

N. Marchand

Introduction

Outline

Mechanics

Cartesian coordinates

Orientat

Frames

Newton

UAV's model

Recreational break

Legislation

EU legislation

Sub-categories of operations

Categories of UAVs

Introduction to control

Main components

Control loops

Attitude control

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Robotics

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INTRODUCTION

- Historical perspective
 - First use of the word Robot (means forced labor or serf in Czech) in the play R.U.R. (Rossum's Universal Robots) by Karel Capek (1890-1938) in January 1921.
 - In R.U.R., Capek poses a paradise, where the machines initially bring so many benefits but in the end bring an equal amount of blight in the form of unemployment and social unrest



Metropolis, Fritz Lang, 1927

- Science fiction
 - Often a bad image: men against robots, dystopic society, etc. More and more a good image.

Formal definition (Robot Institute of America)

A reprogrammable, multifunctional manipulator designed to move material, parts, tools, or specialized devices through various programmed motions for the performance of a variety of tasks

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• Robots have a bad image (1930-1960)

- Robots take human works
- Robots are dangerous since potentially independent and more intelligent than we are

ROBOTS AND THEIR IMAGE

- Robots have a better image (1960-today)
 - Robots can make things that human can not do (space, etc.)
 - Human can do things that robots can not do (we still are clever)
 - Robots can be games
 - Robots can be good or bad





ROBOTICS INDUSTRY: WHERE ? (1/MANY)

• Number of robots for every 10 000 workers:

nits per 10,000 employees

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- 70% of robots in companies with more than 1000 employees
- 17% of robots in companies with less than 300 employees
- In 2002, 95% of robots > 30k€ and 32% of robots > 60k€
- 79% of decrease of the mean price between 1990 and 2002
- Average price in 2018: 45k€ (63k€ in 2009)
- Big robots manufacturers: ABB (S), KUKA (G), Fanuc (JP), etc.



ROBOTICS INDUSTRY: HOW MUCH ? (2/MANY)

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• Robot quality, evolution:

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- Decrease of the price, increase of the quality
- "The Impact of Industrial Robots on EU Employment and Wages: A Local Labour Market Approach", F. Chiacchio, G. Petropoulos and D. Pichler, 2018

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ROBOTICS INDUSTRY: FOR WHAT ? (3/many)

• In which industry sectors:

Robot industry market projections through 2035



Service sector

- Agri., ,forestry and fisheries sector
- Robtech (RT)* products

Manufacturing sector

* "Robot technology"

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ROBOTICS INDUSTRY: FOR WHAT ? (3/MANY)

• In which industry sectors:



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• In which industry sectors:

5000

4000

3000

2000

1000

0

Nombre de robots installés dans les usines françaises

ROBOTICS INDUSTRY: FOR WHAT ?



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(3/MANY)



ROBOTICS INDUSTRY: EXAMPLES (4/many)

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Vacuum cleaner (Kärcher)



Micromanipulator



Surgical robot



Forest robot



Kuka robot for automotive industry







Hollywood robots

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ROBOTICS INDUSTRY: NANO ROBOTICS (5/MANY)

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• Example of nano manipulation



Nano house from FEMTO-ST (France)

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ROBOTICS INDUSTRY: EVOLUTION (6/MANY)

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GROWTH OF INDUSTRIAL ROBOTICS WORLDWIDE & CHINA (THOUSANDS)

Source: IFR World Robotics, 2018. *Forecasted

ESTIMATED ANNUAL SUPPLY OF INDUSTRIAL ROBOTS (2008-2021)



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Robotics industry: UAVs (7/many)

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Robotics industry: UAVs (8/many)

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• Development of the drone's industry:



- Very competitive market with a high technological level of integration
- Commercial margin of 10% to 15% (more than 50% on iPhone)

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ROBOTICS INDUSTRY: UAVS (10/many)

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Environmental

Monitoring

Disaster

Response

Public Land Management

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Police & Fire

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Traffic Monitoring

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Robolution (1/5)

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- Robotics enables 90% of cost reduction (60% for delocation)
- Each new robot destroys 6.2 jobs [MIT/Boston 1990-2007, 2017]
- 47% of jobs in the US, 50% of jobs in Europe have a high risk of being replaced by robots in the next 20 years [Oxford, 2013] ... but only 9% according [OCDE, 2016]
- Poor countries are more vulnerable, especially world factories (85% of the jobs in Ethiopia, 77% in Chine [World Bank])
- Sectors with high impact: Administration et Production
- Winner sectors: Finance, Maths/Sciences, Education
- No link between unemployment and robots
- Helps to relocate jobs in countries where the consumers are
- Very few studies on created jobs (compared to destroyed jobs)
- 800 000 direct jobs in robotics in 2020 and more than 2 millions in connected domains (electronic, energy, agriculture, etc.)



ROBOLUTION (2/5)

Japan*

14,00%

12.00%

10,00%

8,00%

6.00%

4.00%

2,00%

0.00%

14.00%

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450.000

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300.000

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60.000

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20.000

180.000

160.000

140.000

120.000

100.000

80.000

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1995

0

1995 2000 2005 2010 2015

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USA

2005 2010 2015

Number of robots

Unemployement (%)

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2000 Robotics

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Routine manual Non-routine manual Routine cognitive Non-routine analytic Non-routine interpersonal 70 65 The dilemma for education and training: The skills that are easiest to teach and test are 55 also the ones that are easiest to digitise, automate and outsource 50 45 40 35 1960 1970 1980 1990 2000 2006

ROBOLUTION (3/5)

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ROBOLUTION (4/5)



		*****		(
1983	90	2000	10	14

Sources: US Population Survey; Federal Reserve Bank of St Louis

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Attitude control • What about the previous industrial revolution ?

• Machines have created more jobs than they have replaced in the last 140 years

ROBOLUTION (5/5)

- Working is getting less and less exhausting
- Increase of new jobs (+580% éducation)
- But we had fears, as in any big change periods:
 - 1675 : Destruction of machines by weavers (England), 1788 : 2000 workers break weaving machines (France), 1811-1812 : Luddism (Angleterre)
 - 1858 : Karl Marx is prophesies the replacement of the humans by machines
 - 1930 : John Maynard Keynes invents the term "technological unemployment"

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GLOBAL WARMING CHALLENGE (1/6)

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Changes in global surface temperature relative to 1850-1900

a) Change in global surface temperature (decadal average)

as reconstructed (1-2000) and observed (1850-2020)

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b) Change in global surface temperature (annual average) as **observed** and simulated using human & natural and only natural factors (both 1850-2020)

- Global warming challenge
- Exhaustion of raw materials

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Attitude control Human activity impact on climate, biodiversity and health...

GLOBAL WARMING CHALLENGE (2/6)

- Robotic systems require ressources
- Solutions:
 - Economic decrease
 - Including the warming challenge in a strategy of conception
 - Recycling
 - New usages (replacing existing ones)



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GLOBAL WARMING CHALLENGE (3/6)



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GLOBAL WARMING CHALLENGE (4/6)



- 10 UAVs could plant up to 400 000 trees per day
- Much less carbon consuming than other means

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GLOBAL WARMING CHALLENGE (5/6)



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GLOBAL WARMING CHALLENGE (6/6)

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• Basic mechanics for robotics

- Space representation
 - frames, coordinate transformation, etc.

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- Force and torques
- Modelisation
- Control for robots
 - All potential problems:
 - Oscillations, dry friction, saturations, etc.
 - Linear approaches
 - Nonlinear approaches



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Mechanics basis of rigid bodies

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 - Position control

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Position and speed

• The **position** of some point *P* in the **fixed** frame $\mathcal{F}(o, \vec{e_x}, \vec{e_y}, \vec{e_z})$ is the vector $\vec{p} = (x, y, z)^T$



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• The **position** of some point *P* in the **fixed** frame $\mathcal{F}(o, \vec{e_x}, \vec{e_y}, \vec{e_z})$ is the vector $\vec{p} = (x, y, z)^T$

• The **speed** of *P* in \mathcal{F} is the vector $\vec{s} = \dot{\vec{p}} = (\dot{x}, \dot{y}, \dot{z})^T$

POSITION AND SPEED



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Robotics	\bullet A rotation is represented by a 3 \times 3 ma	etrix R such that $R^T=R^{-1}$ and $\det R=1$
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A rotation is represented by a 3 × 3 matrix R such that R^T = R⁻¹ and det R = 1
A rotation of angle θ around:

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- N. Marchand
- A rotation is represented by a 3 × 3 matrix *R* such that *R^T = R⁻¹* and det *R = 1*A rotation of angle θ around:
 - axis $\vec{e_x}$ is given by:

$$R_{\rm x} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$$

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• A rotation is represented by a 3×3 matrix R such that $R^T = R^{-1}$ and det R = 1

• A rotation of angle θ around: N. Marchand • axis $\vec{e_x}$ is given by: $R_{x} = \begin{pmatrix} 1 & 0 & 0\\ 0 & \cos\theta & -\sin\theta\\ 0 & \sin\theta & \cos\theta \end{pmatrix}$ • axis \vec{e}_v is given by: $R_{y} = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$ Orientation Newton Recreational Sub-categories of Main components = nar N. Marchand (gipsa-lab) Robotics ENSE3-ASI 28 / 115



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N. Marchand• A rotation of angle θ around: • axis \vec{e}_x is given by:Introduction Outline $R_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{pmatrix}$ Mechanics Orientation Frames Nexton• axis \vec{e}_y is given by:Cartesian coordinates Orientation Frames Nexton• axis \vec{e}_z is given by:UAV's model Recreational break• axis \vec{e}_z is given by:Legislation Sub-categories of operations Cortexprise of UAVs• axis \vec{e}_z is given by:Littoduction to control Main components Control Legislation Sub-categories of UAVs• axis \vec{e}_z is given by:	Robotics	• A rotation is represented b	y a 3 $ imes$ 3 matrix R such that R	$^{\mathcal{T}}= R^{-1}$ and det $R=1$
Introduction $R_x = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{pmatrix}$ Mechanics • axis \vec{e}_y is given by: Cartesian coordinates Orientation Orientation $R_y = \begin{pmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{pmatrix}$ VAV's model • axis \vec{e}_z is given by: UAV's model • axis \vec{e}_z is given by: Recreational break $R_z = \begin{pmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$ Legislation Sub-categories of operations Sub-categories of UAVs Introduction Main components Control loops Attitude Control loops	N. Marchand	 A rotation of angle θ arou axis e_x is given by: 	ind:	
Outline $\begin{pmatrix} 0 & \sin \theta & \cos \theta \end{pmatrix}$ Mechanics • axis $\vec{e_y}$ is given by: Cartesian coordinates $P_{yy} = \begin{pmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{pmatrix}$ Outline $R_y = \begin{pmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$ VAV's model • axis $\vec{e_z}$ is given by: VAV's model • axis $\vec{e_z}$ is given by: Recreational break $R_z = \begin{pmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$ Legislation Sub-categories of operations Sub-categories of UAVs Introduction Main components Control loops Attitude Attitude	Introduction		$R_{\rm x} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \end{pmatrix}$	
Mechanics • axis \vec{e}_y is given by: Cartesian coordinates $Orientation$ Prames P_{rames} Newton • axis \vec{e}_z is given by: UAV's model • axis \vec{e}_z is given by: Recreational break $P_z = \begin{pmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{pmatrix}$ Legislation EU legistation Subcategories of operations Categories of UAVs Introduction to control Main components Catteride control Main components Cattride Categories of loops	Outline		$\begin{pmatrix} 0 & \sin\theta & \cos\theta \end{pmatrix}$	
• axis $\vec{e_z}$ is given by: Recreational break $R_z = \begin{pmatrix} \cos \theta & -\sin \theta & 0\\ \sin \theta & \cos \theta & 0\\ 0 & 0 & 1 \end{pmatrix}$ Legislation EU legislation Sub-categories of operations Categories of UAVs Introduction to control Main components Control loops Attitude	Mechanics Cartesian coordinates Orientation Frames Newton	• axis $\vec{e_y}$ is given by:	$R_{\rm y} = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$	
Recreational break $R_z = \begin{pmatrix} \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$ Legislation EU legislation Sub-categories of operations Categories of UAVs Introduction to control Main components Control loops Attitude Cutoto	UAV's model	• axis \vec{e}_z is given by:	$(\cos\theta - \sin\theta \ 0)$	
Legislation EU legislation Sub-categories of operations Categories of UAVs Introduction to control Main components Control loops Attitude	Recreational break		$R_z = \begin{pmatrix} \sin\theta & \cos\theta & 0\\ 0 & 0 & 1 \end{pmatrix}$	
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• A rotation is represented by a 3 \times 3 matrix R such that $R^T = R^{-1}$ and det R = 1Robotics • A rotation of angle θ around: N. Marchand • axis \vec{e}_x is given by: $R_{\rm x} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$ • axis \vec{e}_v is given by: $R_{y} = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$ Orientation • axis \vec{e}_{τ} is given by: $R_z = \begin{pmatrix} \cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1 \end{pmatrix}$ Recreational • a unit vector $\vec{u} = (u_x, u_y, u_z)^T$: $\begin{pmatrix} u_x^2 + (1 - u_x^2)c_\theta & u_x u_y(1 - c_\theta) - u_z s_\theta & u_x u_z(1 - c_\theta) + u_y s_\theta \\ u_x u_y(1 - c_\theta) + u_z s_\theta & u_y^2 + (1 - u_y^2)c_\theta & u_y u_z(1 - c_\theta) - u_x s_\theta \\ u_y u_y(1 - c_\theta) - u_y s_\theta & u_y u_y(1 - c_\theta) + u_y s_\theta & u_y^2 + (1 - u_y^2)c_\theta \end{pmatrix}$ with $c_{i} = \cos(\cdot)$ and $s_{i} = \sin(\cdot)$ (and later on $t_{i} = \tan(\cdot)$) Main components

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Robotics	• The scalar product $< v_1, v_2 >$ is defined by: $< v_1, v_2 > := v_1^T v_2 \in \mathbb{R}$
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• The scalar product $< v_1, v_2 >$ is defined by: $< v_1, v_2 > := v_1^T v_2 \in \mathbb{R}$

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$$v_1 \times v_2 := \begin{pmatrix} v_{1y}v_{2z} - v_{1z}v_{2y} \\ v_{1z}v_{2x} - v_{1x}v_{2z} \\ v_{1x}v_{2y} - v_{1y}v_{2x} \end{pmatrix} \in \mathbb{R}^3$$

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• The skew-symmetric matrix associated to a vector $p = (x, y, z)^T$ is:

$$p^{\times} := \begin{pmatrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{pmatrix}$$



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• The set of skew-symmetric matrix with the brackett $[M_1, M_2] = M_1M_2 - M_2M_1$ is called SO(3) and forms an algebra



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- The set of skew-symmetric matrix with the brackett $[M_1, M_2] = M_1M_2 M_2M_1$ is called SO(3) and forms an algebra
- Skew-symmetric matrices and cross product:



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$$v_1 \times v_2 := \begin{pmatrix} v_{1_y} v_{2_z} - v_{1_z} v_{2_y} \\ v_{1_z} v_{2_x} - v_{1_x} v_{2_z} \\ v_{1_x} v_{2_y} - v_{1_y} v_{2_x} \end{pmatrix} \in \mathbb{R}^3$$

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- The set of skew-symmetric matrix with the brackett $[M_1, M_2] = M_1M_2 M_2M_1$ is called SO(3) and forms an algebra
- Skew-symmetric matrices and cross product:

$$v^{\times}u = v \times u$$

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PRODUCTS AND ASSOCIATED TOOLS

• The scalar product $\langle v_1, v_2 \rangle$ is defined by: $\langle v_1, v_2 \rangle := v_1^T v_2 \in \mathbb{R}$

• The cross product $v_1 \times v_2$ is defined by:

$$v_1 \times v_2 := \begin{pmatrix} v_{1_y} v_{2_z} - v_{1_z} v_{2_y} \\ v_{1_z} v_{2_x} - v_{1_x} v_{2_z} \\ v_{1_x} v_{2_y} - v_{1_y} v_{2_x} \end{pmatrix} \in \mathbb{R}^3$$

• The skew-symmetric matrix associated to a vector $p = (x, y, z)^T$ is:

$$p^{\times} := \begin{pmatrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{pmatrix}$$

- The set of skew-symmetric matrix with the brackett $[M_1, M_2] = M_1M_2 M_2M_1$ is called SO(3) and forms an algebra
- Skew-symmetric matrices and cross product:

$$v^{\times}u = v \times u$$

• Skew-symmetric matrices and rotations

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• Skew-symmetric matrices and rotations

$$u^{\times} \sin \theta + (I - uu^{T}) \cos \theta + uu^{T} = \exp((u\theta)^{\times})$$

is the rotation of angle θ leaving axis u fixed

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- Many attitude representations
 - Euler angles



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• Euler angles: 3 angles, 27 possible rotations



L. Euler (1707-1783)

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- Representations with singularities



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W.R. Hamilton (1805-1865)

• Quaternions



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W.R. Hamilton (1805-1865)



- μ fixed by rotation of angle θ
- the quaternion is:

$$q = \begin{pmatrix} u_x \sin \theta/2 \\ u_y \sin \theta/2 \\ u_z \sin \theta/2 \\ \cos \theta/2 \end{pmatrix} = \begin{pmatrix} \vec{q} \\ q_0 \end{pmatrix}$$

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Quaternions



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ATTITUDE REPRESENTATION : ANGULAR VELOCITIES

• The angular velocity $\omega = (\omega_1, \omega_2, \omega_3)^T$ represents the rotation speed w.r.t. each axis of the body frame



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• **Caution:** Angular velocities <u>are not</u> the time derivatives of Euler angles

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Angular velocities are given by:

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 Angular velocities are given by:
 - Rotation matrix:

$$\dot{R} = R\omega^{\times}$$

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$$\dot{R} = R\omega^{\times}$$

• Quaternions :

$$\dot{\vec{q}} = \frac{1}{2} \Omega(\vec{\omega}) q \qquad \text{with} \begin{cases} \Omega(\vec{\omega}) = \begin{pmatrix} 0 & -\vec{\omega}^T \\ \vec{\omega} & -\vec{\omega}^X \end{pmatrix} \\ \exists (q) = \begin{pmatrix} -\vec{q}^T \\ b_{3\times 3} q_0 + \vec{q}^X \end{pmatrix} \end{cases}$$

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MOVING FRAMES





P. Varignon (1654-1722)

Varignon's formula

$$rac{dec{U}}{dt}^{\mathcal{M}} = rac{dec{U}}{dt}^{\mathcal{F}} + \Omega^{\mathcal{F}/\mathcal{M}} imes ec{U}^{\mathcal{F}}$$

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MOVING FRAMES

• $\mathcal{F} := (O, \vec{e_x}, \vec{e_y}, \vec{e_z})$ fixed inertial frame

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MOVING FRAMES

• $\mathcal{F} := (O, \vec{e_x}, \vec{e_y}, \vec{e_z})$ fixed inertial frame • $\mathcal{M} := (M, \vec{t_1}, \vec{t_2}, \vec{t_3})$: mobile frame



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- R: rotation matrix s.t. $\mathcal{M} = R\mathcal{F}$



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- $\Omega^{\mathcal{M}/\mathcal{F}}$: angular velocity matrix of \mathcal{M} w.r.t.



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MOVING FRAMES

• $\mathcal{F} := (O, \vec{e_x}, \vec{e_y}, \vec{e_z})$ fixed frame

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$\begin{array}{l} MOVING \ FRAMES\\ \bullet \ \mathcal{F}:=(\textit{0},\vec{e_x},\vec{e_y},\vec{e_z}) \ fixed \ frame\\ \bullet \ \mathcal{M}:=(\textit{M},\vec{t_1},\vec{t_2},\vec{t_3}): \ mobile \ frame \end{array}$

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MOVING FRAMES

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- $\mathcal{M} := (M, \vec{t_1}, \vec{t_2}, \vec{t_3})$: mobile frame
- R: rotation matrix s.t. $\mathcal{M} = R\mathcal{F}$



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MOVING FRAMES

- $\mathcal{F} := (O, \vec{e_x}, \vec{e_y}, \vec{e_z})$ fixed frame
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- R: rotation matrix s.t. $\mathcal{M} = R\mathcal{F}$
- $\Omega^{\mathcal{M}/\mathcal{F}}$: angular velocity matrix of \mathcal{M} w.r.t. \mathcal{F}



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MOVING FRAMES

 \mathcal{M}

- $\mathcal{F} := (O, \vec{e_x}, \vec{e_y}, \vec{e_z})$ fixed frame
- $\mathcal{M} := (M, \vec{t_1}, \vec{t_2}, \vec{t_3})$: mobile frame
- R: rotation matrix s.t. $\mathcal{M} = R\mathcal{F}$
- $\Omega^{\mathcal{M}/\mathcal{F}}$: angular velocity matrix of \mathcal{M} w.r.t. \mathcal{F}
- Acceleration:

$$\ddot{P}^{\mathcal{F}} := \left(\frac{d\dot{P}^{\mathcal{F}}}{dt}\right)^{\mathcal{F}} = \frac{d\dot{P}^{\mathcal{M}}}{dt}^{\mathcal{F}} + \frac{d\Omega^{\mathcal{M}/\mathcal{F}} \times P^{\mathcal{F}}}{dt}$$

$$\begin{split} \stackrel{\mathcal{M}}{\longrightarrow} &= \ddot{\mathcal{P}}^{\mathcal{M}} + \Omega^{\mathcal{M}/\mathcal{F}} \times \dot{\mathcal{P}}^{\mathcal{M}} \text{ (Varignon's formula)} \\ \frac{\mathcal{M}/\mathcal{F}}{\mathcal{M}} &= \dot{\Omega}^{\mathcal{M}/\mathcal{F}} \times \mathcal{P}^{\mathcal{F}} + \Omega^{\mathcal{M}/\mathcal{F}} \times \dot{\mathcal{P}}^{\mathcal{F}} \\ &= \dot{\Omega}^{\mathcal{M}/\mathcal{F}} \times \mathcal{P}^{\mathcal{F}} + \Omega^{\mathcal{M}/\mathcal{F}} \times \dot{\mathcal{P}}^{\mathcal{M}} + \Omega^{\mathcal{M}/\mathcal{F}} \times (\Omega^{\mathcal{M}/\mathcal{F}} \times \mathcal{P}^{\mathcal{F}}) \end{split}$$

all together:

dP





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Consider[.]

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• a body of mass $m:=\sum m_i$ composed of elements located in $ec{p}_i$ with speed $ec{v}_i$ in $\mathcal F$	
• or a body of mass $m := \int_{body} dm$ composed of elementary part located in \vec{p}_{dm} with speed is	√ _{dm}
in ${\cal F}$	
• $\vec{p} := \frac{\sum_i m_i \vec{p_i}}{m}$ defines the position of its center of mass <i>G</i> in <i>F</i>	
• or $\vec{p} := \frac{\int_{\text{body}} dm \vec{p}_{dm}}{m}$ defines the position of its center of mass G in \mathcal{F}	
• $\vec{v} := \vec{p}$ defines speed of the center of mass	

•
$$\vec{r_i} := (\vec{p_i} - \vec{p}) \; (\text{resp. } \vec{r_{dm}} := (\vec{p_{dm}} - \vec{p}))$$

Linear Momentum

• an inertial frame \mathcal{F}

$$egin{array}{rcl} ec{P} & := & \sum_i m_i ec{v}_i = m ec{v} \in \mathbb{R}^3 \ ec{P} & := & \int_{ ext{body}} ec{v}_{dm} dm \in \mathbb{R}^3 \end{array}$$

Angular Momentum $\vec{L} := \sum_{i} m_{i}(\vec{p}_{i} - \vec{p}) \times \vec{v}_{i}$ $\vec{L} := \int_{\text{body}} (\vec{p}_{dm} - \vec{p}) \times \vec{v}_{dm} dm$ $= \underbrace{\int_{\text{body}} ||\vec{r}_{dm}||^{2} dm}_{J: \text{ moment of inertia}} \vec{\omega}$

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NEWTON'S LAWS

Consider:

- a rigid body
- \bullet an inertial frame ${\cal F}$
- \bullet a moving frame ${\cal M}$ centered in the center of mass and aligned with the main axis of the rigid body
- Let \vec{F}_i 's be forces applying on the body with moment arm \vec{a}_i



Conservation of the angular momentum $\sum \vec{\tau} = \frac{d\vec{L}}{dt}^{\mathcal{F}}$

• In a moving frame (Varignon's formula):

$$\frac{d\vec{L}^{\mathcal{F}}}{dt} = \frac{d\vec{L}^{\mathcal{M}}}{dt} + \Omega \times \vec{L}$$





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- 4 fixed rotors with controlled rotation speed *s_i*
- 4 generated forces F_i



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- 4 fixed rotors with controlled rotation speed *s_i*
- 4 generated forces F_i
- 4 counter-rotating torques Γ_i
- Roll movement

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- 4 generated forces F_i
- 4 counter-rotating torques Γ_i
- Roll movement generated with a dissymmetry between left and right forces:

 $\Gamma_r = l(F_4 - F_2)$



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HOW IT WORKS

- 4 generated forces F_i
- 4 counter-rotating torques Γ_i
- Roll movement generated with a dissymmetry between left and right forces:

 $\Gamma_r = l(F_4 - F_2)$

Pitch movement



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HOW IT WORKS

- 4 generated forces F_i
- 4 counter-rotating torques Γ_i
- Roll movement generated with a dissymmetry between left and right forces:

 $\Gamma_r = l(F_4 - F_2)$

 Pitch movement generated with a dissymmetry between front and rear forces:

$$\Gamma_p = I(F_1 - F_3)$$





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HOW IT WORKS

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- 4 counter-rotating torques Γ_i
- Roll movement generated with a dissymmetry between left and right forces:

 $\Gamma_r = I(F_4 - F_2)$

 Pitch movement generated with a dissymmetry between front and rear forces:

$$\Gamma_p = I(F_1 - F_3)$$

Yaw movement



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- 4 generated forces F_i
- 4 counter-rotating torques Γ_i
- Roll movement generated with a dissymmetry between left and right forces:

 $\Gamma_r = I(F_4 - F_2)$

 Pitch movement generated with a dissymmetry between front and rear forces:

 $\Gamma_p = l(F_1 - F_3)$

• Yaw movement generated with a dissymmetry between front/rear and left/right torques:

$$\Gamma_y = \Gamma_1 + \Gamma_3 - \Gamma_2 - \Gamma_4$$



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• Electrical motor: A 2nd order system with friction and saturation



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ACTUATION AND AERODYNAMICS

• Electrical motor: A 2nd order system with friction and saturation usually *approximated* by a 1^{rst} order system:

$$\dot{s}_i = -\frac{k_m^2}{J_r R} s_i - \frac{1}{J_r} \eta_{\text{oad}} + \frac{k_m}{J_r R} \operatorname{sat}_{\bar{U}_i}(U_i) \quad i \in \{1, 2, 3, 4\}$$

- s_i: rotation speed
- U_i : voltage applied to the motor; real control variable

 τ_{load} : motor load: $\tau_{\text{load}} = k_{gearbox} c_D |s_i| s_i$ with c_D drag coefficient



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ACTUATION AND AERODYNAMICS

• Electrical motor: A 2nd order system with friction and saturation usually *approximated* by a 1^{rst} order system:

$$\dot{s}_i = -rac{k_m^2}{J_r R} s_i - rac{1}{J_r} au_{ ext{load}} + rac{k_m}{J_r R} \operatorname{sat}_{\bar{U}_i}(U_i) \quad i \in \{1, 2, 3, 4\}$$

- *s_i*: rotation speed
- U_i : voltage applied to the motor; real control variable
- τ_{load} : motor load: $\tau_{\text{load}} = k_{gearbox} c_D |s_i| s_i$ with c_D drag coefficient

• Aerodynamical forces and torques: Very complex models exist





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ACTUATION AND AERODYNAMICS

• Electrical motor: A 2nd order system with friction and saturation usually *approximated* by a 1^{rst} order system:

$$\dot{s}_i = -rac{k_m^2}{J_r R} s_i - rac{1}{J_r} au_{ ext{load}} + rac{k_m}{J_r R} \operatorname{sat}_{\bar{U}_i}(U_i) \quad i \in \{1, 2, 3, 4\}$$

- *s_i*: rotation speed
- U_i : voltage applied to the motor; real control variable
- τ_{load} : motor load: $\tau_{\text{load}} = k_{gearbox} c_D |s_i| s_i$ with c_D drag coefficient
- Aerodynamical forces and torques: Very complex models exist but overcomplicated for control, better use the *simplified* model:

$$\begin{array}{rcl} F_i &=& c_T s_i^2 \\ \Gamma_r &=& l c_T (s_4^2 - s_2^2) \\ \Gamma_p &=& l c_T (s_1^2 - s_3^2) \\ \Gamma_y &=& l c_D (s_1^2 + s_3^2 - s_2^2 - s_4^2) \end{array} \qquad i \in \{1,2,3,4\}$$

 c_T : thrust coefficient, c_D : drag coefficient

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Frames and variables

• Two frames

- a fixed frame $\mathcal{E}(\vec{e}_1, \vec{e}_2, \vec{e}_3)$
- a frame attached to the X4 $\mathcal{T}(\vec{t_1}, \vec{t_2}, \vec{t_3})$



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- Frame change
 - a rotation matrix R from $\mathcal T$ to $\mathcal E$



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- Frame change
 - ullet a rotation matrix R from ${\mathcal T}$ to ${\mathcal E}$
 - State variables:



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- Frame change
 - a rotation matrix R from $\mathcal T$ to $\mathcal E$
 - State variables:
 - Cartesian coordinates (in \mathcal{E})
 - position \vec{p}
 - velocity \vec{v}



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Two frames

- a fixed frame $\mathcal{E}(\vec{e_1}, \vec{e_2}, \vec{e_3})$
- a frame attached to the X4 $\mathcal{T}(\vec{t}_1, \vec{t}_2, \vec{t}_3)$
- Frame change
 - a rotation matrix R from \mathcal{T} to \mathcal{E}
 - State variables:
 - Cartesian coordinates (in *E*)
 - position \vec{p}
 - velocity \vec{v}
 - Attitude coordinates:
 - angular velocity $\vec{\omega}$ in the moving frame \mathcal{T}
 - either: Euler angles three successive rotations about \vec{t}_3 , \vec{t}_1 and \vec{t}_3 of angles angles ϕ , θ and ψ giving R
 - or: Quaternion representation $(q_0, \vec{q}) = (\cos \beta/2, \vec{u} \sin \beta/2)$ represent a rotation of angle β about \vec{u}





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$$\begin{cases} p = v \\ m\vec{v} = -mg\vec{e}_3 + R \\ \vec{\tau} : \text{ control thrust} \end{cases} + \vec{F}_{ext}$$

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$$\vec{\vec{p}} = \vec{v}$$

$$\vec{m\vec{v}} = -mg\vec{e}_3 + R \underbrace{\sum_i F_i(s_i)\vec{t}_3}_{i} + \vec{F}_{ext}$$

$$\vec{\vec{T}} : \text{ control thrust}$$

• Attitude:



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$\begin{cases} \dot{\vec{p}} = \vec{v} \\ m\vec{v} = -mg\vec{e}_3 + R \underbrace{\sum_{i} F_i(s_i)\vec{t}_3}_{i} + \vec{F}_{ext} \\ \vec{T} : \text{ control thrust} \end{cases}$

• Rotation matrix formalism:

$$\left\{ \begin{array}{rrr} \dot{R} &=& R\vec{\omega}^{\times} \\ J\dot{\vec{\omega}} &=& -\vec{\omega}^{\times}J\vec{\omega}+\vec{\Gamma}_c+\vec{\Gamma}_{ext} \end{array} \right. \qquad \text{with } \vec{\omega}^{\times} = \begin{pmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{pmatrix}$$

 $\vec{\omega}^{\times}$ is the skew symmetric tensor associated to $\vec{\omega}$



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$$\begin{cases} \vec{p} = \vec{v} \\ m\vec{v} = -mg\vec{e}_3 + R \\ \vdots \\ \vec{T} : \text{ control thrust} \end{cases} + \vec{F}_{ext}$$

Attitude: Rotation matrix formalism:

$$\left\{ \begin{array}{rcl} \dot{R} & = & R\vec{\omega}^{\times} \\ J\dot{\vec{\omega}} & = & -\vec{\omega}^{\times}J\vec{\omega} + \vec{\Gamma}_c + \vec{\Gamma}_{ext} \end{array} \right. \qquad \text{with } \vec{\omega}^{\times} = \left(\begin{matrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{matrix} \right)$$

 $\vec{\omega}^{\times}$ is the skew symmetric tensor associated to $\vec{\omega}$ • Quaternion formalism:

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$$\begin{cases} \dot{q} = \frac{1}{2}\Omega(\vec{\omega})q \\ = \frac{1}{2}\Xi(q)\vec{\omega} \\ J\dot{\vec{\omega}} = -\vec{\omega}^{\times}J\vec{\omega} + \vec{\Gamma}_{c} + \vec{\Gamma}_{ext} \end{cases} \text{ with } \begin{cases} \Omega(\vec{\omega}) = \begin{pmatrix} 0 & -\vec{\omega}^{T} \\ \vec{\omega} & -\vec{\omega}^{\times} \end{pmatrix} \\ \Xi(q) = \begin{pmatrix} 0 & -\vec{\omega}^{T} \\ \vec{\omega} & -\vec{\omega}^{\times} \end{pmatrix} \end{cases}$$

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$\begin{cases} \dot{\vec{p}} = \vec{v} \\ \dot{m}\vec{v} = -mg\vec{e}_3 + R \underbrace{\sum_{i} F_i(s_i)\vec{t}_3}_{\vec{T} : \text{ control thrust}} + \vec{F}_{ext} \end{cases}$

Attitude:

• Rotation matrix formalism:

$$\left\{ \begin{array}{rcl} \dot{R} & = & R\vec{\omega}^{\times} \\ J\vec{\omega} & = & -\vec{\omega}^{\times}J\vec{\omega} + \vec{\Gamma}_{c} + \vec{\Gamma}_{ext} \end{array} \right. \qquad \text{with } \vec{\omega}^{\times} = \left(\begin{matrix} 0 & -\omega_{3} & \omega_{2} \\ \omega_{3} & 0 & -\omega_{1} \\ -\omega_{2} & \omega_{1} & 0 \end{matrix} \right)$$

 $\vec{\omega}^{\times}$ is the skew symmetric tensor associated to $\vec{\omega}$ • Quaternion formalism:

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where
$$\vec{\Gamma}_{c} = \begin{pmatrix} \Gamma_{r}(s_{2}, s_{4}) \\ \Gamma_{p}(s_{1}, s_{3}) \\ \Gamma_{y}(s_{1}, s_{2}, s_{3}, s_{4}) \end{pmatrix}$$
 are the control torques

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THE WRONSKIAN MATRIX

• Consider the 1-2-3 Euler angles (ϕ, θ, ψ)

• The rotation matrix is given by:

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$$R = R_z R_y R_x = \begin{pmatrix} c_\theta c_\phi & s_\phi s_\theta c_\psi - c_\phi s_\psi & c_\phi s_\theta c_\psi + s_\phi s_\psi \\ c_\theta s_\phi & s_\phi s_\theta s_\psi + c_\phi c_\psi & c_\phi s_\theta s_\psi - s_\phi c_\psi \\ -s_\theta & s_\phi c_\theta & c_\phi c_\theta \end{pmatrix}$$

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The wronskian matrix

- Consider the 1-2-3 Euler angles (ϕ, θ, ψ)
- The rotation matrix is given by:

$$R = R_z R_y R_x = \begin{pmatrix} c_\theta c_\phi & s_\phi s_\theta c_\psi - c_\phi s_\psi & c_\phi s_\theta c_\psi + s_\phi s_\psi \\ c_\theta s_\phi & s_\phi s_\theta s_\psi + c_\phi c_\psi & c_\phi s_\theta s_\psi - s_\phi c_\psi \\ -s_\theta & s_\phi c_\theta & c_\phi c_\theta \end{pmatrix}$$

• The relation between the time derivative of the Euler angles and the angular velocity is: $\vec{\omega} = \begin{pmatrix} \dot{\phi} \\ 0 \\ 0 \end{pmatrix} + R_z \begin{pmatrix} 0 \\ \dot{\theta} \\ 0 \end{pmatrix} + R_z R_y \begin{pmatrix} 0 \\ 0 \\ \dot{\psi} \end{pmatrix} = W^{-1} \begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix}$





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The wronskian matrix

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$$R = R_z R_y R_x = \begin{pmatrix} c_\theta c_\phi & s_\phi s_\theta c_\psi - c_\phi s_\psi & c_\phi s_\theta c_\psi + s_\phi s_\psi \\ c_\theta s_\phi & s_\phi s_\theta s_\psi + c_\phi c_\psi & c_\phi s_\theta s_\psi - s_\phi c_\psi \\ -s_\theta & s_\phi c_\theta & c_\phi c_\theta \end{pmatrix}$$

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- W is called the wronskian matrix given by (for 1-2-3 Euler angles):

$$\mathcal{N} = egin{pmatrix} 0 & rac{s_\phi}{c_ heta} & rac{c_\phi}{c_ heta} \ 0 & c_\phi & -s_\phi \ 1 & s_\phi t_ heta & c_\phi t_ heta \end{pmatrix}$$

• This matrix is singular for
$$\theta = \pi/2 + k\pi$$

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The wronskian matrix

- Consider the 3-1-3 Euler angles ($\phi, \theta, \psi)$
- The rotation matrix is given by:

$$R = R_z R_x R_z = \begin{pmatrix} c_{\psi} c_{\phi} - s_{\psi} c_{\theta} s_{\phi} & -c_{\psi} s_{\phi} - s_{\psi} c_{\theta} c_{\phi} & s_{\psi} s_{\theta} \\ s_{\psi} c_{\phi} + c_{\psi} c_{\theta} s_{\phi} & -s_{\psi} s_{\phi} + c_{\psi} c_{\theta} c_{\phi} & -c_{\psi} s_{\theta} \\ s_{\theta} s_{\phi} & s_{\theta} c_{\phi} & c_{\theta} \end{pmatrix}$$

• The relation between the time derivative of the Euler angles and the angular velocity is:

$$\vec{\omega} = \begin{pmatrix} \dot{\phi} \\ 0 \\ 0 \end{pmatrix} + R_x \begin{pmatrix} 0 \\ \dot{\theta} \\ 0 \end{pmatrix} + R_z R_x \begin{pmatrix} 0 \\ 0 \\ \dot{\psi} \end{pmatrix} = W^{-1} \begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix}$$

• W is called the wronskian matrix given by (for 3-1-3 Euler angles):

$$W^{-1} = egin{pmatrix} s_{\psi} s_{ heta} & c_{\psi} & 0 \ c_{\psi} s_{ heta} & -s_{\psi} & 0 \ c_{\phi} & 0 & 1 \end{pmatrix} \qquad W = egin{pmatrix} rac{s_{\psi}}{s_{ heta}} & rac{c_{\psi}}{s_{ heta}} & 0 \ c_{\psi} & -s_{\psi} & 0 \ -rac{s_{\psi} c_{ heta}}{s_{ heta}} & -rac{c_{\psi} c_{ heta}}{s_{ heta}} & 1 \end{pmatrix}$$

• This matrix is singular for $\theta = \mathbf{0} + k\pi$



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$$\begin{aligned} \dot{s}_{i} &= -\frac{k_{m}^{2}}{J_{r}R}s_{i} - \frac{k_{gearbox}c_{D}}{J_{r}}|s_{i}|s_{i} + \frac{k_{m}}{J_{r}R}\operatorname{sat}_{\bar{U}_{i}}(U_{i})\\ \dot{\vec{p}} &= \vec{v}\\ m\dot{\vec{v}} &= -mg\vec{e}_{3} + R\begin{pmatrix}0\\0\\\sum_{i}F_{i}(s_{i})\end{pmatrix}\\ \dot{\vec{R}} &= R\vec{\omega}^{\times}\\ J\dot{\vec{\omega}} &= -\vec{\omega}^{\times}J\vec{\omega} + \begin{pmatrix}\Gamma_{r}(s_{2},s_{4})\\\Gamma_{p}(s_{1},s_{3})\\\Gamma_{y}(s_{1},s_{2},s_{3},s_{4})\end{pmatrix} \end{aligned}$$

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 $\dot{s}_{i} = -\frac{k_{m}^{2}}{J_{r}R}s_{i} - \frac{k_{gearbox}c_{D}}{J_{r}}|s_{i}|s_{i} + \frac{k_{m}}{J_{r}R}\operatorname{sat}_{\bar{U}_{i}}(U_{i})$ $\dot{\vec{p}} = \vec{v}$ $-mg\vec{e}_3 + R\begin{pmatrix} 0\\ \sum F_i(s_i) \end{pmatrix}$ тŸ $\dot{R} = R\vec{\omega}^{\times}$ $J\dot{\vec{\omega}} = -\vec{\omega}^{\times}J\vec{\omega} + \begin{pmatrix} \Gamma_r(s_2, s_4) \\ \Gamma_p(s_1, s_3) \\ \Gamma_y(s_1, s_2, s_3, s_4) \end{pmatrix}$

In red: nonlinearities In blue: where the control variables act

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PARAMETER IDENTIFICATION

• Electrical motor:

- For small input steps, the system behaves very close to a **linear** first order system
- Hence, use linear identification tools
- \bar{U}_i is found on the data-sheet of the motor (damage avoidance)



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- Hence, use linear identification tools
- \bar{U}_i is found on the data-sheet of the motor (damage avoidance)

• Aerodynamical parameters: c_T and c_D

 c_T and c_D measured with specific test beds, depends upon temperature, distance from ground, etc.





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PARAMETER IDENTIFICATION

• Electrical motor:

- For small input steps, the system behaves very close to a **linear** first order system
- Hence, use linear identification tools
- \bar{U}_i is found on the data-sheet of the motor (damage avoidance)

• Aerodynamical parameters: c_T and c_D

 c_T and c_D measured with specific test beds, depends upon temperature, distance from ground, etc.

• Mechanical parameters:

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I length of an arm of the helicopter, easy to measure



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- I_r rotor inertia, hard to have precisely



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The flapping effect

- The thrust was assumed to be $\sum_{i} F_i(s_i) \vec{t}_3$, that is colinear to \vec{t}_3
- It has been proved to be false because it neglects the effect of the apparent wind speed, this is the **flapping effect**



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apparent wind



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The flapping effect

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- It has been proved to be false because it neglects the effect of the apparent wind speed, this is the **flapping effect**
- Higher thrust on one side of the blades
- The thrust becomes $\sum_{i} R_{i}^{\text{flapping}} F_{i}(s_{i}) \vec{t}_{3}$, torques are also modified







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The flapping effect

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MODELING MORE INTO DETAILS: THE FLAPPING EFFECT • The flapping matrix takes can be decomposed :

$$\begin{array}{lll} & \operatorname{pping} & = & R_{x}^{\operatorname{flapping}} \cdot R_{y}^{\operatorname{flapping}} \\ & = & \begin{pmatrix} 1 & 0 & 0 \\ 0 & c(\beta) & -s(\beta) \\ 0 & s(\beta) & c(\beta) \end{pmatrix} \cdot \begin{pmatrix} c(\alpha) & 0 & s(\alpha) \\ 0 & 1 & 0 \\ -s(\alpha) & 0 & c(\alpha) \end{pmatrix} \end{array}$$

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Modeling more into details: the flapping effect

• The flapping matrix takes can be decomposed :

 $R^{\text{flapping}} = R_x^{\text{flapping}} \cdot R_y^{\text{flapping}}$ $= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c(\beta) & -s(\beta) \\ 0 & s(\beta) & c(\beta) \end{pmatrix} \cdot \begin{pmatrix} c(\alpha) & 0 & s(\alpha) \\ 0 & 1 & 0 \\ -s(\alpha) & 0 & c(\alpha) \end{pmatrix}$

 $\bullet \ \alpha \ {\rm and} \ \beta \ {\rm can} \ {\rm be \ composed} \ {\rm as \ follows}$:

$$\begin{array}{rcl} \alpha & = & \alpha_{\mathbf{v}} + \alpha_{\omega} \\ \beta & = & \beta_{\mathbf{v}} + \beta_{\omega} \end{array}$$

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MODELING MORE INTO DETAILS: THE

FLAPPING EFFECT

The flapping matrix takes can be decomposed :

 $\begin{array}{lll} \mathcal{R}^{\text{flapping}} & = & \mathcal{R}_{x}^{\text{flapping}} \cdot \mathcal{R}_{y}^{\text{flapping}} \\ & = & \begin{pmatrix} 1 & 0 & 0 \\ 0 & c(\beta) & -s(\beta) \\ 0 & s(\beta) & c(\beta) \end{pmatrix} \cdot \begin{pmatrix} c(\alpha) & 0 & s(\alpha) \\ 0 & 1 & 0 \\ -s(\alpha) & 0 & c(\alpha) \end{pmatrix}$

 $\bullet \ \alpha \ {\rm and} \ \beta \ {\rm can} \ {\rm be} \ {\rm composed} \ {\rm as} \ {\rm follows}$:

 $\begin{array}{rcl} \alpha & = & \alpha_{\mathbf{v}} + \alpha_{\omega} \\ \beta & = & \beta_{\mathbf{v}} + \beta_{\omega} \end{array}$

• $\alpha_{\rm v}$ and $\beta_{\rm v}$ represent the contribution of the linear speed of the body to the flapping effect



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Modeling more into details: the flapping effect

• The flapping matrix takes can be decomposed :

 $R^{\text{flapping}} = R_x^{\text{flapping}} \cdot R_y^{\text{flapping}}$ $= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c(\beta) & -s(\beta) \\ 0 & s(\beta) & c(\beta) \end{pmatrix} \cdot \begin{pmatrix} c(\alpha) & 0 & s(\alpha) \\ 0 & 1 & 0 \\ -s(\alpha) & 0 & c(\alpha) \end{pmatrix}$

 $\bullet \ \alpha \ {\rm and} \ \beta \ {\rm can} \ {\rm be} \ {\rm composed} \ {\rm as} \ {\rm follows}$:

 $\begin{array}{rcl} \alpha & = & \alpha_{\mathbf{v}} + \alpha_{\omega} \\ \beta & = & \beta_{\mathbf{v}} + \beta_{\omega} \end{array}$

- α_ν and β_ν represent the contribution of the linear speed of the body to the flapping effect
- a_{\u03c6} and b_{\u03c6} represent the contribution of the rotational speed of the body to the flapping effect

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The ground effect

• The thrust was assumed to be $\sum_i F_i(s_i) \vec{t}_3$, with $F_i(s_i) = c_T s_i^2$

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The ground effect

- The thrust was assumed to be $\sum_{i} F_{i}(s_{i}) \vec{t}_{3}$, with $F_{i}(s_{i}) = c_{T} s_{i}^{2}$
- Unfortunately, c_T is not constant but depends upon
 - the density of the air, therefore of the temperature
 - the ground distance : it is the ground effect, $\alpha_g(z) \geq 1$



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ROTORS EFFECTS

• Each rotor may be thought of as a rigid disc rotating around the vertical axis the body frame, with angular velocity *s_i*. The rotor's axis of rotation is itself moving with the angular velocity of the frame. This leads to the following gyroscopic torque :

$$ec{\mathsf{g}}_{\mathsf{gyro}} = \mathit{I_r}ec{\omega} imes ec{t_3} \sum_i (-1) i \ket{s_i}$$

• I_r is the inertia matrix of a rotor



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$$ec{\mathsf{g}}_{\mathsf{gyro}} = \mathit{I_r}ec{\omega} imes ec{t_3} \sum_i (-1) i \ket{s_i}$$

- *I_r* is the inertia matrix of a rotor
- Each rotor produces a counter rotating torque that can be expressed as:

$$s_{res} := \sum_{i} (-1)^{i} |s_{i}|$$

 $\vec{\Gamma}_{I} = I_{r} \dot{s}_{res} \vec{t}_{3}$



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Superposition of thrust center and mass centerModified torque and forces:

$$\dot{J\vec{\omega}} = -\vec{\omega}^{\times}J\vec{\omega} + \vec{\Gamma}_{c} + \vec{\Gamma}_{ext} + \vec{F} \times \vec{PA}$$
(1)

where P is the center of mass and A the point where the thrust force applies



OTHER EFFECTS

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- External forces
- Air friction: $-K_v ||\vec{v}||\vec{v}$
- Many neglected non linear effects



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THE MIXING MATRIX

• The **mixing matrix** M_x links the torques and thrust force to the rotational speed of the rotors

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THE MIXING MATRIX

- The mixing matrix M_{x} links the torques and thrust force to the rotational speed of the rotors
- Depends on the considered configuration (not the same for + or x configuration)







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THE MIXING MATRIX

- The mixing matrix M_x links the torques and thrust force to the rotational speed of the rotors
- Depends on the considered configuration (not the same for + or x configuration)
- For the + configuration presented before, we have:

$$\begin{pmatrix} T \\ \Gamma_r \\ \Gamma_p \\ \Gamma_y \end{pmatrix} = \underbrace{\begin{pmatrix} c_T & c_T & c_T & c_T \\ 0 & -lc_T & 0 & lc_T \\ lc_T & 0 & -lc_T & 0 \\ lc_D & -lc_D & lc_D & -lc_D \end{pmatrix}}_{M_x} \begin{pmatrix} s_1^2 \\ s_2^2 \\ s_3^2 \\ s_4^2 \end{pmatrix}$$



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• Flapping and other effect renders the relation between the rotor's speeds and control thrust and torques complex



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- Flapping and other effect renders the relation between the rotor's speeds and control thrust and torques complex
- With flapping appears coupling phenomenon: the thrust affects the yaw movement and the drag affects thrust/roll/pitch movements



Robotics

• Consider an UAV with $n_r > 3$ rotors

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- where σ_i = 1 if the direction of rotation of the ith rotor is clockwise and σ_i = -1 if it is counterclockwise
- If the number of rotors is even, $\sigma_{i+1} = -\sigma_i$.
- When the number of rotors is odd, $\sigma_{i+1} = -\sigma_i$ except for $i = \frac{n_r - 1}{2}$ where $\sigma_{\frac{n_r - 1}{2} + 1} = \sigma_{\frac{n_r - 1}{2}}.$



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THE MIXING MATRIX: GENERAL CASE

- Consider an UAV with $n_r > 3$ rotors
- Each rotor rotation speed is *s_i* and generates a single thrust and torque:





- where σ_i = 1 if the direction of rotation of the ith rotor is clockwise and σ_i = -1 if it is counterclockwise
- If the number of rotors is even, $\sigma_{i+1} = -\sigma_i$.
- When the number of rotors is odd, $\sigma_{i+1} = -\sigma_i$ except for $i = \frac{n_r - 1}{2}$ where $\sigma_{\frac{n_r - 1}{2} + 1} = \sigma_{\frac{n_r - 1}{2}}.$

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THE MIXING MATRIX: GENERAL CASE

- Consider an UAV with $n_r > 3$ rotors
- Each rotor rotation speed is *s_i* and generates a single thrust and torque:

 $T_i = c_T s_i^2$ $Q_i = l c_D s_i^2$

• The thrust and torques are then:

$$\Gamma_r = I \sum_{i=1}^{n_r} \sin\left[\frac{2\pi(i-1)}{n_r}\right] T_i$$
$$\Gamma_r = I \sum_{i=1}^{n_r} \cos\left[\frac{2\pi(i-1)}{n_r}\right] T_i$$

$$\Gamma_p = l \sum_{i=1}^{l} \cos \left[\frac{2\pi (i-1)}{n_r} \right]^2$$

$$\Gamma_y = \sum_{i=1}^{n_r} \sigma_i Q_i$$
$$T = \sum_{i=1}^{n_r} T_i$$



- where σ_i = 1 if the direction of rotation of the ith rotor is clockwise and σ_i = -1 if it is counterclockwise
- If the number of rotors is even, $\sigma_{i+1} = -\sigma_i.$
- When the number of rotors is odd, $\sigma_{i+1} = -\sigma_i$ except for $i = \frac{n_r - 1}{2}$ where $\sigma_{\frac{n_r - 1}{2} + 1} = \sigma_{\frac{n_r - 1}{2}}$.

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Attitude control

N. Marchand (gipsa-lab)



 $\frac{S_i}{2\pi}$

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N. Marchand (gipsa-lab)

• Consider an UAV with $n_r > 3$ rotors

• Each rotor rotation speed is *s_i* and generates a single thrust and torque



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N. Marchand (gipsa-lab)

• Consider an UAV with $n_r > 3$ rotors

- Each rotor rotation speed is *s_i* and generates a single thrust and torque
- The thrust and torques can be computed as functions of the *s_i*



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THE MIXING MATRIX: GENERAL CASE

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• Consider an UAV with $n_r > 3$ rotors

- Each rotor rotation speed is *s_i* and generates a single thrust and torque
- The thrust and torques can be computed as functions of the *s_i*
- The mixing matrix is:



[2-(n

1)7)

 $\int 0 = c \sin \left[2\pi (i) \right]$

$$\begin{pmatrix} 0 & \dots & c_T \sin \left[\frac{2\pi (r-1)}{n_r} \right] & \dots & c_T \sin \left[\frac{2\pi (n_r-1)}{n_r} \right] \\ c_T & \dots & c_T \cos \left[\frac{2\pi (i-1)}{n_r} \right] & \dots & c_T \cos \left[\frac{2\pi (n_r-1)}{n_r} \right] \\ c_D \sigma_1 & \dots & c_D \sigma_i & \dots & c_D \sigma_{n_r} \\ c_T & \dots & c_T & \dots & c_T \end{pmatrix} \end{pmatrix} \begin{pmatrix} s_1^2 \\ \vdots \\ s_{n_r}^2 \end{pmatrix}$$

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$$=\sum_{i=1}^{n_r} \Xi_i s_i^2$$

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Actuation: depends upon the type of electrical drive you useBody:

$$\vec{p} = \vec{v}$$

$$\vec{m}\vec{v} = -mg\vec{e}_3 - K_v ||\vec{v}|| \vec{v} + R\vec{T} + \vec{F}_{ext}$$

$$\vec{R} = R\vec{\omega}^{\times}$$

$$\vec{J}\vec{\omega} = -\vec{\omega}^{\times}J\vec{\omega} + I_r\dot{s}_{res}\vec{t}_3 + I_r\vec{\omega} \times \vec{t}_3\sum_i (-1)i|s_i| + \vec{\Gamma}_c + \vec{\Gamma}_{ext}$$



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• Thrust:

<u>.</u>

$$\vec{\mathcal{T}} = \sum_{i} R_{i}^{\text{flapping}} \alpha_{g} c_{T} s_{i}^{2} \vec{t}_{3}$$

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$$\vec{R} = R\vec{\omega}^{\times}$$

$$J\vec{\omega} = -\vec{\omega}^{\times}J\vec{\omega} + I_r\dot{s}_{res}\vec{t}_3 + I_r\vec{\omega} \times \vec{t}_3 \sum_i (-1)i|s_i| + \vec{\Gamma}_c + \vec{\Gamma}_{ext}$$

• Thrust:

<u>.</u>

$$\vec{\mathcal{T}} = \sum_{i} R_{i}^{\text{flapping}} \alpha_{g} c_{T} s_{i}^{2} \vec{t}_{3}$$

• Torques:

$$\vec{\Gamma}_{c} = \sum_{i} R_{i}^{\text{flapping}} \alpha_{g} c_{T} s_{i}^{2} \vec{t}_{3} \times p_{rotor_{i}}^{T} + \sum_{i} (-1)^{i+1} c_{D} s_{i}^{2} \vec{t}_{3}$$

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• Controlling a complex system often resume in finding *intermediate control variables*

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 - Actuator control (usually a voltage) u



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- Actuator control (usually a voltage) *u*
- Variables to control (usually a speed or a position) p
- Intermediate control variables C



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TOWARDS CONTROL (1/2)

- Actuator control (usually a voltage) *u*
- Variables to control (usually a speed or a position) p

 $u \rightleftharpoons C \rightleftharpoons p$

• Intermediate control variables C



Robot dynamics



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• Controlling a complex system often resume in finding *intermediate control variables*

TOWARDS CONTROL (1/2)

- Actuator control (usually a voltage) *u*
- Variables to control (usually a speed or a position) p
- Intermediate control variables C

$$u \rightleftharpoons C \rightleftarrows p$$

- Actuator dynamics
- Robot dynamics 🔵
- Key strategy: build inner control loops to simplify the control problem
- Assume the dynamics of inner loops is neglectible w.r.t. outer ones

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TOWARDS CONTROL (2/2)

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N. Marchand (gipsa-lab)

Classic approach:

- \bigcirc First control loop to control the rotation speed s_i of the blades
- s_i and T and $\Gamma_{r,p,y}$ are linked through the mixing matrix:
 - imposing \vec{T} and $\vec{\Gamma}_c$, and s_i is the same
- Attitude control: control the orientation of the UAV using Tand $\vec{\Gamma}_c = (\Gamma_r, \Gamma_p, \Gamma_y)^T$
- O Position control/Trajectory tracking: control the position of the UAV using the orientation



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- Classic approach:
 - \bigcirc First control loop to control the rotation speed s_i of the blades
 - s_i and T and $\Gamma_{r,p,y}$ are linked through the mixing matrix:
 - imposing \vec{T} and $\vec{\Gamma}_c$, and s_i is the same
 - Attitude control: control the orientation of the UAV using Tand $\vec{\Gamma}_c = (\Gamma_r, \Gamma_p, \Gamma_y)^T$
 - O Position control/Trajectory tracking: control the position of the UAV using the orientation
- Many alternatives, for instance:
 - \bigcirc First control loop to control the rotation speed s_i of the blades
 - s_i and T and $\Gamma_{r,p,y}$ are linked through the mixing matrix: imposing T and $\Gamma_{r,p,y}$, and s_i is the same
 - O Control the angular velocity ω of the UAV using T and $\Gamma_{r,p,y}$
 - O Speed control/Trajectory tracking: control the speed of the UAV using the angular velocity ω

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- Actuation: Local inner loop, dynamics can be neglected
 Body:
 - $\begin{cases} \vec{p} = \vec{v} \\ m\vec{v} = -mg\vec{e}_3 + R\vec{T} \\ \dot{R} = R\vec{\omega}^{\times} \\ J\vec{\omega} = -\vec{\omega}^{\times}J\vec{\omega} + \vec{\Gamma}_c \end{cases}$

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Actuation: Local inner loop, dynamics can be neglected
Body:

$$\begin{cases} \vec{p} = \vec{v} \\ m\vec{v} = -mg\vec{e}_3 + R\vec{T} \\ \dot{R} = R\vec{\omega}^{\times} \\ J\dot{\vec{\omega}} = -\vec{\omega}^{\times}J\vec{\omega} + \vec{\Gamma}_c \end{cases}$$

• Control: \vec{T} and $\vec{\Gamma}_c$

Robotics

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Some dynamical system modeling basis

• Linear system:

$$\dot{x} = Ax + Bu$$

• General nonlinear system:

$$\dot{x} = f(x, u)$$

• Affine in the control system:

$$\dot{x} = f(x) + g(x)u$$

- *x* denotes the state of the system: how the system is
- *u* is the control variable of the system: how to *move* the system
- Every nonlinear system can be locally approximated by its linear approximation

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Robotics

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LINEAR APPROXIMATION (1/3)

• Consider the 1-2-3 Euler representation Robotics N. Marchand Newton UAV's model Recreational Sub-categories of

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LINEAR APPROXIMATION (1/3)

- Consider the 1-2-3 Euler representation
- The rotation matrix is:

$$R=egin{pmatrix} c_{ heta}c_{\psi}&s_{\phi}s_{ heta}c_{\psi}-c_{\phi}s_{\psi}&c_{\phi}s_{ heta}c_{\psi}+s_{\phi}s_{\psi}\ c_{ heta}s_{\psi}&s_{\phi}s_{ heta}s_{\psi}+c_{\phi}c_{\psi}&c_{\phi}s_{ heta}s_{\psi}-s_{\phi}c_{\psi}\ -s_{ heta}&s_{\phi}c_{ heta}&c_{\phi}c_{ heta}\end{pmatrix}$$

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LINEAR APPROXIMATION (1/3)

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$$R = egin{pmatrix} c_{ heta}c_{\psi} & s_{\phi}s_{ heta}c_{\psi} - c_{\phi}s_{\psi} & c_{\phi}s_{ heta}c_{\psi} + s_{\phi}s_{\psi} \ c_{ heta}s_{\psi} & s_{\phi}s_{ heta}s_{\psi} + c_{\phi}c_{\psi} & c_{\phi}s_{ heta}s_{\psi} - s_{\phi}c_{\psi} \ -s_{ heta} & s_{\phi}c_{ heta} & c_{\phi}c_{ heta} \end{pmatrix}$$

• We assume that the s_i are controlled or at least join a given reference speed s_i^r sufficiently rapidly to neglect its dynamics



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LINEAR APPROXIMATION (1/3)

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- We assume that the s_i are controlled or at least join a given reference speed s^r_i sufficiently rapidly to neglect its dynamics
 - We take $x := (\phi, \theta, \psi, \dot{\phi}, \dot{\theta}, \dot{\psi}, p^T, v^T)^T$ as state vector

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LINEAR APPROXIMATION (1/3)

- Consider the 1-2-3 Euler representation
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$$R = \begin{pmatrix} c_{\theta}c_{\psi} & s_{\phi}s_{\theta}c_{\psi} - c_{\phi}s_{\psi} & c_{\phi}s_{\theta}c_{\psi} + s_{\phi}s_{\psi} \\ c_{\theta}s_{\psi} & s_{\phi}s_{\theta}s_{\psi} + c_{\phi}c_{\psi} & c_{\phi}s_{\theta}s_{\psi} - s_{\phi}c_{\psi} \\ -s_{\theta} & s_{\phi}c_{\theta} & c_{\phi}c_{\theta} \end{pmatrix}$$

- We assume that the s_i are controlled or at least join a given reference speed s_i^r sufficiently rapidly to neglect its dynamics
- We take $x := (\phi, \theta, \psi, \dot{\phi}, \dot{\theta}, \dot{\psi}, p^T, v^T)^T$ as state vector
- We take $u := (s_1, \ldots, s_{n_r})$ as control variable

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$$R = \begin{pmatrix} c_{\theta}c_{\psi} & s_{\phi}s_{\theta}c_{\psi} - c_{\phi}s_{\psi} & c_{\phi}s_{\theta}c_{\psi} + s_{\phi}s_{\psi} \\ c_{\theta}s_{\psi} & s_{\phi}s_{\theta}s_{\psi} + c_{\phi}c_{\psi} & c_{\phi}s_{\theta}s_{\psi} - s_{\phi}c_{\psi} \\ -s_{\theta} & s_{\phi}c_{\theta} & c_{\phi}c_{\theta} \end{pmatrix}$$

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- We take $x := (\phi, \theta, \psi, \dot{\phi}, \dot{\theta}, \dot{\psi}, p^T, v^T)^T$ as state vector
- We take $u := (s_1, \ldots, s_{n_r})$ as control variable
- We chose a constant reference position x^r of the form (0, 0, ψ^r, 0, 0, 0, p^{rT}, 0)^T: one position and one direction

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LINEAR APPROXIMATION (1/3)

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- We take $x := (\phi, \theta, \psi, \dot{\phi}, \dot{\theta}, \dot{\psi}, p^T, v^T)^T$ as state vector
- We take $u := (s_1, \ldots, s_{n_r})$ as control variable
- We chose a constant reference position x^r of the form (0,0,ψ^r,0,0,0,p^{rT},0)^T: one position and one direction
- The nominal input u^r must compensate the weight of the robot

$$\Xi u^{r} = \begin{pmatrix} \Gamma_{r} \\ \Gamma_{p} \\ \Gamma_{y} \\ T \end{pmatrix}^{r} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ mg \end{pmatrix}$$

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LINEAR APPROXIMATION (2/3)

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• Let $\tilde{x} := x - x^r$ and $\tilde{u} := u - u^r$ denote respectively the variation of the state and the control vectors

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LINEAR APPROXIMATION (2/3)

Let x̃ := x - x^r and ũ̃ := u - u^r denote respectively the variation of the state and the control vectors
Linearizing the system in a neighborhood of x^r, one obtains the following linear system of dimension 12:

$$\dot{\tilde{x}} = A\tilde{x} + B\Xi\tilde{u}$$

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LINEAR APPROXIMATION FIRST PRACTICAL (2/3)

• First practical work: Program on MATLAB/Simulink the nonlinear model of the UAV and build a first control based on its linearization

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LINEAR APPROXIMATION FIRST PRACTICAL (2/3)

- First practical work: Program on MATLAB/Simulink the nonlinear model of the UAV and build a first control based on its linearization
- the numerical values are:

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LINEAR APPROXIMATION FIRST PRACTICAL (2/3)

• First practical work: Program on MATLAB/Simulink the nonlinear model of the UAV and build a first control based on its linearization

Robotics

- the numerical values are:
 - Motor parameters:

parameter	description	value	unit
k _m	motor constant	4.3×10^{-3}	N.m/A
Jr	rotor inertia	3.4×10^{-5}	J.g.m ²
R	motor resistance	0.67	Ω
k _{gearbox}	gearbox ratio	2.7×10^{-3}	-
\bar{U}_i	maximal voltage	12	V

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k _m	motor constant	$4.3 imes 10^{-3}$	N.m/A
Jr	rotor inertia	3.4×10^{-5}	J.g.m ²
R	motor resistance	0.67	Ω
k _{gearbox}	gearbox ratio	2.7×10^{-3}	-
\bar{U}_i	maximal voltage	12	V

• Aerodynamical parameters:

parameter	description	value
CT CT	thrust coefficient	$3.8 imes10^{-6}$
c _D	drag coefficient	$2.9 imes10^{-5}$

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LINEAR APPROXIMATION FIRST PRACTICAL (2/3)

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R	motor resistance	0.67	Ω
k _{gearbox}	gearbox ratio	2.7×10^{-3}	-
\bar{U}_i	maximal voltage	12	V

• Aerodynamical parameters:

parameter	description	value
CT CT	thrust coefficient	$3.8 imes10^{-6}$
c _D	drag coefficient	$2.9 imes10^{-5}$

Body parameters:

parameter	description	value	unit
J	inertia matrix	$\begin{pmatrix} 14.6 \times 10^{-3} & 0 & 0 \\ 0 & 7.8 \times 10^{-3} & 0 \\ 0 & 0 & 7.8 \times 10^{-3} \end{pmatrix}$	kg.m ²
m	mass of the UAV	0.458	kg
1	radius of the UAV	22.5	cm
g	gravity	9.81	m/s ²

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Attitude control

• Born in the 50s, aiming to *autonomously moving* robots



Grey Walter's "Turtle" (machina speculatrix): attracted by light

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Attitude control

• Born in the 50s, aiming to autonomous mobile robots



John Hopkins Univ. "Beast" robot: first use of transistor based sensing (ultrasound and photodiodes)



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• Born in the 50s, aiming to autonomous mobile robots



Shakey robot from Stanford Univ. Platform used to show first results on AI (1969)

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• Bio inspired locomotion: first biped robot



Honda E0 first biped robot (1986)

Robotics

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• Bio inspired locomotion: first biped walk

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Rabbit robot CNRS-Grenoble (2004)

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Robotics

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• Bio inspired locomotion: more about mobility



Boston Dynamics (SoftBank)

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• SLAM: Simultaneous localization and mapping

MOBILE ROBOTICS



https://github.com/erik-nelson/blam

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• SLAM: Simultaneous localization and mapping

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Aerial robotics



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LEGISLATION ON UAVS

- Vocabulary:
 - \bullet UAS (Unmanned Aircraft System): Unmanned Aerial Vehicle + ground station + communication system
 - RPAS (Remotely Piloted Aircraft System): UAS with a remote ground pilot

• ICAO (International Civil Aviation Organization):

- Adopts for international aviation: standards and recommended practices concerning air navigation, its infrastructure, flight inspection, prevention of unlawful interference, and facilitation of border-crossing procedures
- In charge of RPAS since 2008
- Creation in 2014 of the "RPAS Panel": integration of RPAS in the "IFR traffic"^a
- $\bullet\,$ In 2016: the states member of the ICAO officially ask to give rules on how to handle RPAS
- End 2016: ICAO produce the "UAS Toolkit" and RPAS are included in the GASP (Global Aviation Safety Plan)
- 2015: ICAO has also to give rules for UAS... still working on it
- To learn more: https://bit.ly/38MpA6F





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LEGISLATION ON UAVS

- Each state translate the ICAO recommandation into the law
- The ICAO UAS Toolkit can be consulted here: https://bit.ly/31wBx3U
- Helps states to built their laws and operator to develop safe UAS
- Each state have transposed the recommandations: https://bit.ly/31BAvU9





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EU LEGISLATION ON UAVS

• European community

- July 4, 2018: Regulation (EU) 2018/1139: creation of the European Union Aviation Safety Agency
- Regulations (EU) 2019/945 and (EU) 2020/105 transposed in each country legislation : 5 class of UAS
- To learn more: https://bit.ly/2IFeaqj (fr) of https://bit.ly/2IJUzpp (eng)

• France

- In France, legislation is detailed and explained by DGAC (Direction Générale de l'Aviation Civile)
- The new European legislation applies as of January 1, 2020
- The old French legislation is still applicable until July 31, 2020
- 5 class of UAS, no more distinction between professional and non professional usage of UAS
- A new "open class" of UAS gather professional and non professional usage of UAS below 25kg
- To learn more: https://cutt.ly/qg7TSy9







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EU LEGISLATION ON UAVS

Main dispositions

• Three type of operations:

- Open category: flight with direct view on the UAV, non dense geographic areas, low aerial traffic \rightarrow low risk for people and goods
- Specific category: flights not in the previous category (higher density, aerial traffic, etc.), without direct view on the UAV \rightarrow moderate risk for people and goods
- Certified category: risky operations (taxis, dangerous goods, etc.)

• UAV over 25kg

- Only in the Specific or Certified categories
- Immatriculation of the UAV and license plate (at least 10cm×5cm)
- Geofensing (prevent unauthorized areas)
- Each flight must have an authorization of the DSAC (direction de la sécurité de l'aviation civile)
- The pilot must have an appropriate "permit"
- Very similar to plane's legislation (flight plan, etc.)
- UAV under 25kg
 - Usually in the Open category (except for special cases)
 - 3 sub-categories of operations: A1, A2 and A3
 - 5 categories of UAVs: C0, C1, C2, C3 and C4



Sub-categories of operations

Robotics

N. Marchand	Sub- category	Distance to people	UAS	ldentif. geofensing	Pilot formation
Introduction		Isolated neonle on	CO (m < 250g)	no	 Read the manual given by the manufacturer
Outline	A1	the ground	DIY UAV (m < 250g)		 Formation and exam on Fox AlphaTango
Mechanics					recommended
Cartesian coordinates Orientation	A1	Close to people	C1 (m < 900g)	yes	 Read the manual given by the manufacturer
Newton					 Formation and exam on
UAV's model					mandatory
Recreational break	A2	Minimal distance to people: 30 <i>m</i> 5 <i>m</i> if low speed mode exists	C2 (m < 4kg)	yes	 Read the manual given by the manufacturer
Legislation					 Formation and exam on
EU legislation					Fox AlphaTango
Sub-categories of operations					mandatory
Categories of UAVs					 Mandatory autoformation (inline, declarative)
Introduction to control					 Mandatory theoretical avamentions the "browst
Main components					d'antitude de pilote à
Control loops					distance"
Autout		1	1	1	

control

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Sub-categories of operations

Robotics

N. Marchand	Sub- category	Distance to people	UAS, Identif. and geofensing	Pilot formation
Introduction			 DIY UAV (250g < m < 25kg): electronic identification if m > 800 g 	
Outline			111 > 000g	
Mechanics		Far from people	 C1 (m < 900g): electronic identification 	 Read the manual given by the manufacturer
Cartesian coordinates	Δ3	1. 150 6	• C^{2} $(m < 4ka)$: electronic	a Formation and even on
Orientation	/15	d > 150m from inhab- itants/workers/etc	identification	Fox AlphaTango
Frames			Identification	
Newton			 C3 (m < 25kg): electronic 	mandatory
UAV's model			identification	
Recreational			 C4 (m < 25kg): electronic identification if m > 800g 	

Legislation

EU legislation

Sub-categories of operations

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Main rules:

- No flight over people
- I always see my UAV (no night flight)
- I respect the maximum flight height, inhabited aircraft always have the priority
- . I can have an automatic piloting system if i can take back the control at anytime
- I must be static to pilot an UAV (not in a moving vehicle)
- Nothing must fall from my UAV, i can not transport dangerous goods
- https://fox-alphatango.aviation-civile.gouv.fr/

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Robotics

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Categories of UAVs

Robotics

N Marchand	Pictogramme	Nom de la	Exigences principales	
	d'identification	classe		
Introduction			 Masse maximum au décollage de 250 g 	
Introduction	\sim		 Vitesse maximum verticale (vol en palier) de 19 m/s 	
Outline		Classe C0	• En cas de suivi du sujet (Follow me) la distance maximum par	
Mechanics				
Cartesian coordinates			 Avoir une tension nominale ne dépassant pas 24 volts en continu 	
Orientation			 Masse maximum au décollage de 900 g 	
Frames			 Vitesse maximum verticale (vol en palier) de 19 m/s 	
UAV's model			 Comporter un numéro de série physique conforme à la norme ANSI/CTA-206 	
Recreational break			 Le drone doit permettre l'identification à distance en temps réel pendant toute la durée du vol 	
Legislation		Classa C1	• Etre équipé d'un système géovigilance permettant la limitation de	
EU legislation		Classe C1	l'espace aérien (position, altitude \Rightarrow Règlement (UE) 2019/947).	
Sub-categories of operations			 En cas de suivi du sujet (Follow me) la distance maximum par rapport au pilote devra être de 50 m 	
Categories of UAVs			- Eta équiné de ferre (menere melilité accentibilité)	
Introduction			• Etre equipe de reux (manoevrabilite, peceptibilite)	
to control			 Avoir une tensesion nominale ne dépassant pas 24 volts en continu 	
Main components			 Donner au pilote à distance un signal d'alerte clair lorsque la 	
Control loops			batterie du drone ou sa station de contrôle atteint un niveau bas	
Attitude				

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Robotics

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EU LEGISLATION FOR UAVS BELOW 25KG A focus on geofensing, class C1 and above

- Geofensing prevents the UAV from flying into certain areas
- Geofensing forces the UAV to stay in authorized areas
- Example: airports



- The flying zone are divided in zones
 - forbidden zone
 - or with restricted altitude
- Can be obtained : https://www.geoportail.gouv.fr/ donnees/restrictions-pour-drones-de-loisir



EU LEGISLATION FOR UAVS BELOW 25KG Typical flying zones

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EU LEGISLATION FOR UAVS BELOW 25KG

Typical flying zones

• Sensible infrastructures



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Robotics

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EU LEGISLATION FOR UAVS BELOW 25KG Typical flying zones

Robotics

Natural areas

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Control loops

Attitude control

Col du Louto Bourg-d'Oisans Col d'Ornon Saint-Christophe Monetier-les-Bains en-Oisans Barre des Écrins Briançon La Mure Entraigues AMont Pelvoux Vallouise L'Argentière-La Chapelle-en-Valgaudemar - la-Bessée A Vieux Chaillol Pont-du-Fossé • Orcières Saint-Bonnet Le Mourre Froid en-Champsaur Châteauroux-les-Alpes HAUTES-ALPES • Embrun Réserve intégrale du Lauvitel • GAP Cœur de parc Savines-le-Lac Aire optimale d'adhésion Réserves naturelles

SAVOIE , Coldu Galibie

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Robotics

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Categories of UAVs

Robotics

N. Marchand	Pictogramme	Nom de la	Exigences principales
	d'identification	classe	
Introduction			 Masse maximum au décollage de 4 Kg
Outline			 Etre équipé d'un mode à base vitesse sélectionnable par le pilote à distance et limitant la vitesse horizontale à 3 m/s maximum
Mechanics			• Avoir une tension nominale ne dépassant pas 48 volts en continu
Cartesian coordinates			 Comporter un numéro de série physique conforme à la norme
Orientation			ANSI/CTA-206
Frames			• Le drone doit permettre l'identification à distance en temps réel
Newton			pendant toute la durée du vol
UAV's model	(2)	Classe C2	• Donner au pilote à distance un signal d'alerte clair lorsque la
Recreational	\sim		batterie du drone ou sa station de contrôle atteint un niveau bas
break			Avoir un niveau de puissance acoustique LWA pondéré (*) apposée ave la dessa se (av sus can exclusive)
Legislation			sur le drone et/ou sur son emballage
EU legislation			 Etre équipé d'un système géovigilance permettant la limitation de
Sub-categories of			l'espace aérien (position, altitude \Rightarrow Règlement (UE) 2019/947).
Categories of UAVs			 En cas de suivi du sujet (Follow me) la distance maximum par rapport au pilote devra être de 50 m
Introduction			 Etre équipé de feux (manoevrabilité, peceptibilité)
to control		1	

Control loops

Attitude control

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Robotics

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Categories of UAVs

Robotics

N. Marchand	Pictogramme	Nom de la	Exigences principales	
N. Marchanu	d'identification	classe		
Introduction			 Masse maximum au décollage de 25 Kg 	
Outline			 Avoir un niveau de puissance acoustique LWA pondéré apposée sur le drone et/ou sur son emballage 	
Mechanics			• Avoir une tension nominale ne dépassant pas 48 volts en continu	
Cartesian coordinates			• Etre équipé d'un système géovigilance permettant la limitation de	
Orientation	\frown		l'espace aérien (position, altitude \Rightarrow Réglement d'éxécution (UE)	
Frames	3	Classe C3	2019/947).	
Newton UAV's model			 Le drone doit permettre l'identification à distance en temps réel pendant toute la durée du vol 	
Recreational break			 Donner au pilote à distance un signal d'alerte clair lorsque la batterie du drone ou sa station de contrôle atteint un niveau bas 	
Legislation			 Comporter un numéro de série physique conforme à la norme ANSI/CTA-206 	
EU legislation			ANSI/ C1A-200	
Sub-categories of operations		Classe CA	 Masse maximum au décollage de 25 Kg 	
Categories of UAVs		Classe C+	 Ne pas être doté de modes de contrôle automatique 	

Main components

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- Outline
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- Cartesian coordinate Orientation
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- Recreational break
- Legislation
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- Sub-categories of operations
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- Main components Control loops
- Attitude control

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 - Mechanics basis of rigid bodies
 - UAV's model
 - Recreational break
 - Legislation aspects
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MAIN COMPONENTS OF UAS

• Ground: UAV base station, remote control + ground PC • UAV:

Frame

Motor (brushless most of the time)

ESC (Electronic Speed Controller)

Flight controller card



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First Person View camera: fixed or pitch compensated

Camera

• UAV:

Blades

Battery

Image/IA card



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• Actuation: Local inner loop, dynamics can be neglected. Handled by the ESC

MAIN CONTROL LOOPS IN A UAV

• Body:

$$\begin{cases} \dot{\vec{p}} = \vec{v} \\ m\vec{v} = -mg\vec{e}_3 + R\vec{T} \\ \dot{\vec{R}} = R\vec{\omega}^{\times} \\ J\dot{\vec{\omega}} = -\vec{\omega}^{\times}J\vec{\omega} + \vec{\Gamma}_c \end{cases}$$

• **Control**:
$$\vec{T}$$
 and $\vec{\Gamma}_c$









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MAIN POSSIBLE CONTROL LOOPS IN A UAV





MAIN POSSIBLE CONTROL LOOPS IN A UAV



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• The dynamics of faster loops are neglected by slower ones

- Autonomous UAVs: control of \vec{p}
- Remote pilots:
 - control of R for beginners
 - control of $\vec{\omega}$ for advanced pilots



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Some dynamical system modeling basis

• Linear system:

$$\dot{x} = Ax + Bu$$

• General nonlinear system:

$$\dot{x} = f(x, u)$$

• Affine in the control system:

$$\dot{x} = f(x) + g(x)u$$

- *x* denotes the state of the system: how the system is
- *u* is the control variable of the system: how to *move* the system
- Every nonlinear system can be locally approximated by its linear approximation

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• Generalized notion of energy: Lyapunov function

Some basis of control: Lyapunov functions

• A Lyapunov function $V : \mathbb{R}^n \to \mathbb{R}^+$ is such that:

•
$$V(x) > 0$$
 for all $x \neq 0$

•
$$V(0) = 0$$

- Stability and Lyapunov functions:
 - A system will converge to an equilibrium if it can only loose energy
 - or equivalently if $\dot{V}(x) < 0$ for all $x \neq 0$
- Therefore the aim will be more or less to find u(x) such that

$$\dot{V} = \frac{\partial V(x)}{\partial x} \dot{x} < 0$$

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Angular velocity control

$$\begin{cases} \dot{R} = R\vec{\omega}^{\times} \\ J\vec{\omega} = -\vec{\omega}^{\times}J\vec{\omega} + \vec{\Gamma}_{c} \end{cases}$$

• $\vec{\Gamma}_c$ as control variable

• Linear approximation

$$\frac{\dot{\vec{\omega}}}{\vec{\omega} - \vec{\omega}_{SS}} = J^{-1} \vec{\Gamma}_c \quad \text{close to steady state } \vec{\omega}_{SS} = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix}^T$$
$$\frac{\dot{\vec{\omega}}}{\vec{\omega} - \vec{\omega}_{SS}} = J^{-1} \begin{bmatrix} -\vec{\omega}_{SS}^{\times} J(\vec{\omega} - \vec{\omega}_{SS}) + \vec{\Gamma}_c \end{bmatrix} \quad \text{close to } \vec{\omega}_{SS}$$

• Take
$$u := \vec{\Gamma}_c$$
 and $x := \vec{\omega} - \vec{\omega}_{SS}$

• Then one has
$$\dot{x} = A(\omega_{SS})x + Bu$$
 $(B = J^{-1})$

- Linear system
- Easy PID, optimal control
- Nonlinear control

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- $\vec{\Gamma}_c$ as control variable
- Linear approximation
- Nonlinear control
 - Take $V = \vec{\omega}^T J \vec{\omega}$ as Lyapunov function
 - its time derivative gives:

$$\begin{split} \dot{V} &= \vec{\omega}^T J \dot{\vec{\omega}} + \dot{\vec{\omega}}^T J \vec{\omega} \\ &= \vec{\omega}^T [-\vec{\omega}^{\times} J \vec{\omega} + \vec{\Gamma}_c] + [-\vec{\omega}^{\times} J \vec{\omega} + \vec{\Gamma}_c]^T \vec{\omega} \\ &= 2\vec{\omega}^T \vec{\Gamma}_c \end{split}$$

- Any control of the form $\vec{\Gamma}_c = -k_P \vec{\omega}$ with $k_P > 0$:
 - is such that $\dot{V} < 0$ for $\vec{\omega} \neq 0$
 - $\bullet~$ stabilizes $\vec{\omega}~$ to zero
- A P-controller stabilizes the angular velocity

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- $\vec{\Gamma}_c$ as control variable
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- Nonlinear control
 - Take $V = \vec{e}^T J \vec{e}$ with $\vec{e} = (\vec{\omega} \vec{\omega}_{SS})$ as Lyapunov function
 - its time derivative gives:

$$\dot{V} = 2\vec{e}^T J \dot{\vec{\omega}} = 2\vec{e}^T \vec{\Gamma}_c$$

- Any control of the form $\vec{\Gamma}_c = -k_P \vec{e}$ with $k_P > 0$:
 - is such that $\dot{V} < 0$ for $\vec{e} \neq 0$
 - stabilizes $\vec{\omega}$ to $\vec{\omega}_{SS}$
- A P-controller stabilizes the angular velocity

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- $\vec{\Gamma}_c$ as control variable
- Linear approximation
- Nonlinear control
 - Take $V = \vec{\omega}^T J \vec{\omega} + \vec{\Omega}^T Q \vec{\Omega}$ as Lyapunov function with • $\vec{\Omega} = \int \vec{\omega}$ and Q > 0
 - its time derivative gives:

$$\dot{V} = 2\vec{\omega}^T J \dot{\vec{\omega}} + 2\vec{\omega}^T Q \vec{\Omega} = 2\vec{\omega}^T [\vec{\Gamma}_c + Q \vec{\Omega}]$$

- Any control of the form $ec{\Gamma}_c = -k_Pec{\omega} Qec{\Omega}$ with $k_P > 0$
 - is such that $\dot{V} < 0$ for $\vec{\omega} \neq 0$
 - stabilizes $\vec{\omega}$ and $\vec{\Omega}$ to zero
- A PI-like controller stabilizes the angular velocity

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$$\dot{R} = R\vec{\omega}^{\times}$$
$$J\vec{\omega} = -\vec{\omega}^{\times}J\vec{\omega} + \vec{\Gamma}_{c}$$

- $\vec{\Gamma}_c$ as control variable
- Linear approximation
- Nonlinear control
 - Take $V = \vec{\omega}^T J \vec{\omega} + \vec{\Omega}^T Q \vec{\Omega}$ as • $\vec{\Omega} = \int \vec{\omega}$ and Q > 0
 - its time derivative gives:

LaSalle's Invariance principle

- V: a Lyapunov function
- $\dot{V} \leq 0$ everywhere

•
$$S := \{x \text{ s.t. } V(x) = 0\}$$

- If \mathcal{I} is the largest invariant set in \mathcal{S}
- $\bullet~$ Then every trajectory converge to ${\cal I}$

$$\begin{aligned} & \ell &= 2\vec{\omega}^T J \dot{\vec{\omega}} + 2\vec{\omega}^T Q \vec{\Omega} \\ & = 2\vec{\omega}^T [\vec{\Gamma}_c + Q \vec{\Omega}] \end{aligned}$$

• Any control of the form $\vec{\Gamma}_c = -k_P \vec{\omega} - Q \vec{\Omega}$ with $k_P > 0$ • is such that $\dot{V} < 0$ for $\vec{\omega} \neq 0$: $S = \left\{ (\vec{\omega}, \vec{\Omega}) \text{ s.t. } \vec{\omega} = 0 \right\}$

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- stabilizes $\vec{\omega}$ and $\vec{\Omega}$ to zero: $\mathcal{I} = \left\{ (\vec{\omega}, \vec{\Omega}) \text{ s.t. } \vec{\omega} = \vec{\Omega} = \vec{0} \right\}$
- A PI-like controller stabilizes the angular velocity

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- $\vec{\Gamma}_c$ as control variable
- Linear approximation
- Nonlinear control
 - Take $V = \vec{e}^T J \vec{e} + \vec{E}^T Q \vec{E}$ as Lyapunov function with

•
$$\vec{e} = \vec{\omega} - \vec{\omega}_{SS}$$

• $\vec{E} = \int \vec{e}$ and $Q > 0$

• its time derivative gives:

$$\dot{V} = 2\vec{e}^T J \vec{\omega} + 2\vec{e}^T Q \vec{E} = 2\vec{e}^T [\Gamma_c + Q \vec{E}]$$

- Any control of the form $ec{\Gamma}_c = k_P ec{e} Q ec{E}$ with $k_P > 0$
 - is such that $\dot{V} < 0$ for $\vec{e} \neq 0$
 - stabilizes $\vec{\omega}$ and $\vec{\Omega}$ to zero
- A PI-like controller stabilizes the angular velocity

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- $\vec{\Gamma}_c$ as control variable
- Linear approximation
- Nonlinear control

• Take
$$V = \vec{e}^T J \vec{e} + \vec{E}^T Q \vec{E}$$
 as L
• $\vec{e} = \vec{\omega} - \vec{\omega}_{SS}$

$$\vec{V} = 2\vec{e}^T J \vec{\omega} + 2\vec{e}^T Q \vec{E}$$
$$= 2\vec{e}^T [\Gamma_c + Q\vec{E}]$$

- Any control of the form $\vec{\Gamma}_c = -k_P \vec{e} Q\vec{E}$ with $k_P > 0$ • is such that $\dot{V} < 0$ for $\vec{e} \neq 0$: $S = \left\{ (\vec{e}, \vec{E}) \text{ s.t. } \vec{e} = 0 \right\}$
 - stabilizes $\vec{\omega}$ and $\vec{\Omega}$ to zero: $\mathcal{I} = \left\{ (\vec{e}, \vec{E}) \text{ s.t. } \vec{e} = \vec{E} = 0 \right\}$
- A PI-like controller stabilizes the angular velocity

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- Then every trajectory converge to I

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$$\begin{array}{rcl} \dot{R} & = & R\vec{\omega}^{\times} \\ J\dot{\vec{\omega}} & = & -\vec{\omega}^{\times}J\vec{\omega} + \vec{\Gamma}_{c} \end{array}$$

$$\begin{cases} \dot{q} = \frac{1}{2} \begin{pmatrix} -\vec{q}^T \\ f_{3\times 3}q_0 + \vec{q}^{\times} \end{pmatrix} \vec{\omega} \qquad \vec{\omega} = W^{-1} \begin{pmatrix} \phi \\ \dot{\theta} \\ \dot{J} \\ \vec{\omega} = -\vec{\omega}^{\times} J \\ \vec{\omega} + \vec{\Gamma}_c \\ q^T q = q_0^2 + \vec{q}^T \vec{q} = 1 \end{cases}$$

- $\vec{\Gamma}_c$ as control variable
- Linear approximation
 - Close to $\phi = \theta = \psi = 0$, $W \approx I$
 - Therefore for small angles and angular velocities, the attitude behaves like three double integrators on each roll, pitch and yaw axis

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- Easy PID, optimal control, etc.
- Nonlinear control



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Attitude control $\left\{ \begin{array}{rcl} \dot{R} &=& R\vec{\omega}^{\times} \\ J\dot{\vec{\omega}} &=& -\vec{\omega}^{\times}J\vec{\omega}+\vec{\Gamma}_{c} \end{array} \right.$

$$\begin{cases} \dot{q} = \frac{1}{2} \begin{pmatrix} -\vec{q}^T \\ l_{3\times 3}q_0 + \vec{q}^\times \end{pmatrix} \vec{\omega} & \vec{\omega} = W^{-1} \begin{pmatrix} \phi \\ \dot{\theta} \\ j \vdots \\ J \vdots \\ q^T q = q_0^2 + \vec{q}^T \vec{q} = 1 \end{cases}$$

- $\vec{\Gamma}_c$ as control variable
- Linear approximation
- Nonlinear control
 - Take $V = \frac{1}{2}\vec{\omega}^T J\vec{\omega} + k((1-q_0)^2 + \vec{q}^T \vec{q})$ as Lyapunov function • Note that $V = \frac{1}{2}\vec{\omega}^T J\vec{\omega} + 2k(1-q_0)$
 - its time derivative gives:

$$\dot{V} = \vec{\omega}^T J \dot{\vec{\omega}} - 2k \dot{q}_0 = \vec{\omega}^T \vec{\Gamma}_c + k \vec{q}^T \vec{\omega} = \vec{\omega}^T \left[\vec{\Gamma}_c + k \vec{q} \right]$$

- Any control of the form $\vec{\Gamma}_c = -k_P \vec{\omega} k \vec{q}$ with $k_P > 0$:
 - is such that $\dot{V} < 0$ for $\vec{\omega} \neq 0$
 - stabilizes $\vec{\omega}$ and \vec{q} to zero
- A PI-like controller stabilizes the attitude

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Attitude control

 $\begin{cases} \dot{R} = R\vec{\omega}^{\times} \\ J\dot{\vec{\omega}} = -\vec{\omega}^{\times}J\vec{\omega} + \vec{\Gamma}_{c} \end{cases}$

$$\begin{cases} \dot{q} = \frac{1}{2} \begin{pmatrix} -\vec{q}^T \\ l_{3\times 3q_0} + \vec{q}^\times \end{pmatrix} \vec{\omega} & \vec{\omega} = W^{-1} \begin{pmatrix} \phi \\ \dot{\theta} \\ \dot{J} \vdots & = -\vec{\omega}^\times J \vec{\omega} + \vec{\Gamma}_c \\ q^T q = q_0^2 + \vec{q}^T \vec{q} = 1 \end{cases}$$

- $\vec{\Gamma}_c$ as control variable
- Linear approximation
- Nonlinear control

• Take
$$V = \frac{1}{2} \vec{\omega}^T J \vec{\omega} + k((1 - q_0))$$

- Note that $V = \frac{1}{2}\vec{\omega}^T J\vec{\omega} + 2k(1)$
- its time derivative gives:

LaSalle's Invariance principle

• V: a Lyapunov function

$$V \leq 0$$
 everywhere

•
$$S := \{x \text{ s.t. } V(x) = 0\}$$

• If \mathcal{I} is the largest invariant set in \mathcal{S}

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Then every trajectory converge to I

$$= \vec{\omega}^T J \vec{\omega} - 2k \dot{q}_0$$
$$= \vec{\omega}^T \vec{\Gamma}_c + k \vec{q}^T \vec{\omega} = \vec{\omega}^T \left[\vec{\Gamma}_c + k \vec{q} \right]$$

Any control of the form Γ_c = -k_P ω - kq with k_P > 0:
is such that V < 0 for ω ≠ 0: S = {(ω, q) s.t. ω = 0}
stabilizes ω and q to zero : I = {(ω, q) s.t. ω = q = 0}

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• A PI-like controller stabilizes the attitude

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$$\begin{array}{rcl} \dot{R} &=& R\vec{\omega}^{\times} \\ J\vec{\omega} &=& -\vec{\omega}^{\times}J\vec{\omega} + \vec{\Gamma}_{c} \\ \vec{\omega} &=& -\vec{\omega}^{\times}J\vec{\omega} + \vec{\Gamma}_{c} \\ J\vec{\omega} &=& -\vec{\omega}^{\times}J\vec{\omega} + \vec{\Gamma}_{c} \\ q^{T}q &= q_{0}^{2} + \vec{q}^{T}\vec{q} = 1 \end{array} \qquad \qquad \vec{\omega} = W^{-1} \begin{pmatrix} \phi \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix}$$

- $\vec{\Gamma}_c$ as control variable
- We want a control such that $\Gamma_{c_{1,2,3}} \in [-\overline{\Gamma}_{c_{1,2,3}}, \overline{\Gamma}_{c_{1,2,3}}]$
- Nonlinear control
 - Take $V = \frac{1}{2}\vec{\omega}^T J \vec{\omega} + 2k(1-q_0)$ as Lyapunov function
 - its time derivative gives:

$$\begin{split} \dot{V} &= \vec{\omega}^{T} \left[\vec{\Gamma}_{c} + k\vec{q} \right] \\ &= \omega_{1} \Gamma_{c_{1}} + kq_{1}\omega_{1} + \omega_{2} \Gamma_{c_{2}} + kq_{2}\omega_{2} + \omega_{3} \Gamma_{c_{3}} + kq_{3}\omega_{3} \\ &= \dot{V}_{1} + \dot{V}_{2} + \dot{V}_{3} \end{split}$$

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Full bounded attitude control

 $\begin{cases} \dot{R} = R\vec{\omega}^{\times} \\ J\dot{\vec{\omega}} = -\vec{\omega}^{\times}J\vec{\omega} + \vec{\Gamma}_{c} \\ J\dot{\vec{\omega}} = -\vec{\omega}^{\times}J\vec{\omega} + \vec{\sigma}_{c} \end{cases} \qquad \begin{cases} \dot{q} = \frac{1}{2} \begin{pmatrix} -\vec{q}^{T} \\ J_{3\times3}q_{0} + \vec{q}^{\times} \end{pmatrix} \vec{\omega} & \vec{\omega} = W^{-1} \begin{pmatrix} \phi \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix}$

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• $\vec{\Gamma}_c$ as control variable • We want a control such that $\Gamma_{c_{1,2,3}} \in [-\vec{\Gamma}_{c_{1,2,3}}, \vec{\Gamma}_{c_{1,2,3}}]$ • Nonlinear control • Take $V = \frac{1}{2}\vec{\omega}^T J \vec{\omega} + 2k(1 - q_0)$ as Lyapunov function

• its time derivative gives:

$$\begin{split} \dot{V} &= \vec{\omega}^{T} \left[\vec{\Gamma}_{c} + k\vec{q} \right] \\ &= \omega_{1} \Gamma_{c_{1}} + kq_{1}\omega_{1} + \omega_{2} \Gamma_{c_{2}} + kq_{2}\omega_{2} + \omega_{3} \Gamma_{c_{3}} + kq_{3}\omega_{3} \\ &= \dot{V}_{1} + \dot{V}_{2} + \dot{V}_{3} \end{split}$$

- Any control of the form $\Gamma_{c_i} = -\operatorname{sat}_{\overline{\Gamma}_{c_i}} (k_P \omega_i + kq_i)$ with $k_P > 0$:
 - is such that $\dot{V}_i < 0$ for $\omega_i \notin \mathcal{J} := \left[-\frac{k}{k_P}, \frac{k}{k_P} \right] \Rightarrow \omega_i \to \mathcal{J}$
 - $\omega_i \in \mathcal{J} \Rightarrow k_P \omega_i \in [-k, k] \Rightarrow k_P \omega_i + kq_i \in [-2k, 2k] \xrightarrow{\text{if } 2k \leq \overline{\Gamma}_{c_i}} k_P \omega_i + kq_i \in [-\overline{\Gamma}_{c_i}, \overline{\Gamma}_{c_i}]$
 - The control is not saturated: $\Gamma_{c_i} = -(k_P \omega_i + kq_i)$

•
$$V = -k_P \vec{\omega}' \vec{\omega} < 0$$
 for $\omega \neq 0$

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FULL BOUNDED ATTITUDE CONTROL

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Attitude control

 $\begin{cases} \dot{R} = R\vec{\omega}^{\times} \\ j\dot{\vec{\omega}} = -\vec{\omega}^{\times}J\vec{\omega} + \vec{\Gamma}_{c} \\ j\dot{\vec{\omega}} = -\vec{\omega}^{\times}J\vec{\omega} + \vec{\Gamma}_{c} \end{cases} \qquad \begin{cases} \dot{q} = \frac{1}{2} \begin{pmatrix} -\vec{q}^{T} \\ j_{3\times3}q_{0} + \vec{q}^{\times} \end{pmatrix} \vec{\omega} & \vec{\omega} = W^{-1} \begin{pmatrix} \phi \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} \\ j\dot{\vec{\omega}} = -\vec{\omega}^{\times}J\vec{\omega} + \vec{\Gamma}_{c} \\ q^{T}q = q_{c}^{2} + \vec{q}^{T}\vec{q} = 1 \end{cases}$

- $\vec{\Gamma}_{c}$ as control variable
- Nonlinear control

• Take
$$V = \frac{1}{2}\vec{\omega}^T J\vec{\omega} + 2k(1-q_0)$$

its time derivative gives:

$$\dot{V} = \vec{\omega}^T \begin{bmatrix} \vec{\Gamma}_c \\ = \omega_1 \Gamma_{c_1} + \\ = \dot{V}_1 + \dot{V}_2 + V_3 \end{bmatrix}$$

• We want a control such that $\Gamma_{c_{1,2}}$ LaSalle's Invariance principle

V: a Lyapunov function

$$V \leq 0$$
 everywhere

•
$$S := \{x \text{ s.t. } V(x) = 0\}$$

- If \mathcal{I} is the largest invariant set in \mathcal{S}
- Then every trajectory converge to \mathcal{I}

• Any control of the form
$$\Gamma_{c_i} = -\operatorname{sat}_{\overline{\Gamma}_{c_i}} (k_P \omega_i + kq_i)$$
 with $k_P > 0$:

• is such that
$$\dot{V}_i < 0$$
 for $\omega_i \notin \mathcal{J} := \left[-\frac{k}{k_P}, \frac{k}{k_P} \right] \Rightarrow \omega_i \to \mathcal{J}$

$$\omega_i \in \mathcal{J} \Rightarrow k_P \omega_i \in [-k,k] \Rightarrow k_P \omega_i + kq_i \in [-2k,2k] \xrightarrow{\text{if } 2k \leq \overline{\Gamma}_{c_i}} k_P \omega_i + kq_i \in [-\overline{\Gamma}_{c_i},\overline{\Gamma}_{c_i}]$$

• The control is not saturated:
$$\Gamma_{c_i} = -(k_P \omega_i + kq_i)$$

•
$$V = -k_P \vec{\omega}' \vec{\omega} < 0$$
 for $\omega \neq 0$

- stabilizes $\vec{\omega}$ and \vec{q} to zero
- A saturated PI-like controller stabilizes the attitude

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SPEED/POSITION CONTROL



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Speed control

- We assume that the attitude loop is infinitely fast: $R \to R_d$ (subscript d stands for "desired")
- The model becomes:

$$\begin{cases} m\dot{\vec{v}} = -mg\vec{e_3} + R_d\vec{T} \end{cases}$$

- R_d is a control variable (as T)
- Detailing the equations:

$$\begin{cases} \dot{v}_x = -\left[\cos\varphi_d \sin\theta_d \cos\psi_d + \sin\varphi_d \sin\psi_d\right] T/m := u_x \\ \dot{v}_y = -\left[\cos\varphi_d \sin\theta_d \sin\psi_d - \sin\varphi_d \cos\psi_d\right] T/m := u_y \\ \dot{v}_z - g + \left[\cos\varphi_d \cos\theta_d\right] T/m := u_z \end{cases}$$

• Assume i could choose (u_x, u_y, u_z) the way i would like :

ſ	u_x	=	$-k_x(v_x-v_x^d)$	ſ	$v_x \rightarrow v_x^d$
ł	u_y	=	$-k_y(v_y-v_y^d)$	⇔{	$v_y \rightarrow v_y^d$
l	u_z	=	$-k_z(v_z - v_z^d)$	l	$v_z \rightarrow v_z^d$

with k > 0• Take

 $\begin{array}{ll} \alpha := \sin \varphi_d & \Rightarrow & \cos \varphi_d = \pm \sqrt{1 - \alpha^2} \\ \beta := \sin \theta_d & \Rightarrow & \cos \theta_d = \pm \sqrt{1 - \beta^2} \end{array}$

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$\begin{array}{lll} u_{x} & := & \pm \sqrt{1 - \alpha^{2}} \beta T/m \\ u_{y} & := & \alpha T/m \\ u_{z} & := & \left(\pm \sqrt{1 - \alpha^{2}} \cdot \mp \sqrt{1 - \beta^{2}} \cdot T/m \right) - g \end{array}$

 $\alpha := \sin \varphi_d \quad \Rightarrow \quad \cos \varphi_d = \pm \sqrt{1 - \alpha^2}$

 $\beta := \sin \theta_d \Rightarrow \cos \theta_d = \pm \sqrt{1 - \beta^2}$

$$\beta := \pm \left[\left(\frac{g + u_z}{u_x} \right)^2 + 1 \right]^{\frac{1}{2}}$$
$$T = \pm m \sqrt{\frac{u_x^2}{\beta^2} + u_y^2}$$
$$\alpha = u_y \cdot \frac{m}{T}$$

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• If $u_x \neq 0$:

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Speed control

$$\begin{split} \beta &:= & \pm \left[\left(\frac{g + u_z}{u_x} \right)^2 + 1 \right]^{\frac{1}{2}} \\ T &= & \pm m \sqrt{\frac{u_x^2}{\beta^2} + u_y^2} \\ \alpha &= & u_y \cdot \frac{m}{T} \end{split}$$

• T is always positive therefore:

$$T = \pm m \sqrt{\frac{u_x^2}{\beta^2} + u_y^2}$$

•
$$\alpha$$
 is also unique and $\varphi_d = \arcsin \alpha \in [-\pi/2, \pi/2]$
• $\theta_d = \arcsin \beta \in [-\pi/2, \pi/2]$ of opposite sign of u_x

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POSITION CONTROL

- We assume that the attitude loop is *infinitely* fast: $R \rightarrow R_d$ (subscript d stands for "desired")
- The model becomes:

$$\dot{\vec{p}} = \vec{v}$$
$$m\vec{v} = -mg\vec{e}_3 + R_d\vec{T}$$

- R_d is a control variable (as T)
- Detailing the equations:

$$\begin{cases} \ddot{x} = -\left[\cos\varphi_d\sin\theta_d\cos\psi_d + \sin\varphi_d\sin\psi_d\right]T/m := u_x \\ \ddot{y} = -\left[\cos\varphi_d\sin\theta_d\sin\psi_d - \sin\varphi_d\cos\psi_d\right]T/m := u_y \\ \ddot{z} - -g + \left[\cos\varphi_d\cos\theta_d\right]T/m := u_z \end{cases}$$

• Assume i could choose (u_x, u_y, u_z) the way i would like :

ſ	<i>u_x</i>	=	$k_{\dot{x}}v_x + k_x(x-x^d)$	ſ	$x \to x^d$
ł	u _y	=	$k_{\dot{y}}v_y + k_y(y - y^d)$	⇔{	$y \rightarrow y^d$
l	u_z	=	$k_{\dot{z}}v_z + k_z(z-z^d)$	l	$z \rightarrow z^d$

with an appropriate choice of the controller parameters Take

$$\begin{array}{ll} \alpha := \sin \varphi_d & \Rightarrow & \cos \varphi_d = \pm \sqrt{1 - \alpha^2} \\ \beta := \sin \theta_d & \Rightarrow & \cos \theta_d = \pm \sqrt{1 - \beta^2} \end{array}$$

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 $\begin{array}{rcl} \alpha := \sin \varphi_d & \Rightarrow & \cos \varphi_d = \pm \sqrt{1 - \alpha^2} \\ \beta := \sin \theta_d & \Rightarrow & \cos \theta_d = \pm \sqrt{1 - \beta^2} \end{array}$

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$\begin{array}{lll} u_{x} & := & \pm \sqrt{1 - \alpha^{2} \beta T / m} \\ u_{y} & := & \alpha T / m \\ u_{z} & := & \left(\pm \sqrt{1 - \alpha^{2}} \cdot \mp \sqrt{1 - \beta^{2}} \cdot T / m \right) - g \end{array}$

$$\beta := \pm \left[\left(\frac{g + u_z}{u_x} \right)^2 + 1 \right]^{\frac{1}{2}}$$
$$T = \pm m \sqrt{\frac{u_x^2}{\beta^2} + u_y^2}$$
$$\alpha = u_y \cdot \frac{m}{T}$$

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$\beta := \pm \left[\left(\frac{g + u_z}{u_x} \right)^2 + 1 \right]^{\frac{1}{2}}$ $T = \pm m \sqrt{\frac{u_x^2}{\beta^2} + u_y^2}$ $\alpha = u_y \cdot \frac{m}{T}$

• T is always positive therefore:

• If $u_x \neq 0$:

$$T = \pm m \sqrt{\frac{u_x^2}{\beta^2} + u_y^2}$$

•
$$\alpha$$
 is also unique and $\varphi_d = \arcsin \alpha \in [-\pi/2, \pi/2]$
• $\theta_d = \arcsin \beta \in [-\pi/2, \pi/2]$ of opposite sign of u_x

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TRAJECTORY TRACKING

- We assume that the attitude loop is infinitely fast: $R \rightarrow R_d$ (subscript d stands for "desired")
- The model becomes:

- R_d is a control variable (as T)
- Detailing the equations:

$$\begin{cases} \ddot{x} = -\left[\cos\varphi_d \sin\theta_d \cos\psi_d + \sin\varphi_d \sin\psi_d\right]\right] T/m := u_x \\ \ddot{y} = -\left[\cos\varphi_d \sin\theta_d \sin\psi_d - \sin\varphi_d \cos\psi_d\right]\right] T/m := u_y \\ \ddot{z} - -g + \left[\cos\varphi_d \cos\theta_d\right] T/m := u_z \end{cases}$$

- Given a trajectory to track: $(x^d(t),y^d(t),z^d(t))$ and its time derivative $(v^d_x(t),v^d_y(t),v^d_z(t))$
- Assume i could choose (ux, uy, uz) the way i would like :

$$\begin{cases} u_x &= k_{\dot{x}}(v_x - v_x^d) + k_x(x - x^d) \\ u_y &= k_{\dot{y}}(v_y - v_y^d) + k_y(y - y^d) \\ u_z &= k_{\dot{z}}(v_z - v_z^d) + k_z(z - z^d) \end{cases} \Leftrightarrow \begin{cases} x \to x^d(t) \\ y \to y^d(t) \\ z \to z^d(t) \end{cases}$$

with an appropriate choice of the controller parameters Take

$$\begin{array}{ll} \alpha := \sin \varphi_d & \Rightarrow & \cos \varphi_d = \pm \sqrt{1 - \alpha^2} \\ \beta := \sin \theta_d & \Rightarrow & \cos \theta_d = \pm \sqrt{1 - \beta^2} \end{array}$$

N. Marchand (gipsa-lab)

Robotics

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POSITION CONTROL

Robotics

N. Marchand

Introduction

Outline

It gives:

Take

Mechanics

- Cartesian coordinat
- Orientatio
- Frames
- Newton

UAV's model

Recreational break

• If $u_x \neq 0$:

Legislation

EU legislation

Sub-categories of operations

Categories of UAVs

Introduction to control

Main components

Control loops

Attitude control

Resition Marchand (gipsa-lab)

$\begin{array}{lll} u_{x} & := & \pm \sqrt{1 - \alpha^{2} \beta T / m} \\ u_{y} & := & \alpha T / m \\ u_{z} & := & \left(\pm \sqrt{1 - \alpha^{2}} \cdot \mp \sqrt{1 - \beta^{2}} \cdot T / m \right) - g \end{array}$

 $\alpha := \sin \varphi_d \quad \Rightarrow \quad \cos \varphi_d = \pm \sqrt{1 - \alpha^2}$

 $\beta := \sin \theta_d \Rightarrow \cos \theta_d = \pm \sqrt{1 - \beta^2}$

$$\beta := \pm \left[\left(\frac{g + u_z}{u_x} \right)^2 + 1 \right]^{\frac{1}{2}}$$
$$T = \pm m \sqrt{\frac{u_x^2}{\beta^2} + u_y^2}$$
$$\alpha = u_y \cdot \frac{m}{T}$$

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$\beta := \pm \left[\left(\frac{g + u_z}{u_x} \right)^2 + 1 \right]^{\frac{1}{2}}$ $T = \pm m \sqrt{\frac{u_x^2}{\beta^2} + u_y^2}$ m

$$\alpha = u_y \cdot \frac{m}{T}$$

• If $u_x \neq 0$:

$$T = \pm m \sqrt{\frac{u_x^2}{\beta^2} + u_y^2}$$

•
$$\alpha$$
 is also unique and $\varphi_d = \arcsin \alpha \in [-\pi/2, \pi/2]$
• $\theta_d = \arcsin \beta \in [-\pi/2, \pi/2]$ of opposite sign of u_x

Robotics

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