\frac{k_m^2}{J_r R} s_i - \frac{1}{J_r} \eta_{\text{oad}} + \frac{k_m}{J_r R} \operatorname{sat}_{\bar{U}_i}(U_i)\right\}$$

• If one wants to **act on the X4 with desired forces** F_i^d , it is necessary to be able to **set the rotors speeds** s_i **to** s_i^d with

$$s_i^d = \sqrt{rac{1}{b}F_i^d}$$

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• We go back to the X4 example and focus on the rotors:

$$\left\{\dot{s}_i = -\frac{k_m^2}{J_r R} s_i - \frac{1}{J_r} \tau_{\mathsf{load}} + \frac{k_m}{J_r R} \operatorname{sat}_{\bar{U}_i}(U_i)\right\}$$

• If one wants to **act on the X4 with desired forces** F_i^d , it is necessary to be able to **set the rotors speeds** s_i **to** s_i^d with

$$s_i^d = \sqrt{rac{1}{b}F_i^d}$$

• A usual way to control the electrical motor consist in

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• We go back to the X4 example and focus on the rotors:

$$\left\{\dot{s}_i = -\frac{k_m^2}{J_r R} s_i - \frac{1}{J_r} \tau_{\mathsf{load}} + \frac{k_m}{J_r R} \operatorname{sat}_{\bar{U}_i}(U_i)\right\}$$

• If one wants to **act on the X4 with desired forces** F_i^d , it is necessary to be able to **set the rotors speeds** s_i **to** s_i^d with

$$s_i^d = \sqrt{rac{1}{b}F_i^d}$$

A usual way to control the electrical motor consist in
 taking τ_{load} as un unknown load

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INNER CONTROL LOOP Anti-windup PID

• We go back to the X4 example and focus on the rotors:

$$\left\{\dot{s}_i = -\frac{k_m^2}{J_r R} s_i - \frac{1}{J_r} \tau_{\mathsf{load}} + \frac{k_m}{J_r R} \operatorname{sat}_{\bar{U}_i}(U_i)\right\}$$

• If one wants to **act on the X4 with desired forces** F_i^d , it is necessary to be able to **set the rotors speeds** s_i **to** s_i^d with

$$s_i^d = \sqrt{rac{1}{b}F_i^d}$$

- A usual way to control the electrical motor consist in
 - taking τ_{load} as un unknown load
 - neglecting the voltage limitations \bar{U}_i

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• The so obtained system is linear

$$\frac{s_i(s)}{U_i(s)} = \frac{\frac{1}{k_m}}{1 + \frac{J_r R}{k_m^2}s} = \frac{G}{1 + \tau s}$$

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• The so obtained system is linear

$$\frac{s_i(s)}{U_i(s)} = \frac{\frac{1}{k_m}}{1 + \frac{J_r R}{k_m^2} s} = \frac{G}{1 + \tau s}$$

• Define a **PI controller** for it:

$$C(s) = K_p + \frac{K_i}{s}$$

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INNER CONTROL LOOP Anti-windup PID

• The so obtained system is linear

$$\frac{s_i(s)}{U_i(s)} = \frac{\frac{1}{k_m}}{1 + \frac{J_r R}{k_m^2} s} = \frac{G}{1 + \tau s}$$

• Define a **PI controller** for it:

$$C(s) = K_p + \frac{K_i}{s}$$

• Taking $K_i = \frac{1}{\tau_{CL}G}$ and $K_p = \tau K_i$, the closed loop system is: $\frac{s_i(s)}{U_i(s)} = \frac{1}{1 + \tau_{CL}s}$

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Anti-windup PID

INNER CONTROL LOOP

• Make a step that **compensates the weight**, that is such that $s_i^d = \sqrt{\frac{mg}{4b}}$ so that $\sum_i F_i^d = mg$

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• Make a step that compensates the weight, that is such that $s_i^d = \sqrt{\frac{mg}{4b}}$ so that $\sum_i F_i^d = mg$

INNER CONTROL LOOP Anti-windup PID

• Taking $\tau_{CL} = 50$ ms, one gets without saturations



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• Make a step that compensates the weight, that is such that $s_i^d = \sqrt{\frac{mg}{4b}}$ so that $\sum_i F_i^d = mg$

INNER CONTROL LOOP Anti-windup PID

• Taking $\tau_{CL} = 50$ ms, one gets with saturations



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• The result could be worse:

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• The result could be worse:



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• For $u \in [-1.2, 1.2]$, the closed-loop behavior is:





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• For $u \in [-1.2, 1.2]$, the closed-loop behavior is:





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Visual servoing • Saturations may lead to instability especially in the presence of integrators in the loop



• For $u \in [-1.2, 1.2]$, the closed-loop behavior is:



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• Consider a linear system with a PID controller:



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• Consider a linear system with a PID controller:

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• Consider a linear system with a PID controller:



• The instability comes from the integration of the error

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• Consider a linear system with a PID controller:



- The instability comes from the **integration** of the error
- Key idea: soften the integral effect when the control is saturated

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• Structure of the PID controller with anti-windup:



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Anti-windup PID

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• Structure of the **PID controller with anti-windup**:

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Visual servoing • Structure of the PID controller with anti-windup:



• If $u = \bar{u}$, that is if u is not saturated, then the PID controller with anti-windup is identical to the classical PID controller



Anti-windup PID

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• Structure of the PID controller with anti-windup:



- If $u = \bar{u}$, that is if u is not saturated, then the PID controller with anti-windup is identical to the classical PID controller
- If u is saturated $(u \neq \bar{u})$, K_s tunes the reduction of the integral effect of the PID



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Anti-windup PID

INNER CONTROL LOOP

• Make a step that **compensates the weight**, that is such that $s_i^d = \sqrt{\frac{mg}{4b}}$ so that $\sum_i F_i^d = mg$

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• Make a step that compensates the weight, that is such that $s_i^d = \sqrt{\frac{mg}{4b}}$ so that $\sum F_i^d = mg$

INNER CONTROL LOOP Anti-windup PID

• Taking $\tau_{CL} = 50$ ms, one gets without anti-windup



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• Make a step that compensates the weight, that is such that $s_i^d = \sqrt{\frac{mg}{4b}}$ so that $\sum F_i^d = mg$

INNER CONTROL LOOP Anti-windup PID

• Taking $\tau_{CL} = 50$ ms, one gets with anti-windup



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Towards gain scheduling

• Take again $\tau_{CL} = 50$ ms and a PI controller tuned at s_i^d

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INNER CONTROL LOOP

Towards gain scheduling

• Take again $\tau_{CL} = 50$ ms and a PI controller tuned at s_i^d

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• Make speed steps of different level



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INNER CONTROL LOOP

Towards gain scheduling

- Take again $\tau_{CL} = 50$ ms and a PI controller tuned at s_i^d
- Make speed steps of different level



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Towards gain scheduling • Take again $\tau_{CL} = 50$ ms and a PI controller tuned at the current s_i

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Towards gain scheduling • Take again $\tau_{CI} = 50$ ms and a PI controller tuned at the current s_i

• Make speed steps of different level

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Towards gain scheduling • Take again $\tau_{CI} = 50$ ms and a PI controller tuned at the current s_i

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• The rotors are now well controlled...almost

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• Make speed steps of different level

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• We assume we can measure every thing (thanks to people like Hassen) !

Assumptions and principles

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Assumptions and principles

- We assume we can measure every thing (thanks to people like Hassen) !
- Three embedded loops in a control strategy as follows



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Assumptions and principles

- We assume we can measure every thing (thanks to people like Hassen) !
- Three embedded loops in a control strategy as follows





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Actuator control

- Closing the actuator loop will
 - increase the precision of the forces and torques generated
 - allow non identical actuators
 - render the system more reactive
 - face battery drop problem and the non constant gain of the open loop system



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- A speed measure is required, usually optic or magnetic
- Two usual approaches: sliding mode and PI controller with anti-windup

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Attitude control

- Required by most of the higher level control strategies
- Basic for stable remote piloting
- Embedded in all commercial platforms

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Attitude control

- Required by most of the higher level control strategies
- Basic for stable remote piloting
- Embedded in all commercial platforms
- Consist in controlling only the part of the model corresponding to the angular motion:

$$\dot{R} = R\vec{\omega}^{\times} J\vec{\omega} = -\vec{\omega}^{\times}J\vec{\omega} + \vec{\Gamma}_{c} + \vec{\Gamma}_{ext}$$

$$\begin{cases} \dot{\vec{q}} = \frac{1}{2}\Omega(\vec{\omega})\vec{q} \\ = \frac{1}{2}\Xi(q)\vec{\omega} \\ J\vec{\omega} = -\vec{\omega}^{\times}J\vec{\omega} + \vec{\Gamma}_{c} + \vec{\Gamma}_{ext} \end{cases}$$

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Attitude control

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- Consist in controlling only the part of the model corresponding to the angular motion:

$$\begin{cases} \dot{R} = R\vec{\omega}^{\times} \\ J\dot{\vec{\omega}} = -\vec{\omega}^{\times}J\vec{\omega} + \vec{\Gamma}_{c} + \vec{\Gamma}_{ext} \end{cases} \begin{cases} \dot{\vec{q}} = \frac{1}{2}\Omega(\vec{\omega})\vec{q} \\ = \frac{1}{2}\Xi(q)\vec{\omega} \\ J\dot{\vec{\omega}} = -\vec{\omega}^{\times}J\vec{\omega} + \vec{\Gamma}_{c} + \vec{\Gamma}_{ext} \end{cases}$$

- Linearization of the rotational dynamics gives three second order integrators
- Most of the applied strategies are PID controllers based on the linearization. Some sliding mode approaches. Few nonlinear approaches.

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Attitude control

- Required by most of the higher level control strategies
- Basic for stable remote piloting
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- Consist in controlling only the part of the model corresponding to the angular motion:

$$\begin{vmatrix} \dot{R} &= R\vec{\omega}^{\times} \\ J\vec{\omega} &= -\vec{\omega}^{\times}J\vec{\omega} + \vec{\Gamma}_{c} + \vec{\Gamma}_{ext} \end{vmatrix} \begin{cases} \dot{\vec{q}} &= \frac{1}{2}\Omega(\vec{\omega})\vec{q} \\ &= \frac{1}{2}\Xi(q)\vec{\omega} \\ J\vec{\omega} &= -\vec{\omega}^{\times}J\vec{\omega} + \vec{\Gamma}_{c} + \vec{\Gamma}_{ext} \end{cases}$$

- Linearization of the rotational dynamics gives three second order integrators
- Most of the applied strategies are PID controllers based on the linearization. Some sliding mode approaches. Few nonlinear approaches.
- Valid only around zero angles position, but
 - robust
 - easy to tune
 - can handle saturation
 - can be adaptive

• ...

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R : Rotation matrix from $\mathcal T$ to $\mathcal E$



 \mathcal{T} : Mobile frame

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$\vec{p} = \vec{v}$ $\vec{v} = -g\vec{e}_3 + \frac{1}{m}R\vec{f} - c\vec{v}$ \vec{q}_0 $\vec{q}_v = \frac{1}{2}\begin{pmatrix} -q_v^T\\ I_3q_0 - q_v^X \end{pmatrix} \omega$ $\vec{\omega} = J^{-1}(\vec{\tau} - \omega \times J\omega)$

R : Rotation matrix from \mathcal{T} to \mathcal{E} $\vec{f}, \vec{\tau}$: Aerodynamic force and torque in \mathcal{T} \vec{p}, \vec{v} : Linear position and velocity in \mathcal{E}



- \mathcal{T} : Mobile frame
- m : Body's mass

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- c: Viscosity coefficient
- g : Gravity

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R : Rotation matrix from \mathcal{T} to \mathcal{E} $\vec{f}, \vec{\tau}$: Aerodynamic force and torque in \mathcal{T} \vec{p}, \vec{v} : Linear position and velocity in \mathcal{E} $q = [q_0 q_v]^T$: Quaternion q_v^{\times} : Skew symmetric matrix associated to q_v ω : Angular velocity in \mathcal{T}



- $\mathcal{T}:\mathsf{Mobile}\;\mathsf{frame}$
- m : Body's mass
- c: Viscosity coefficient
- g : Gravity
- I_3 : Identity matrix
- J: Inertia matrix

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Visual servoing $\dot{\vec{p}} = \vec{v}$ $\dot{\vec{v}} = -g\vec{e}_3 + \frac{1}{m}R\vec{f} - c\vec{v}$ $\begin{pmatrix} \dot{q}_0 \\ \dot{q}_v \end{pmatrix} = \frac{1}{2} \begin{pmatrix} -q_v^T \\ I_3q_0 - q_v^X \end{pmatrix} \omega$ $\dot{\omega} = J^{-1}(\vec{\tau} - \omega \times J\omega)$

R : Rotation matrix from \mathcal{T} to \mathcal{E} $\vec{f}, \vec{\tau}$: Aerodynamic force and torque in \mathcal{T} \vec{p}, \vec{v} : Linear position and velocity in \mathcal{E} $q = [q_0 q_v]^T$: Quaternion q_v^{\times} : Skew symmetric matrix associated to q_v ω : Angular velocity in \mathcal{T}



Fixed frame

- $\mathcal{T}:\mathsf{Mobile}\;\mathsf{frame}$
- m : Body's mass
- c: Viscosity coefficient
- g : Gravity
- I₃ : Identity matrix
- J : Inertia matrix

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 $\phi^{wing}(\phi_0^{wing}, t)$ $\psi^{wing}(\psi_0^{wing}, t)$

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 $\phi^{wing}(\phi_0^{wing}, t)$ aerodynamic forces and torques $\psi^{wing}(\psi_0^{wing}, t)$ $(\vec{f}, \vec{\tau})$

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A saturation based attitude control

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 $\phi^{wing}(\phi_0^{wing}, t)$ average averaged aerodynamic over aerodynamic forces and torques a wingbeat forces and torques $\psi^{\text{wing}}(\psi^{\text{wing}}_{2}, t)$ period $(\vec{f}, \vec{\tau})$ $(\vec{T}, \vec{\Gamma}_c)$ θ^{wing} Λ, Λ^{-1} $(\vec{T},\vec{\Gamma}_{c}) = \Lambda(\phi_{0}^{wing},\psi_{0}^{wing})^{d,g}$

Find $\vec{T} = \vec{T}(\vec{p}, \vec{v}, \vec{q}, \vec{\omega})$ and $\vec{\Gamma}_c = \vec{\Gamma}_c(\vec{p}, \vec{v}, \vec{q}, \vec{\omega})$ such that the system has the desired behavior

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Averaged forces and torques $(\vec{T}, \vec{\Gamma}_c)$

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 $\begin{array}{l} \text{Constraints} \\ 0 \leq \phi_0^{wing} \leq \tilde{\phi}_0^{wing} \\ -\tilde{\psi}_0^{wing} \leq \psi_0^{wing} \leq \tilde{\psi}_0^{wing} \end{array}$



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 $\begin{array}{ccc} \text{Wings angles} & \text{Averaged} \\ \text{amplitudes} & & & \text{forces and torques} \\ (\phi_0^{wing}, \psi_0^{wing})^{l,r} & & (\vec{T}, \vec{\Gamma}_c) \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & \\ & & & & \\ & & & \\ & & & & \\ & & & \\ & & & \\ & & & \\ & & & & \\ & & & \\ & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & &$

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• Physical limitations: wings angles saturation

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$$\begin{array}{rcl} 0 & \leq & (\phi_0^{\textit{wing}})^{l,r} & \leq & \widetilde{\phi}_0^{\textit{wing}} \\ \widetilde{\psi}_0^{\textit{wing}} & \leq & (\psi_0^{\textit{wing}})^{l,r} & \leq & \widetilde{\psi}_0^{\textit{wing}} \end{array}$$

$$\beta x \left[\phi_0^{\text{wing}, r^2} \cos \psi_0^d - \phi_0^{\text{wing}, l^2} \cos \psi_0^g \right] = \Gamma_r^{\text{max}}$$
$$\alpha x \left[\phi_0^{\text{wing}, r^2} \sin \psi_0^d - \phi_0^{\text{wing}, l^2} \sin \psi_0^g \right] = \Gamma_y^{\text{max}}$$

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$$\begin{array}{rcl} 0 & \leq & (\phi_0^{\rm wing})^{l,r} & \leq & \widetilde{\phi}_0^{\rm wing} \\ \tilde{\psi}_0^{\rm wing} & \leq & (\psi_0^{\rm wing})^{l,r} & \leq & \widetilde{\psi}_0^{\rm wing} \end{array}$$

$$\beta x \left[\phi_0^{\text{wing}, r^2} \cos \psi_0^d - \phi_0^{\text{wing}, l^2} \cos \psi_0^g \right] = \Gamma_r^{\text{max}}$$
$$\alpha x \left[\phi_0^{\text{wing}, r^2} \sin \psi_0^d - \phi_0^{\text{wing}, l^2} \sin \psi_0^g \right] = \Gamma_y^{\text{max}}$$

• Coupled saturations of the control torques



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• Physical limitations: wings angles saturation

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$\begin{array}{rcl} 0 & \leq & (\phi_0^{\rm wing})^{l,r} & \leq & \tilde{\phi}_0^{\rm wing} \\ -\tilde{\psi}_0^{\rm wing} & \leq & (\psi_0^{\rm wing})^{l,r} & \leq & \tilde{\psi}_0^{\rm wing} \end{array}$

$$\beta x \left[\phi_0^{\text{wing}, r^2} \cos \psi_0^d - \phi_0^{\text{wing}, l^2} \cos \psi_0^g \right] = \Gamma_r^{\text{max}} \\ \alpha x \left[\phi_0^{\text{wing}, r^2} \sin \psi_0^d - \phi_0^{\text{wing}, l^2} \sin \psi_0^g \right] = \Gamma_y^{\text{max}}$$



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$\begin{array}{rcl} 0 & \leq & (\phi_0^{\rm wing})^{l,r} & \leq & \tilde{\phi}_0^{\rm wing} \\ -\tilde{\psi}_0^{\rm wing} & \leq & (\psi_0^{\rm wing})^{l,r} & \leq & \tilde{\psi}_0^{\rm wing} \end{array}$

$$\beta x \left[\phi_0^{\text{wing}, r^2} \cos \psi_0^d - \phi_0^{\text{wing}, l^2} \cos \psi_0^g \right] = \Gamma_r^{\text{max}} \\ \alpha x \left[\phi_0^{\text{wing}, r^2} \sin \psi_0^d - \phi_0^{\text{wing}, l^2} \sin \psi_0^g \right] = \Gamma_y^{\text{max}}$$



Admissible saturation set Reduced to an ellipse in order to simplify the computations

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$\begin{array}{rcl} 0 & \leq & (\phi_0^{\rm wing})^{l,r} & \leq & \tilde{\phi}_0^{\rm wing} \\ -\tilde{\psi}_0^{\rm wing} & \leq & (\psi_0^{\rm wing})^{l,r} & \leq & \tilde{\psi}_0^{\rm wing} \end{array}$

$$\beta x \left[\phi_0^{\text{wing}, r^2} \cos \psi_0^d - \phi_0^{\text{wing}, l^2} \cos \psi_0^g \right] = \Gamma_r^{\max}$$
$$\alpha x \left[\phi_0^{\text{wing}, r^2} \sin \psi_0^d - \phi_0^{\text{wing}, l^2} \sin \psi_0^g \right] = \Gamma_y^{\max}$$



Admissible saturation set Reduced to an ellipse in order to simplify the computations Nul yaw torque for a maximal roll torque

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• Physical limitations: wings angles saturation

$$\begin{array}{rclcrcl} 0 & \leq & (\phi_0^{\rm wing})^{l,r} & \leq & \tilde{\phi}_0^{\rm wing} \\ -\tilde{\psi}_0^{\rm wing} & \leq & (\psi_0^{\rm wing})^{l,r} & \leq & \tilde{\psi}_0^{\rm wing} \end{array}$$

$$\beta x \left[\phi_0^{\text{wing}, r^2} \cos \psi_0^d - \phi_0^{\text{wing}, l^2} \cos \psi_0^g \right] = \Gamma_r^{\max}$$
$$\alpha x \left[\phi_0^{\text{wing}, r^2} \sin \psi_0^d - \phi_0^{\text{wing}, l^2} \sin \psi_0^g \right] = \Gamma_y^{\max}$$



 $\Gamma_r^{max}(N.m)$

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Admissible saturation set Reduced to an ellipse in order to simplify the computations Nul yaw torque for a maximal roll torque Roll stabilization is preferred in order to bring the body to a horizontal position

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Robotics

x 10⁻⁵



Robotics

• Attitude with reference vectors:

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Current vector: $s_k^T = R s_k^{\mathcal{E}}$ Desired vector: $s_{k_d}^T = R_d s_k^{\mathcal{E}}$

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Attitude with reference vectors:

Current vector: $s_k^T = R s_k^{\mathcal{E}}$ Desired vector: $s_{k_d}^T = R_d s_k^{\mathcal{E}}$

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- Yaw control is impossible
- Add another sensor giving a non collinear measurement (magnetometer for example)

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Visual servoing $R_d \overline{s}_k^c \overline{s}_k^T$

Attitude with reference vectors:

Current vector: $s_k^T = R s_k^{\mathcal{E}}$ Desired vector: $s_{k_d}^T = R_d s_k^{\mathcal{E}}$

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- Yaw control is impossible
- Add another sensor giving a non collinear measurement (magnetometer for example)
- Number of non collinear sensors $n \ge 2$



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• Attitude with reference vectors:

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Attitude error.

Current vector: $s_k^T = R s_k^{\mathcal{E}}$ Desired vector: $s_{k_d}^T = R_d s_k^{\mathcal{E}}$

 $\vec{\zeta} = \frac{\Delta^{-1}}{n} \sum_{k=1}^{n} \vec{s}_{k}^{\mathcal{T}} \times R_{d} \vec{s}_{k}^{\mathcal{E}}$

- Δ : positive diagonal matrix
- n : sensors number
- R_d : desired orientation of \mathcal{T} relatively to \mathcal{E})

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• Attitude with reference vectors:

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Attitude error:

$$R_d \tilde{s}_k^{\mathcal{E}} \tilde{s}_k^{\mathcal{T}}$$

Current vector: $s_k^T = R s_k^{\mathcal{E}}$ Desired vector: $s_{k_d}^T = R_d s_k^{\mathcal{E}}$

$$\vec{\zeta} = \frac{\Delta^{-1}}{n} \sum_{k=1}^{n} \vec{s}_{k}^{\mathcal{T}} \times R_{d} \vec{s}_{k}^{\mathcal{E}}$$

- Δ : positive diagonal matrix
- n : sensors number
- R_d : desired orientation of \mathcal{T} relatively to \mathcal{E})

• If $\vec{s}_k^{\mathcal{T}}$ and $R_d \vec{s}_k^{\mathcal{E}}$ are collinear, then $\vec{\zeta} = 0$

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A saturation based attitude control

• Sensors used: Rate gyros and reference sensors

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A saturation based attitude control

• Sensors used: Rate gyros and reference sensors

Control torques

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$$ar{ au}_j = -\operatorname{sat}_{ ilde{ au}_j}(\lambda_j[ar{\omega}_{G_j} +
ho_jar{\zeta}_j)]) \qquad j = \{1, 2, 3\}$$

- λ_j, ρ_j : positive tuning parameters sat $\tilde{\tau}_j$: saturation function
 - $\bar{\omega}_{G_i}$: averaged angular velocity (measured by the rate gyros)
 - : averaged attitude error



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• Sensors used: Rate gyros and reference sensors

Control torques

$$ar{ au}_j = -\operatorname{sat}_{ ilde{ au}_j}(\lambda_j[ar{\omega}_{G_j} +
ho_jar{\zeta}_j)]) \qquad j = \{1, 2, 3\}$$

- $\begin{array}{lll} \lambda_j, \rho_j &: \text{ positive tuning parameters} \\ \mathrm{sat}_{\tilde{\tau}_j} &: \text{ saturation function} \\ \bar{\omega}_{G_i} &: \text{ averaged angular velocity (measured by the rate gyros)} \end{array}$
 - f_{j} : averaged attitude error
- Stability proved (rigid body) using Lyapunov function:

$$V = \frac{1}{2}\vec{\omega}^{T}J\vec{\omega} + \frac{1}{n}\sum_{k=1}^{n}(1 - \vec{s}_{k}^{mT}R_{d}\vec{s}_{k}^{f})$$

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A saturation based attitude control

• Sensors used: Rate gyros and reference sensors

Control torques

$$ar{ au}_j = -\operatorname{sat}_{ ilde{ au}_j}(\lambda_j[ar{\omega}_{G_j} +
ho_jar{\zeta}_j)]) \qquad j = \{1, 2, 3\}$$

- λ_j, ρ_j : positive tuning parameters
- $\operatorname{sat}_{\tilde{\tau}_i}$: saturation function
- $\bar{\omega}_{G_i}$: averaged angular velocity (measured by the rate gyros)
 - *j* : averaged attitude error
- Stability proved (rigid body) using Lyapunov function:

$$V = \frac{1}{2}\vec{\omega}^T J\vec{\omega} + \frac{1}{n} \sum_{k=1}^n (1 - \vec{s}_k^{mT} R_d \vec{s}_k^f)$$

• Generalized PID controller, *almost* global stability, simpler version using quaternion exists, stability independent from the knowledge of *J*, robust to velocity sensor saturation.

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A saturation based attitude control

Initial orientation: $(\phi, \theta, \psi) = (70, -50, 30)^{\circ}$

Angles (Roll, pitch, yaw)



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A saturation based attitude control Angular velocities

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Linearization

- First thing people want to try ?
- Many possible approaches
- Taking

$$\boldsymbol{x} := (\boldsymbol{\phi}, \boldsymbol{\theta}, \boldsymbol{\psi}, \dot{\boldsymbol{\phi}}, \dot{\boldsymbol{\theta}}, \dot{\boldsymbol{\psi}}, \boldsymbol{p}^{\mathsf{T}}, \boldsymbol{v}^{\mathsf{T}})^{\mathsf{T}}$$

the linearization of the linear and angular dynamics around some reference x^r of the form $(0, 0, \psi^r, 0, 0, 0, p^{r^T}, 0)^T$ is given by :

$$\dot{\tilde{x}} = A\tilde{x} + B\tilde{u}$$

with the following matrices A and B:

$$A := \begin{pmatrix} 0_{3\times3} & l_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & l_{3\times3} \\ 0 & g & 0 \\ -g & 0 & 0 & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \end{pmatrix}, \qquad B := \begin{pmatrix} 0_{3\times3} & 0_{3\times1} \\ J^{-1} & 0_{3\times1} \\ 0_{3\times3} & 0_{3\times1} \\ 0 \\ 0_{3\times3} & 0_{3\times1} \\ 0 \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \end{pmatrix}$$

• Linear control is always possible but not very suitable

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Position control

A robotic oriented nonlinear approach

- Based on fast attitude and actuator loops
- Angle tracking is assumed to be perfect
- The aim is to bring the UAV to $\vec{p}^{\star} = (p_1^{\star} \quad p_2^{\star} \quad p_3^{\star})^T$ Filter the position target $\vec{p}_f^{\star} = \frac{\vec{p}^{\star}}{(\tau_f s + 1)^3}$, \vec{p}_f^{\star} must be C^3
- Let the tilde denote the error, for instance x̃ = x_f[⋆] − x
- With PIDs controllers, define an acceleration target on the two first direction (i = 1, 2):

$$[\ddot{p}_{i_{f}}]^{\star} = k_{P} \ddot{p}_{i} + k_{I} \int_{0}^{t} \tilde{p}_{i} dt + k_{D} \left(\dot{p}_{i_{f}}^{\star} - v_{i} \right) + \ddot{p}_{i_{f}}^{\star}$$

• With a PID controller, compute the thrust control:

$$T^{\star} = \frac{k_{P}\tilde{p}_{3} + k_{I}\int_{0}^{t}\tilde{p}_{3}dt + k_{D}\left(\dot{p}_{3_{f}}^{\star} - v_{3}\right)}{c_{\phi}c_{\theta}} + \frac{m}{c_{\phi}c_{\theta}}\left(g + \ddot{p}_{3_{f}}^{\star}\right)$$

- Yaw angle \u03c6 can be stabilized to any direction independently
- Compute the roll and pitch control:

$$\begin{array}{lll} \phi^{\star} & = & \sin^{-1}\left(\frac{m}{T^{\star}}([\ddot{p}_{1_{\ell}}]^{\star}s_{\psi}-[\ddot{p}_{2_{\ell}}]^{\star}c_{\psi})\right) \\ \theta^{\star} & = & \sin^{-1}\left(\frac{m}{T^{\star}}([\ddot{p}_{1_{\ell}}]^{\star}c_{\psi}+[\ddot{p}_{2_{\ell}}]^{\star}s_{\psi})\right) \end{array}$$

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Position control



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• Actuator control *u_i*

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• Characteristic variables:

- Actuator control *u_i*
- Actuator torques C_i
- Angles θ_i

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JOINTED-ARM ROBOTS

- Characteristic variables:
 - Actuator control *u_i*
 - Actuator torques C_i
 - Angles θ_i
 - Spatial position X_i
- Controlling a robot is equivalent to mastering the relation

$$u_i \rightleftharpoons C_i \rightleftharpoons \theta_i \rightleftharpoons X_i$$

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- Characteristic variables:
 - Actuator control *u_i*
 - Actuator torques C_i
 - Angles θ_i

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- Spatial position X_i
- Controlling a robot is equivalent to mastering the relation

$$u_i \rightleftharpoons C_i \rightleftharpoons \theta_i \rightleftharpoons X_i$$
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JOINTED-ARM ROBOTS

- Characteristic variables:
 - Actuator control u;
 - Actuator torques C_i
 - Angles θ_i

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- Spatial position X_i
- Controlling a robot is equivalent to mastering the relation

$$u_i \rightleftharpoons C_i \rightleftharpoons \theta_i \rightleftharpoons X_i$$
• Actuator dynamics • Robot dynamics •

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• Consist in finding the relations $X_i = f_i(\theta_i)$

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Geometrical model of robots

• Consist in finding the relations $X_i = f_i(\theta_i)$

Sometimes called "forward kinematics"

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- Consist in finding the relations $X_i = f_i(\theta_i)$
- Sometimes called "forward kinematics"
- That gives $X_n = f(\theta_i, \dots, \theta_n)$, the position of the extremity of the arm as a functions of the control angles (and of the robot parameters)

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- The aim is then to deduce the θ_i^r 's using f^{-1} (inversion)



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- Assumptions:



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- The aim is then to deduce the θ_i^r 's using f^{-1} (inversion)
- Assumptions:
 - The model must be quite precise



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- The aim is then to deduce the θ_i^r 's using f^{-1} (inversion)
- Assumptions:
 - The model must be quite precise
 - no friction, no drift, no backlash, no dead zone, ...



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- Consist in finding the relations $X_i = f_i(\theta_i)$
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- The aim is then to deduce the θ_i^r 's using f^{-1} (inversion)
- Assumptions:
 - The model must be quite precise
 - no friction, no drift, no backlash, no dead zone, ...
 - The dynamical phenomena must be negligible



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- Consist in finding the relations $X_i = f_i(\theta_i)$
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- The aim is then to deduce the θ_i^r 's using f^{-1} (inversion)
- Assumptions:
 - The model must be quite precise
 - no friction, no drift, no backlash, no dead zone, ...
 - The dynamical phenomena must be negligible
 - mass effect fully compensated by the inner-loop



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- Consist in finding the relations $X_i = f_i(\theta_i)$
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- The aim is then to deduce the θ_i^r 's using f^{-1} (inversion)
- Assumptions:
 - The model must be quite precise
 - no friction, no drift, no backlash, no dead zone, ...
 - The dynamical phenomena must be negligible
 - mass effect fully compensated by the inner-loop
 - few flexibility of the arms (not for spatial robotic arms !)



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- Assumptions:
 - The model must be quite precise
 - no friction, no drift, no backlash, no dead zone, ...
 - The dynamical phenomena must be negligible
 - mass effect fully compensated by the inner-loop
 - few flexibility of the arms (not for spatial robotic arms !)
 - Sufficiently simple model to be online inverted



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- Assumptions:
 - The model must be quite precise
 - no friction, no drift, no backlash, no dead zone, ...
 - The dynamical phenomena must be negligible
 - mass effect fully compensated by the inner-loop
 - few flexibility of the arms (not for spatial robotic arms !)
 - Sufficiently simple model to be online inverted
 - The model must be invertible



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- Consist in finding the relations $X_i = f_i(\theta_i)$
- Sometimes called "forward kinematics"
- That gives $X_n = f(\theta_i, \dots, \theta_n)$, the position of the extremity of the arm as a functions of the control angles (and of the robot parameters)
- The aim is then to deduce the θ_i^r 's using f^{-1} (inversion)
- Assumptions:
 - The model must be quite precise
 - no friction, no drift, no backlash, no dead zone, ...
 - The dynamical phenomena must be negligible
 - mass effect fully compensated by the inner-loop
 - few flexibility of the arms (not for spatial robotic arms !)
 - Sufficiently simple model to be online inverted
 - The model must be invertible
- Despite the limitations, this approach is widely used (oversized robots)

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• Let X be the orientation and position of the last segment in \mathcal{R}_0 (usually variable to control)

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- Let X be the orientation and position of the last segment in \mathcal{R}_0 (usually variable to control)
- **Orientation**: for any \vec{v}

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- Let X be the orientation and position of the last segment in \mathcal{R}_0 (usually variable to control)
- **Orientation**: for any \vec{v}
 - $\vec{v}(\mathcal{R}_i) = R_{i-1}^i \vec{v}(\mathcal{R}_{i-1})$



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- **Orientation**: for any \vec{v}
 - $\vec{v}(\mathcal{R}_i) = R_{i-1}^i \vec{v}(\mathcal{R}_{i-1})$ • $\vec{v}(\mathcal{R}_i) = \prod_{k-1}^i R_{k-1}^k \vec{v}(\mathcal{R}_0) = R_0^i \vec{v}(\mathcal{R}_0)$

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- **Position**: for any point *C*



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 - $\vec{v}(\mathcal{R}_i) = R_{i-1}^i \vec{v}(\mathcal{R}_{i-1})$

•
$$\vec{v}(\mathcal{R}_i) = \prod_{k=1} R_{k-1}^k \vec{v}(\mathcal{R}_0) = R_0^i \vec{v}(\mathcal{R}_0)$$

• **Position**: for any point *C*

•
$$\overrightarrow{O_0C}(\mathcal{R}_0) = \overrightarrow{O_0O_i}(\mathcal{R}_0) + \overrightarrow{O_iC}(\mathcal{R}_0) = \overrightarrow{O_0O_i}(\mathcal{R}_0) + R_i^0 \overrightarrow{O_iC}(\mathcal{R}_i)$$

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- Let X be the orientation and position of the last segment in \mathcal{R}_0 (usually variable to control)
- **Orientation**: for any \vec{v}
 - $\vec{v}(\mathcal{R}_i) = R_{i-1}^i \vec{v}(\mathcal{R}_{i-1})$

•
$$\vec{v}(\mathcal{R}_i) = \prod_{k=1} R_{k-1}^k \vec{v}(\mathcal{R}_0) = R_0^i \vec{v}(\mathcal{R}_0)$$

• **Position**: for any point *C*

•
$$\overrightarrow{O_0C}(\mathcal{R}_0) = \overrightarrow{O_0O'}(\mathcal{R}_0) + \overrightarrow{O_1C}(\mathcal{R}_0) = \overrightarrow{O_0O'}(\mathcal{R}_0) + R_i^0 \overrightarrow{O_1C}(\mathcal{R}_i)$$

• $\overrightarrow{O_0C}(\mathcal{R}_0) = \overrightarrow{O_0O'}(\mathcal{R}_0) + R_1^0 \overrightarrow{O_1O'}(\mathcal{R}_1) + \dots + R_{i-1}^0 \overrightarrow{O_{i-1}O'}(\mathcal{R}_{i-1}) + R_i^0 \overrightarrow{O_iC}(\mathcal{R}_i)$

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 $R_0 \begin{array}{c} o_0 & o_1 \\ \hline R_1 & o_2 \end{array}$

- Let X be the orientation and position of the last segment in \mathcal{R}_0 (usually variable to control)
- **Orientation**: for any \vec{v}
 - $\vec{v}(\mathcal{R}_i) = R_{i-1}^i \vec{v}(\mathcal{R}_{i-1})$

•
$$\vec{v}(\mathcal{R}_i) = \prod_{k=1} R_{k-1}^k \vec{v}(\mathcal{R}_0) = R_0^i \vec{v}(\mathcal{R}_0)$$

• **Position**: for any point *C*

$$\begin{array}{l} \bullet \quad \overline{O_0 \overrightarrow{C}}(\mathcal{R}_0) = \overline{O_0 \overrightarrow{O_i}}(\mathcal{R}_0) + \overline{O_i \overrightarrow{C}}(\mathcal{R}_0) = \overline{O_0 \overrightarrow{O_i}}(\mathcal{R}_0) + R_i^0 \overline{O_i \overrightarrow{C}}(\mathcal{R}_i) \\ \bullet \quad \overline{O_0 \overrightarrow{C}}(\mathcal{R}_0) = \overline{O_0 \overrightarrow{O_i}}(\mathcal{R}_0) + R_1^0 \overline{O_1 \overrightarrow{O_i}}(\mathcal{R}_1) + \dots + R_{i-1}^0 \overline{O_{i-1} \overrightarrow{O_i}}(\mathcal{R}_{i-1}) + R_i^0 \overline{O_i \overrightarrow{C}}(\mathcal{R}_i) \end{array}$$

• where R_i^{i+1} is the rotation matrix from \mathcal{R}_i to \mathcal{R}_{i+1} :

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• **Position**: for any point *C*

•
$$\overline{O_0}\overline{\mathcal{C}}(\mathcal{R}_0) = \overline{O_0}\overline{\mathcal{O}}_i(\mathcal{R}_0) + \overline{O_i}\overline{\mathcal{C}}(\mathcal{R}_0) = \overline{O_0}\overline{\mathcal{O}}_i(\mathcal{R}_0) + R_i^0\overline{O_i}\overline{\mathcal{C}}(\mathcal{R}_i)$$

• $\overline{O_0}\overline{\mathcal{C}}(\mathcal{R}_0) = \overline{O_0}\overline{\mathcal{O}}_i(\mathcal{R}_0) + R_1^0\overline{O_1}\overline{\mathcal{O}}_2(\mathcal{R}_1) + \dots + R_{i-1}^0\overline{O_{i-1}}\overline{\mathcal{O}}_i(\mathcal{R}_{i-1}) + R_i^0\overline{O_i}\overline{\mathcal{C}}(\mathcal{R}_i)$

• where R_i^{i+1} is the rotation matrix from \mathcal{R}_i to \mathcal{R}_{i+1} :

•
$$R_i^{i+1} = {R_{i+1}^i}^T$$
, $\det R_i^{i+1} = 1$

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Computation of the geometrical model

Combination of rotations and translations

• Easy way to compute the geometrical model: homogeneous coordinates

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COMPUTATION OF THE GEOMETRICAL MODEL Combination of rotations and translations

- Easy way to compute the geometrical model: homogeneous coordinates
- Let $\vec{v} := \begin{pmatrix} v_1 & v_2 & v_3 \end{pmatrix}$, then it is equivalent to the 4-dimension vector \vec{V} with $\omega = 1$: $V = \begin{pmatrix} v_1 \omega \\ v_2 \omega \\ v_3 \omega \\ v_3 \omega \end{pmatrix}$

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- Let $\vec{v} := \begin{pmatrix} v_1 & v_2 & v_3 \end{pmatrix}$, then it is equivalent to the 4-dimension vector \vec{V} with $\omega = 1$: $V = \begin{pmatrix} v_1 \omega \\ v_2 \omega \\ v_3 \omega \\ \omega \end{pmatrix}$

• Translation: a translation of vector $\begin{pmatrix} a & b & c \end{pmatrix}$ is given by: $Trans = \begin{pmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \end{pmatrix}$

$$\mathsf{rans} = \begin{pmatrix} 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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COMPUTATION OF THE GEOMETRICAL MODEL Combination of rotations and translations

- Easy way to compute the geometrical model: homogeneous coordinates
- Let $\vec{v} := \begin{pmatrix} v_1 & v_2 & v_3 \end{pmatrix}$, then it is equivalent to the 4-dimension vector \vec{V} with $\omega = 1$: $V = \begin{pmatrix} v_1 \omega \\ v_2 \omega \\ v_3 \omega \\ \omega \end{pmatrix}$

• **Translation**: a translation of vector $\begin{pmatrix} a & b & c \end{pmatrix}$ is given by:

$$\text{Trans} = \begin{pmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

• Rotation: a rotation of matrix R is given by:

$$\mathsf{Rot} = egin{pmatrix} R & \mathsf{0}_{3 imes 1} \ \mathsf{0}_{1 imes 3} & 1 \end{pmatrix}$$

Note that still
$$R^{-1}=R^{\mathcal{T}}$$
 and $det(R)=1$

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• Consider two successive articulations

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- Consider two successive articulations
- Then, to go from O_k to O_{k+1} and from \mathcal{R}_k to \mathcal{R}_{k+1} , one does successively:



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- Consider two successive articulations
- Then, to go from O_k to O_{k+1} and from \mathcal{R}_k to \mathcal{R}_{k+1} , one does successively:
 - One rotation around z_k of angle θ_{k+1}



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- Consider two successive articulations
- Then, to go from O_k to O_{k+1} and from \mathcal{R}_k to \mathcal{R}_{k+1} , one does successively:
 - One rotation around z_k of angle θ_{k+1}
 - One translation along z_k of distance d_{k+1}

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 - One translation along x_{k+1} of distance a_{k+1}



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 - One rotation around z_k of angle θ_{k+1}
 - One translation along z_k of distance d_{k+1}
 - One translation along x_{k+1} of distance a_{k+1}
 - One rotation around x_{k+1} of angle α_{k+1}

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- Consider two successive articulations
- Then, to go from O_k to O_{k+1} and from \mathcal{R}_k to \mathcal{R}_{k+1} , one does successively:
 - One rotation around z_k of angle θ_{k+1}
 - One translation along z_k of distance d_{k+1}
 - One translation along x_{k+1} of distance a_{k+1}
 - One rotation around x_{k+1} of angle α_{k+1}
- The DH parametrization always exists and is unique

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Control with the geometrical model

• Compute the set of θ_i^r corresponding to the reference X^r

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Control with the geometrical model

Compute the set of θ^r_i corresponding to the reference X^r
θ_i as a function of X^r is often called "inverse kinematics"

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• Compute the set of θ_i^r corresponding to the reference X^r

CONTROL WITH THE GEOMETRICAL MODEL

- θ_i as a function of X^r is often called "inverse kinematics"
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- Drawbacks: the actuators are in closed loop but the robot is in open-loop



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- Make a step in the inner control loop to go from θ_i^0 to θ_i^r
- **Drawbacks:** the actuators are in closed loop but the robot is in open-loop
 - what about the speed ?



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• dry friction if multiple X^d



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 - dry friction if multiple X^d
 - what about the influence of the weight (that depends upon the configuration)

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- Make a step in the inner control loop to go from θ_i^0 to θ_i^r
- Drawbacks: the actuators are in closed loop but the robot is in open-loop
 - what about the speed ?
 - the trajectory is not well defined (obstacle avoidance, etc.)
 - dry friction if multiple X^d
 - what about the influence of the weight (that depends upon the configuration)
 - inertia may cause overshoot or oscillations

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 - One rotation around z_k of angle θ_{k+1} :

$$R_1 = \begin{pmatrix} c\theta_{k+1} & -s\theta_{k+1} & 0 & 0\\ s\theta_{k+1} & c\theta_{k+1} & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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Exercise

- Compute the matrix transformation of the Denavit-Hartenberg's convention
 - One rotation around z_k of angle θ_{k+1} :

• One translation along Z_k of distance d_{k+1} $T_1 = \begin{pmatrix} zy_{k+1} & -sy_{k+1} & 0 & 0 \\ s\theta_{k+1} & c\theta_{k+1} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ $T_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_{k+1} \\ 0 & 0 & 0 & 1 \end{pmatrix}$

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convention

• One translation along z_k of distance d_{k+1}

• One rotation around z_k of angle θ_{k+1} :

 $T_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_{k+1} \\ 0 & 0 & 0 & 1 \end{pmatrix}$

 $R_1 = \begin{pmatrix} c\theta_{k+1} & -s\theta_{k+1} & 0 & 0\\ s\theta_{k+1} & c\theta_{k+1} & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1 \end{pmatrix}$

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- One translation along x_{k+1} of distance a_{k+1}
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$$R_2 = \begin{pmatrix} 0 & c\alpha_{k+1} & -s\alpha_{k+1} & 0\\ 0 & s\alpha_{k+1} & c\alpha_{k+1} & 0\\ 0 & 0 & 0 & 1 \end{pmatrix}$$

• The matrix transformation of the Denavit-Hartenberg's convention is: $R_2 \cdot T_2 \cdot T_1 \cdot R_1$

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KINEMATIC MODEL OF ROBOTS

- Express the infinitesimal mouvement dX as a function of speed of the actuators $\frac{dA}{dA}$
- Sometimes called "velocity kinematics"
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KINEMATIC MODEL OF ROBOTS

- Express the infinitesimal mouvement dX as a function of speed of the actuators $\frac{d\theta}{dt}$
- Sometimes called "velocity kinematics"
- Assumes that, thanks to inner-loops, actuators speeds can be assumed to be control variables

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$$\dot{X} = \frac{\partial f}{\partial \theta} \dot{\theta}$$



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- J can be decomposed into J_v and J_ω so that:

$$\begin{array}{llll} \dot{x}_{n}^{\mathcal{R}_{f}} & = & J_{v}\dot{\theta} \\ \omega_{n}^{\mathcal{R}_{f}} & = & J_{\omega}\dot{\theta} \end{array}$$

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- J can be decomposed into J_v and J_ω so that:

• The kinematic model can also be obtained using the composition of speed and decomposing the Denavit-Hartenberg's parametrization:

$$R(z,\theta)T(z,d)T(x^+,a)R(x^+,\alpha)$$

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• Fastidious in many cases but systematic ! See books for that

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KINEMATIC MODEL OF ROBOTS

• Kinematic model can be used if "it can be stopped quasi instantaneously" (quickly w.r.t. the tasks to be done)

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KINEMATIC MODEL OF ROBOTS

- Kinematic model can be used if "it can be stopped quasi instantaneously" (quickly w.r.t. the tasks to be done)
- As for geometrical model, the dynamics has to be neglected

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 - J has more columns than rows: add a criterium to find the optimal path



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• The kinematic model is a state space representation of a controlled system

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• Example: the car in the plane

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Example: the car in the plane Characterizing variables (state variables): x, y and θ



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• Example: the car in the plane

- Characterizing variables (state variables): x, y and θ
- Control variables: speed of each wheels V_r and V_l



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- What is the kinematic model of the car ?



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Example of kinematic model

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 - What is the kinematic model of the car ?
 - What is the expression of the Jacobian of this robot ?





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 - What is the expression of the Jacobian of this robot ?
 - Is this system underactuated or overactuated ? Explain why $\cos \theta$ $J = \frac{1}{2} \begin{pmatrix} \sin \theta & \sin \theta \\ -\frac{2}{2} & \frac{2}{2} \end{pmatrix}$







Relation between workspace forces and joint torques

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• The workspace forces and joint torques are linked with the relation:

$$\tau = J_v^T F$$

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Relation between workspace forces and joint torques

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Visual servoing • The workspace forces and joint torques are linked with the relation:

$$\tau = J_v^T F$$

• the Jacobian must be derived at each origin O_i of each link frame



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Visual servoing When a robot is given by its kinematic model $\dot{X} = J\dot{ heta}$

- J is usually $n \times p$ with $X \in \mathbb{R}^n$ and $\theta \in \mathbb{R}^p$
- r = p n is called the kinematic redundancy number

Kinematic redundancy

- When r < 0, the robot is underactuated, usually the case with mobile robots ⇒ advanced control
- When r > 0, the robot is overactuated. It has redundancy. For a robot with redundancy, one can write:

• $J = \begin{pmatrix} J_n & J_{p-n} \end{pmatrix}$ with J_n invertible

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Visual servoing Control with J^t Take a robot given by its kinematic model $\dot{X}=J\dot{\theta}$

Control through the kinematic equation

- Control with J^t
 - Apply a fictive force $F = K(X X_d)$ with K positive and symmetric
 - Take $\dot{\theta} = J^t F = J^t K(X X_d) = J^t Ke$
 - Then the elastic potential $\Phi(e) = \frac{1}{2}e^t K e$ is such that

$$\dot{\Phi}(e) = -e^t K J J^t K e < 0$$

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Control with J^+ Take a robot given by its kinematic model $\dot{X} = J\dot{\theta}$

Control through the kinematic equation

- Control with $J^+ := J^t (JJ^t)^{-1}$
 - J⁺ is the Moore-Penrose pseudo-inverse (pinv in Matlab)
 - Can be obtained through SVD decomposition. $J = U\Delta V^t$, Δ diagonal $\implies J^+ = V\Delta^+ U^t$, Δ^+ is the inverse of the nonzero coefficient of Δ
 - Taking $\dot{\theta} = J^+ \dot{X}$ minimizes the energy $\dot{\theta}^t \dot{\theta}$
 - Taking $\dot{\theta} = J_M^+ \dot{X}$ with $J_M^+ := M^{-1} J^t (JM^{-1} J^t)^{-1}$ minimizes the kinetic energy $T = \frac{1}{2} \dot{\theta}^t M(\theta) \dot{\theta}$



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DYNAMICAL MODEL OF ROBOTS

• Express the accelerations of movement as a function of the actuation variables





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DYNAMICAL MODEL OF ROBOTS

- Express the accelerations of movement as a function of the actuation variables
- The dynamical model is obtained writing the mechanical equations of the system (conservation of momentum)



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• simplifications are required:



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 - based on relative speed of the \neq parts of the robot



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- simplifications are required:
 - based on relative speed of the \neq parts of the robot
 - thanks to inner-loops that can render parts instantaneous w.r.t. other parts of the robot



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- Almost never used for arm-robots



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- Very complex and most of the time impossible to control (too complex to design a control)
- simplifications are required:
 - based on relative speed of the \neq parts of the robot
 - thanks to inner-loops that can render parts instantaneous w.r.t. other parts of the robot
- Almost never used for arm-robots
- Widely used for flying or diving robots (UAVs, AUVs, etc.) or walking robots



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DYNAMICAL MODELS OF ROBOTS *n*-link manipulator

• The dynamical equations are of the form:

 $D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = r$

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q are the generalized coordinates



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•
$$C(q,\dot{q})\dot{q} = \sum_{i}\sum_{j}c_{ij}(q)\dot{q}_{i}\dot{q}_{j}$$



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$$C(q,\dot{q})\dot{q} = \sum_{i}\sum_{j}c_{ij}(q)\dot{q}_{i}\dot{q}_{j}$$

• Centrifugal effect when i = j (term in \dot{q}_i^2)

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• Coriolis effect when $i \neq j$ (terms in $\dot{q}_i \dot{q}_j$)



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- Centrifugal effect when i = j (term in \dot{q}_i^2)
- Coriolis effect when $i \neq j$ (terms in $\dot{q}_i \dot{q}_j$)
- An important literature on the control of this type of systems can be found

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DYNAMICAL MODELS OF ROBOTS flying and diving robots

• The dynamical equations are of the form:

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 $\begin{cases} \dot{\vec{p}} = \vec{v} \\ m \dot{\vec{v}} = -mg \vec{e}_3 + R \begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix} \\ \dot{\vec{R}} = R \vec{\omega}^{\times} \\ J \dot{\vec{\omega}} = -\vec{\omega}^{\times} J \vec{\omega} + \begin{pmatrix} \Gamma_r \\ \Gamma_\rho \\ \Gamma_y \end{pmatrix} \end{cases}$

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• The number of available controls depends upon the system

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DIFFERENT MODELS OF ROBOTS

• Geometrical model (or forward kinematic model):

Position of the robot = f(position of the actuators)

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DIFFERENT MODELS OF ROBOTS

• Geometrical model (or forward kinematic model):

Position of the robot = f(position of the actuators)

Inverse geometrical model (or inverse kinematic model):
 Position of the actuators = f(position of the robot)



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• Geometrical model (or forward kinematic model):

Position of the robot = f(position of the actuators)

- Inverse geometrical model (or inverse kinematic model):
 Position of the actuators = f(position of the robot)
- Kinematic model (state space representation) (or velocity kinematic model):

Speed of the robot = f(position, actuation speed)



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DIFFERENT MODELS OF ROBOTS

• Geometrical model (or forward kinematic model):

Position of the robot = f(position of the actuators)

- Inverse geometrical model (or inverse kinematic model):
 Position of the actuators = f(position of the robot)
- Kinematic model (state space representation) (or velocity kinematic model):

Speed of the robot = f(position, actuation speed)

• Dynamical model (state space representation):

Robot acceleration = f(position and speed, forces/torques)



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• Need to choose a path for the end effector that avoids

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Need to choose a path for the end effector that avoids collisions

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• Need to choose a path for the end effector that avoids

- collisions
- singularities of the robot

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PATH PLANNING

- Need to choose a path for the end effector that avoids
 - collisions
 - singularities of the robot
- Collision are easy to characterize in the workspace but may need to be transformed in the configuration space


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• Need to choose a path for the end effector that avoids

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- collisions
- singularities of the robot
- Collision are easy to characterize in the workspace but may need to be transformed in the configuration space
- The complexity of obstacle avoidance grows exponentially with the number of DOF



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• Need to choose a path for the end effector that avoids

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- collisions
- singularities of the robot
- Collision are easy to characterize in the workspace but may need to be transformed in the configuration space
- The complexity of obstacle avoidance grows exponentially with the number of DOF
- The method used are (usually):

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• Need to choose a path for the end effector that avoids

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- singularities of the robot
- Collision are easy to characterize in the workspace but may need to be transformed in the configuration space
- The complexity of obstacle avoidance grows exponentially with the number of DOF
- The method used are (usually):
 - Potential field: renders the obstacle repulsive

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• Need to choose a path for the end effector that avoids

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- collisions
- singularities of the robot
- Collision are easy to characterize in the workspace but may need to be transformed in the configuration space
- The complexity of obstacle avoidance grows exponentially with the number of DOF
- The method used are (usually):
 - Potential field: renders the obstacle repulsive
 - Gradient descent or Probabilistic roadmaps to generate the path



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• The workspace is the volume *W* the end effector can reach. Usually divided into:

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 - Reachable



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- The workspace is the volume *W* the end effector can reach. Usually divided into:
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 - Dexterous

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- Reachable
- Dexterous
- The "configuration" is the "location" of all points of the robot

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WORKSPACE AND OBSTACLES

- The workspace is the volume W the end effector can reach. Usually divided into:
 - Reachable
 - Dexterous
- The "configuration" is the "location" of all points of the robot
 - Configuration answers the question: where is the robot



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• The workspace is the volume *W* the end effector can reach. Usually divided into:

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- Reachable
- Dexterous
- The "configuration" is the "location" of all points of the robot
 - Configuration answers the question: where is the robot
 - The configuration can be adapted to the problem: from the set of all points of the robot to the sole the effector

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• The workspace is the volume *W* the end effector can reach. Usually divided into:

WORKSPACE AND OBSTACLES

- Reachable
- Dexterous
- The "configuration" is the "location" of all points of the robot
 - Configuration answers the question: where is the robot
 - The configuration can be adapted to the problem: from the set of all points of the robot to the sole the effector
 - The θ_i's are sufficient to characterize the configuration of an arm robot for arm robots



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 - The θ_i's are sufficient to characterize the configuration of an arm robot for arm robots
- $\bullet\,$ The set of $\theta_i{\,}'{\rm s}$ corresponding to a possible configuration is noted Q

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WORKSPACE AND OBSTACLES

- Obstacles are denotes O_i and the set of obstacle is $O = \cup O_i$
- Let $\theta \in Q$ and $C(\theta)$ denote the corresponding configuration

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- Obstacles are denotes O_i and the set of obstacle is $O = \cup O_i$
- Let $\theta \in Q$ and $C(\theta)$ denote the corresponding configuration
- Then the workspace can be divided into:
 - the collision-free configuration subspace $Q_f = \{\theta \in Q | C(\theta) \cap O = \emptyset\}$



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• Obstacles are denotes O_i and the set of obstacle is $O = \bigcup O_i$

WORKSPACE AND OBSTACLES

- Let $\theta \in Q$ and $C(\theta)$ denote the corresponding configuration
- Then the workspace can be divided into:
 - the collision-free configuration subspace $Q_f = \{\theta \in Q | C(\theta) \cap O = \emptyset\}$
 - the collision configuration subspace
 - $Q_c = \{ \theta \in Q | C(\theta) \cap O \neq \emptyset \}$



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EXAMPLE: THE CAR



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EXAMPLE: THE CAR

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EXAMPLE: THE CAR



• The collision configuration subspace is the convex hull in which the robot and an obstacle make vertex to vertex contact



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EXAMPLE: THE CAR



- The collision configuration subspace is the convex hull in which the robot and an obstacle make vertex to vertex contact
- Can be much more complicate to obtain

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EXAMPLE: THE CAR



- The collision configuration subspace is the convex hull in which the robot and an obstacle make vertex to vertex contact
- Can be much more complicate to obtain
- Numerical simulation can easily solve this problem (systematic simulation)



EXAMPLE: ARM ROBOT

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A RECALL ON GRADIENT DESCENT • $F(x, y) = \sin(\frac{1}{2}x^2 - \frac{1}{4}y^2 + 3)\cos(2x + 1 - e^y)$

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A RECALL ON GRADIENT DESCENT • $F(x, y) = \sin(\frac{1}{2}x^2 - \frac{1}{4}y^2 + 3)\cos(2x + 1 - e^y)$ • z := (x, y), F(x, y) = F(z)

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A RECALL ON GRADIENT DESCENT

$$F(x,y) = \sin(\frac{1}{2}x^2 - \frac{1}{4}y^2 + 3)\cos(2x + 1 - e^y)$$

•
$$z := (x, y), F(x, y) = F(z)$$

• Aim: finding z^* such that $F(z^*)$ is minimum



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$$z := (x, y), F(x, y) = F(z)$$

• Aim: finding z^* such that $F(z^*)$ is minimum



• Maximum/minimum obtained iteratively by :

$$z_{k+1} = z_k - \gamma \nabla F(z_k)$$

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A RECALL ON GRADIENT DESCENT • $F(x, y) = \sin(\frac{1}{2}x^2 - \frac{1}{4}y^2 + 3)\cos(2x + 1 - e^y)$

$$z := (x, y), F(x, y) = F(z)$$

• Aim: finding z^* such that $F(z^*)$ is minimum



• Maximum/minimum obtained iteratively by :

$$z_{k+1} = z_k - \gamma \nabla F(z_k)$$

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A RECALL ON GRADIENT DESCENT

About the stop criteria

• Many solutions to stop the iteration

$$z_{k+1} = z_k - \gamma \nabla F(z_k)$$

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A RECALL ON GRADIENT DESCENT About the stop criteria

• Many solutions to stop the iteration

$$z_{k+1} = z_k - \gamma \nabla F(z_k)$$

Better from the criteria point of view:
 stops if F(z_{k+1}) > F(z_k)

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A RECALL ON GRADIENT DESCENT About the stop criteria

• Many solutions to stop the iteration

$$z_{k+1} = z_k - \gamma \nabla F(z_k)$$

Better from the criteria point of view: stops if F(z_{k+1}) > F(z_k)
No more improvement in the criteria:

stops if
$$|F(z_{k+1}) - F(z_k)| < \varepsilon$$

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A RECALL ON GRADIENT DESCENT About the stop criteria

• Many solutions to stop the iteration

$$z_{k+1} = z_k - \gamma \nabla F(z_k)$$

- Better from the criteria point of view:
 stops if F(z_{k+1}) > F(z_k)
- No more improvement in the criteria: stops if |F(z_{k+1}) - F(z_k)| < ε
 No more slope (almost the same as previous condition)
 - stops if $||\nabla F(z_k)|| < \varepsilon$



A RECALL ON GRADIENT DESCENT

About the step size γ

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$\bullet~{\rm On}$ the step size γ

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A RECALL ON GRADIENT DESCENT About the step size γ

- $\bullet\,$ On the step size $\gamma\,$
- Newton-Euler method: H, Hessian of F

$$z_{k+1} = z_k - \nabla F(z_k) H(x_k)^{-1}$$

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A RECALL ON GRADIENT DESCENT About the step size γ

- On the step size γ
- Newton-Euler method: H, Hessian of F

$$z_{k+1} = z_k - \nabla F(z_k) H(x_k)^{-1}$$

$$z_{k+1} = z_k - \rho_k B_k \nabla F(z_k)$$

B_k: approximation of the Hessian
http://en.wikipedia.org/wiki/Quasi-Newton_method)

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PATH PLANNING PROBLEM FORMULATION

• Want to go from one configuration θ_0 (position) to another one θ_f

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PATH PLANNING PROBLEM FORMULATION

- Want to go from one configuration θ_0 (position) to another one θ_f
- We define a continuous function $\gamma : [0,1] \rightarrow Q_f$ such that

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PATH PLANNING PROBLEM FORMULATION

- Want to go from one configuration θ_0 (position) to another one θ_f
- We define a continuous function $\gamma: [0,1] \rightarrow Q_f$ such that

•
$$\gamma(0) = heta_0$$
 and $\gamma(1) = heta_f$



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PATH PLANNING PROBLEM FORMULATION

- Want to go from one configuration θ_0 (position) to another one θ_f
- We define a continuous function $\gamma : [0,1] \rightarrow Q_f$ such that

•
$$\gamma(0) = \theta_0$$
 and $\gamma(1) = \theta_0$

• γ will represent a configuration between the initial configuration and the final



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PATH PLANNING PROBLEM FORMULATION

- Want to go from one configuration θ_0 (position) to another one θ_f
- We define a continuous function $\gamma : [0,1] \rightarrow Q_f$ such that
 - $\gamma(0) = heta_0$ and $\gamma(1) = heta_f$
- γ will represent a configuration between the initial configuration and the final
- The aim will be to fin successive γ that remain in Q_f : $\tau \to \gamma(\tau)$ is a path from θ_0 to θ_f



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PATH PLANNING PROBLEM FORMULATION

- Want to go from one configuration θ_0 (position) to another one θ_f
- We define a continuous function $\gamma : [0,1] \rightarrow Q_f$ such that

•
$$\gamma(0) = \theta_0$$
 and $\gamma(1) = \theta_0$

- $\bullet ~\gamma$ will represent a configuration between the initial configuration and the final
- The aim will be to fin successive γ that remain in Q_f :

 $au
ightarrow \gamma(au)$ is a path from $heta_0$ to $heta_f$

• We define a potential field (criterium):

$$U(\theta) = U_{att}(\theta) + U_{rep}(\theta)$$

The aim will be to minimize the criterium

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PATH PLANNING PROBLEM FORMULATION

- Want to go from one configuration θ_0 (position) to another one θ_f
- We define a continuous function $\gamma : [0,1] \rightarrow Q_f$ such that

•
$$\gamma(0) = \theta_0$$
 and $\gamma(1) = \theta_f$

- $\bullet ~\gamma$ will represent a configuration between the initial configuration and the final
- The aim will be to fin successive γ that remain in Q_f :

 $au
ightarrow \gamma(au)$ is a path from $heta_0$ to $heta_f$

• We define a potential field (criterium):

$$U(\theta) = U_{att}(\theta) + U_{rep}(\theta)$$

• $U_{att}(\theta)$ will attract γ to θ_f : the goal configuration

The aim will be to minimize the criterium

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PATH PLANNING PROBLEM FORMULATION

- Want to go from one configuration θ_0 (position) to another one θ_f
- We define a continuous function $\gamma : [0,1] \rightarrow Q_f$ such that

•
$$\gamma(0) = \theta_0$$
 and $\gamma(1) = \theta_f$

- $\bullet ~\gamma$ will represent a configuration between the initial configuration and the final
- The aim will be to fin successive γ that remain in Q_f :

 $au
ightarrow \gamma(au)$ is a path from $heta_0$ to $heta_f$

• We define a potential field (criterium):

$$U(\theta) = U_{att}(\theta) + U_{rep}(\theta)$$

U_{att}(θ) will attract γ to θ_f: the goal configuration
 U_{rep}(θ) will repulse the system away from obstacle
 The aim will be to minimize the criterium

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SIMPLE EXEMPLE OF OBJECTIVE FUNCTION

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• Take $U_{att}(\theta) = ||\theta - \theta_f||$: U_{att} is the distance to the final destination

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SIMPLE EXEMPLE OF OBJECTIVE FUNCTION

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• Take $U_{att}(\theta) = ||\theta - \theta_f||$: U_{att} is the distance to the final destination

• Take
$$U_{rep}(\theta) = \frac{1}{d(\theta, Q_c)}$$
: U_{rep} is infinite if there is a risk of obstacle

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ATTRACTIVE/REPULSIVE FIELDS

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• Trying to minimize or maximize the distance is not necessary appropriate

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ATTRACTIVE/REPULSIVE FIELDS

- Trying to minimize or maximize the distance is not necessary appropriate
- Inappropriate criterium may:



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- Inappropriate criterium may:
 - generate local minima

ATTRACTIVE/REPULSIVE FIELDS



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- Trying to minimize or maximize the distance is not necessary appropriate
- Inappropriate criterium may:
 - generate local minima
 - be delicate to minimize

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ATTRACTIVE/REPULSIVE FIELDS

- Inappropriate criterium may:
 - generate local minima
 - be delicate to minimize
 - have singularities



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• Trying to minimize or maximize the distance is not necessary appropriate

ATTRACTIVE/REPULSIVE FIELDS

- Inappropriate criterium may:
 - generate local minima
 - be delicate to minimize
 - have singularities
- The main problem consist in finding a criterium that will be convex (or close to)



MANIPULATOR ROBOTS

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• We define a potential field for each articulation

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MANIPULATOR ROBOTS

- We define a potential field for each articulation
- The attractive field is a monotonically increasing function of the distance of the *i*th frame to the goal position



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Visual servoing • We define a potential field for each articulation

• The attractive field is a monotonically increasing function of the distance of the *i*th frame to the goal position

MANIPULATOR ROBOTS

• The attractive field applies a fictitious force that push the manipulator into its goal position



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• We define a potential field for each articulation

• The attractive field is a monotonically increasing function of the distance of the *i*th frame to the goal position

MANIPULATOR ROBOTS

- The attractive field applies a fictitious force that push the manipulator into its goal position
- The repulsive field will create a fictitious force that will prevent collisions by repelling the robot from the obstacles



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ATTRACTIVE FIELDS

• Simple potential field: *conic well potential*

$$U_{att_i}(\theta) = \zeta_i ||O_i(\theta) - O_i(\theta_f)||$$

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ATTRACTIVE FIELDS

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• The corresponding force is:

$$F_{att_i}(\theta) = -\zeta_i \nabla ||O_i(\theta) - O_i(\theta_f)|| = -\zeta_i \frac{O_i(\theta) - O_i(\theta_f)}{||O_i(\theta) - O_i(\theta_f)||}$$

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• it is a ζ_i -norm vector pointing to the objective



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• it is a ζ_i -norm vector pointing to the objective

• has a singularity at the objective



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- it is a ζ_i -norm vector pointing to the objective
- has a singularity at the objective
- ζ_i is a ponderation between articulations



ATTRACTIVE FIELDS

• Instead we use: parabolic well potential

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$$U_{att_i}(\theta) = \frac{1}{2}\zeta_i ||O_i(\theta) - O_i(\theta_f)||^2$$

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ATTRACTIVE FIELDS

• Instead we use: parabolic well potential

$$U_{att_i}(heta) = rac{1}{2} \zeta_i ||O_i(heta) - O_i(heta_f)||^2$$

• The corresponding force is:

$$F_{att_i}(\theta) = -\nabla ||O_i(\theta) - O_i(\theta_f)|| = -\zeta_i(O_i(\theta) - O_i(\theta_f))$$

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• this force is defined everywhere



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- this force is defined everywhere
- Or the hybrid potential:



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- Or the hybrid potential:

• $U_{att_i}(\theta) = rac{1}{2}\zeta_i \left|\left|O_i(\theta) - O_i(\theta_f)\right|\right|^2$ if $\left|\left|O_i(\theta) - O_i(\theta_f)\right|\right| \le d$

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 - $U_{att_i}(\theta) = \frac{1}{2}\zeta_i ||O_i(\theta) O_i(\theta_f)||^2$ if $||O_i(\theta) O_i(\theta_f)|| \le d$ • $U_{att_i}(\theta) = -d\zeta_i ||O_i(\theta) - O_i(\theta_f)|| - \frac{1}{2}\zeta_i d^2$ if $||O_i(\theta) - O_i(\theta_f)|| \le d$
- The corresponding force is:
 - $F_{att_i}(\theta) = -\zeta_i(O_i(\theta) O_i(\theta_f))$ if $||O_i(\theta) O_i(\theta_f)|| \le d$

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ATTRACTIVE FIELDS

• Instead we use: parabolic well potential

$$U_{att_i}(heta) = rac{1}{2} \zeta_i ||O_i(heta) - O_i(heta_f)||^2$$

• The corresponding force is:

$$F_{att_i}(\theta) = -\nabla ||O_i(\theta) - O_i(\theta_f)|| = -\zeta_i(O_i(\theta) - O_i(\theta_f))$$

- this force is defined everywhere
- Or the hybrid potential:
 - $U_{att_i}(\theta) = \frac{1}{2}\zeta_i ||O_i(\theta) O_i(\theta_f)||^2$ if $||O_i(\theta) O_i(\theta_f)|| \le d$ • $U_{att_i}(\theta) = -d\zeta_i ||O_i(\theta) - O_i(\theta_f)|| - \frac{1}{2}\zeta_i d^2$ if $||O_i(\theta) - O_i(\theta_f)|| \le d$
- The corresponding force is:
 - $F_{att_i}(\theta) = -\zeta_i(O_i(\theta) O_i(\theta_f))$ if $||O_i(\theta) O_i(\theta_f)|| \le d$ • $F_{att_i}(\theta) = -d\zeta_i \frac{O_i(\theta) - O_i(\theta_f)}{||O_i(\theta) - O_i(\theta_f)||}$ if $||O_i(\theta) - O_i(\theta_f)|| \le d$

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REPULSIVE FIELDS

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• Again, one repulsive field by articulation is given

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• Again, one repulsive field by articulation is given

Repulsive Fields

• Should *strongly* repel the robot close to obstacles

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REPULSIVE FIELDS

- Again, one repulsive field by articulation is given
- Should strongly repel the robot close to obstacles
- Usually, should not have any influence far from the obstacle



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REPULSIVE FIELDS

- Again, one repulsive field by articulation is given
- Should strongly repel the robot close to obstacles
- Usually, should not have any influence far from the obstacle
- First define a radius of influence $\rho_i > \rho_0$



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REPULSIVE FIELDS

- Again, one repulsive field by articulation is given
- Should strongly repel the robot close to obstacles
- Usually, should not have any influence far from the obstacle
- First define a radius of influence $\rho_i > \rho_0$
- Define the repulsive field:



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REPULSIVE FIELDS

- Again, one repulsive field by articulation is given
- Should strongly repel the robot close to obstacles
- Usually, should not have any influence far from the obstacle
- First define a radius of influence $\rho_i > \rho_0$
- Define the repulsive field:

• $U_{rep_i}(\theta) = 0$ if $d(\theta, O) > \rho_i$

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REPULSIVE FIELDS

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- Usually, should not have any influence far from the obstacle
- First define a radius of influence $\rho_i > \rho_0$
- Define the repulsive field:

•
$$U_{rep_i}(\theta) = 0$$
 if $d(\theta, O) > \rho_i$
• $U_{rep_i}(\theta) = \frac{\zeta_i}{2} \left(\frac{1}{d(\theta, O)} - \frac{1}{\rho_0}\right)^2$ if $d(\theta, O) \le \rho_i$



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Repulsive fields

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• The corresponding fictive force is:

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Repulsive fields

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• The corresponding fictive force is:

•
$$F_{rep_i}(heta) = 0$$
 if $d(heta, O) >
ho_i$

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• Again, one repulsive field by articulation is given

Repulsive fields

- Should strongly repel the robot close to obstacles
- Usually, should not have any influence far from the obstacle
- First define a radius of influence $\rho_i > \rho_0$
- Define the repulsive field:

•
$$U_{rep_i}(\theta) = 0$$
 if $d(\theta, O) > \rho_i$
• $U_{rep_i}(\theta) = \frac{\zeta_i}{2} \left(\frac{1}{d(\theta, O)} - \frac{1}{\rho_0}\right)^2$ if $d(\theta, O) \le \rho_i$

• The corresponding fictive force is:

•
$$F_{rep_i}(\theta) = 0$$
 if $d(\theta, O) > \rho_i$
• $F_{rep_i}(\theta) = -\zeta_i \left(\frac{1}{d(\theta, O)} - \frac{1}{\rho_0}\right) d(\theta, O)^{-2} \nabla d(\theta, O)$ if $d(\theta, O) \le \rho_i$

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• The total joint torques acting on a robot is the sum of the torques from all attractive and repulsive potentials:

FROM ATTRACTIVE/REPULSIVE FORCES TO ACTUATOR TORQUES

$$au(heta) = \sum_{i} J_{O_i}^{T}(heta) \left(F_{\textit{att}_i}(heta) + F_{\textit{rep}_i}(heta)
ight)$$

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GRADIENT DESCENT

• Now that we can formulate the total torques acting on the joints in the configuration space due to the artificial potentials, we can formulate a path planning algorithm



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GRADIENT DESCENT

Now that we can formulate the total torques acting on the joints in the configuration space due to the artificial potentials, we can formulate a path planning algorithm
 First, determine your initial configuration



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GRADIENT DESCENT

- Now that we can formulate the total torques acting on the joints in the configuration space due to the artificial potentials, we can formulate a path planning algorithm
 - First, determine your initial configuration
 - Second, given a desired point in the workspace, calculate the final configuration using the inverse kinematics: Use this to create an attractive potential field



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GRADIENT DESCENT

- Now that we can formulate the total torques acting on the joints in the configuration space due to the artificial potentials, we can formulate a path planning algorithm
 - First, determine your initial configuration
 - Second, given a desired point in the workspace, calculate the final configuration using the inverse kinematics: Use this to create an attractive potential field
 - Ocate obstacles in the workspace: Create a repulsive potential field



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GRADIENT DESCENT

- Now that we can formulate the total torques acting on the joints in the configuration space due to the artificial potentials, we can formulate a path planning algorithm
 - First, determine your initial configuration
 - Second, given a desired point in the workspace, calculate the final configuration using the inverse kinematics: Use this to create an attractive potential field
 - Ocate obstacles in the workspace: Create a repulsive potential field
 - Sum the joint torques in the configuration space



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GRADIENT DESCENT

- Now that we can formulate the total torques acting on the joints in the configuration space due to the artificial potentials, we can formulate a path planning algorithm
 - First, determine your initial configuration
 - Second, given a desired point in the workspace, calculate the final configuration using the inverse kinematics: Use this to create an attractive potential field
 - Ocate obstacles in the workspace: Create a repulsive potential field
 - Sum the joint torques in the configuration space
 - **O** Use gradient descent to reach your target configuration



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$\bullet \quad i=0, \ \theta[0]=\theta_0$

- 2 if $||\theta[i] \theta_f|| > \varepsilon$, then:
 - $\theta[i+1] =$ $\theta[i] + \alpha[i] \frac{\tau(\theta[i])}{||\tau(\theta[i])||}$ • i = i+1
 - goto 2

else:

• return $\theta[0], \ldots, \theta[i]$

- Many other algorithm are possible
 - steepest descent (gradient) (Euler)
 - Newton
 - ... see optimization books
- the θ[0],...,θ[i] are the successive configuration to track = path
- It is possible to add random to escape local minima

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• Randomly sample the configuration space

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- Randomly sample the configuration space
- Enables to roughly separate Q_f from O



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Randomly sample the configuration space

PROBABILISTIC ROADMAP

- Enables to roughly separate Q_f from O
- Discards the points "too close" from O



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Randomly sample the configuration space

- Enables to roughly separate Q_f from O
- Discards the points "too close" from O
- Connect using straight line segments that do not intersect obstacles

PROBABILISTIC ROADMAP



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• Randomly sample the configuration space

- Enables to roughly separate Q_f from O
- Discards the points "too close" from O
- Connect using straight line segments that do not intersect obstacles

PROBABILISTIC ROADMAP

• Eventually resample until Q_f is sufficiently covered



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Visual servoing

Randomly sample the configuration space

- Enables to roughly separate Q_f from O
- Discards the points "too close" from O
- Connect using straight line segments that do not intersect obstacles

PROBABILISTIC ROADMAP

- Eventually resample until Q_f is sufficiently covered
- Chose the path in the connected space



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Some final remarks

• All the previous methods assume an a priori knowledge of the environnement

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Some final remarks

- All the previous methods assume an a priori knowledge of the environnement
- Predictive control can also be used to handle constraints "on line"

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Some final remarks

- All the previous methods assume an a priori knowledge of the environnement
- Predictive control can also be used to handle constraints "on line"
- Adding fictive force is a very power tool also widely used in formation control or robotics with communication constraints (mainly range)



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Mobile robotics

Visual servoing

• Born in the 50s, aiming to autonomously moving robots



Grey Walter's "Turtle" (machina speculatrix): attracted by light

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Visual servoing

• Born in the 50s, aiming to autonomous mobile robots



John Hopkins Univ. "Beast" robot: first use of transistor based sensing (ultrasound and photodiodes)

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• Born in the 50s, aiming to autonomous mobile robots



Shakey robot from Stanford Univ. Platform used to show first results on AI (1969)

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• Bio inspired locomotion: first biped robot



Honda E0 first biped robot (1986)

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• Bio inspired locomotion: first biped walk

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Rabbit robot CNRS-Grenoble (2004)

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• Bio inspired locomotion: more about mobility



Boston Dynamics (SoftBank)

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• SLAM: Simultaneous localization and mapping

MOBILE ROBOTICS



https://github.com/erik-nelson/blam

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• SLAM: Simultaneous localization and mapping

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BASIS OF NAVIGATION

Navigation gathers different problems

- Approach of a given visible target, going to the target. Each new sensing produces an action. Typically what some insects do. Usually based on a gradient approach
- Guidance Ability to go to some position characterized by a visible environnement.

Usually based on a gradient approach

To goal navigation In that case, the target don't need to be visible but the robot has a representation of the world. *Graph or gradient approach*

Topological navigation Same as previous one with a memory of the possible the spatial relationship between positions: the robot can go back) *Graph or gradient approach*

Metric navigation Same as above but the robot is capable to memorize the metric positions: the robot can go back to a point without taking the same path.

- The 3 first strategies: reactiv navigation
- The 2 last enable trajectory planification also called path planning

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BASIS OF NAVIGATION

Three key words of navigation

- Navigation relies on
 - Perception where am i ? Planification where should i go ? Action how can i move ?
- The order of Perception/Planification/Action is not trivial
- Sometimes it may be necessary to move to see where to go: perception depends upon control
- Sometimes it may be necessary see to know where to go: navigation depends upon perception



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BASIS OF NAVIGATION

the perception

• Two kind of perceptions:

Proprioceptive information Everything that the robot can measure independently from the environnement, typically the rotation of its wheels, accelerometers, gyrometers, etc.

Exterioceptive information Everything that the robot sense in the outside world, typically distance to obstacles. Sensors are cameras, infrared/laser/ultra sound sensors, etc.

• Two type of problems

Perception variability The perception of the same place can vary (e.g. because of the sun)

Perceptual aliasing The same perception signals can correspond to 2 different places

• Perception is merged via a fusion step



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Different kind of usage of perception information Direct

BASIS OF NAVIGATION the perception



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• Different kind of usage of perception information

BASIS OF NAVIGATION the perception

- Direct
- To built a metric map



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• Different kind of usage of perception information

BASIS OF NAVIGATION the perception

- Direct
- To built a metric map
- To built a metric map with objects



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• Different kind of usage of perception information

BASIS OF NAVIGATION the perception

- Direct
- To built a metric map
- To built a metric map with objects
- To built a metric map with objects of known typology



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BASIS OF NAVIGATION

Navigation key words of navigation

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INTRODUCTION TO VISUAL SERVOING

• An arm robot equipped with a camera

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- An arm robot equipped with a camera
- Aim: bring the final effector to a given predefined configuration

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- Two possible configurations



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• Eye to hand configuration



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INTRODUCTION TO VISUAL SERVOING The key points

- Being able to extract feature from the image: "recognize" points of the object
- Being able to characterize the relation between the robot movement and the image changes



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Image based visual servoing

THE INTERACTION MATRIX

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The interaction matrix links the mouvement of O_c (lateral and rotational) to the movement of the feature points (f_i^c)

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A short mathematical background

$$e(t) = s(q(t), a) - s^{\star}$$

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• s denotes the current feature depending upon



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• Positioning error:

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- s denotes the current feature depending upon
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- s* denotes the target feature



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- s^{*} denotes the target feature
- The relation between the image and the real world is given by the interaction matrix:

$$\dot{s} = L_s \nu_c$$

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$$\dot{s} = L_s \nu_c$$

where

• $\nu_c := (v_c, \omega_c) = (\text{linear veloc}_{cam frame}, \text{angular veloc}_{cam frame})$

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$$\dot{s} = L_s \nu_c$$

where

ν_c := (ν_c, ω_c) = (linear veloc_{cam frame}, angular veloc_{cam frame})
 L_s ∈ ℝ^{k×6}: interaction matrix (Jacobian)



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CONTROL IN VISUAL SERVOING

A simple control approach

• Coupling the error and the interaction relation, one gets:

$$\dot{e} = L_s \nu_c$$

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A simple control approach

• Coupling the error and the interaction relation, one gets:

 $\dot{e} = L_s \nu_c$

• Take the linear velocities and angular velocities as control variable

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CONTROL IN VISUAL SERVOING

A simple control approach

Coupling the error and the interaction relation, one gets:

 $\dot{e} = L_s \nu_c$

- Take the linear velocities and angular velocities as control variable
- Let $L_s^+ := (L_s^T L_s)^{-1} L_s^T$ be the Moore–Penrose pseudo-inverse of L_s



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Coupling the error and the interaction relation, one gets:

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- Take the linear velocities and angular velocities as control variable
 - Let $L_s^+ := (L_s^T L_s)^{-1} L_s^T$ be the Moore–Penrose pseudo-inverse of L_s
 - To force an exponential decrease of the error:

$$\dot{e} = -\lambda e$$

we must chose

$$\nu_{c} := -\lambda L_{s}^{+} e$$

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 - To force an exponential decrease of the error:

$$\dot{e} = -\lambda e$$

we must chose

$$\nu_c := -\lambda L_s^+ e$$

• Practically, *L_s* is never known perfectly and we use an approximation

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IMAGE-BASED VISUAL SERVOING

• Take a 3D point of coordinates P = (X, Y, Z) in the camera frame

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IMAGE-BASED VISUAL SERVOING

• Take a 3D point of coordinates P = (X, Y, Z) in the camera frame

• Its coordinates in the image will be p = (x, y):

$$x = X/Z = (u - c_u)/f\alpha$$

$$y = Y/Z = (v - c_v)/f$$

where f is the focal length, α is the ratio of the pixel dimensions, c_u and c_v are the coordinates of the principal point.



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where f is the focal length, α is the ratio of the pixel dimensions, c_u and c_v are the coordinates of the principal point.

Derivating, we get

$$\dot{x} = \dot{X}/Z - X\dot{Z}/Z^2 = (\dot{X} - x\dot{Z})/Z$$

$$\dot{y} = \dot{Y}/Z - Y\dot{Z}/Z^2 = (\dot{Y} - y\dot{Z})/Z$$

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IMAGE-BASED VISUAL SERVOING

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• Using the Varignon's formula

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IMAGE-BASED VISUAL SERVOING

• Using the Varignon's formula

$$\dot{X} = -v_c - \omega_c^{\times} X$$

• Mixing the two last equation, we get the interaction matrix form *P*

$$\dot{p} = L_p \nu_c$$

with

$$L_{p} = \begin{pmatrix} -1/Z & 0 & x/Z & xy & -(1+x^{2}) & y \\ 0 & -1/Z & y/Z & 1+y^{2} & -xy & -x \end{pmatrix}$$

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• Z is the depth and is usually not known



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- Z is the depth and is usually not known
- To control six degrees of freedom, at least three points are required (p₁, p₂, p₃)

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Workspace and obstacles

path planning

Mobile robotics

Visual servoing

N. Marchand (gipsa-lab)

IMAGE-BASED VISUAL SERVOING

• Using the Varignon's formula

$$\dot{X} = -v_c - \omega_c^{\times} X$$

• Mixing the two last equation, we get the interaction matrix form *P*

$$\dot{p} = L_p \nu_c$$

with

$$L_{p} = \begin{pmatrix} -1/Z & 0 & x/Z & xy & -(1+x^{2}) & y \\ 0 & -1/Z & y/Z & 1+y^{2} & -xy & -x \end{pmatrix}$$

- Z is the depth and is usually not known
- To control six degrees of freedom, at least three points are required (p₁, p₂, p₃)
- Camera parameters can be obtained by calibration


IMAGE-BASED VISUAL STEREO SERVOING

Robotics

N. Marchand

Introduction

Modeling

Cartesian coordinates

Orientation

Frames

Newton

Χ4

Kinematics

Arm robots Inner-loop Geometrical m Kinematic mod

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N. Marchand (gipsa-lab)

• We assume now that we have two cameras

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