Coverage control via gossip communication

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joint work with F. Bullo (UC Santa Barbara), R. Carli (University of Padova) and J.W. Durham (Kiva Systems)

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Coordination in multi-agent systems

What kind of systems?

- each agent **senses** its immediate environment,
- **communicates** with others,
- **processes** information gathered, and
- **takes local action** in response

What kind of tasks?

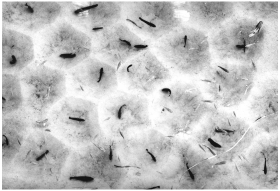
1. coordinated motion: rendezvous, flocking, formation
2. cooperative sensing: surveillance, exploration, search and rescue
3. cooperative material handling and transportation
4. cooperative deployment and coverage control

What is coverage control?

Distributed, dynamical deployment of robots/agents in some environment, optimizing the resulting "coverage"

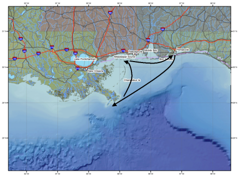
- Robots have limited mobility, communication, sensing, computation, memory capacities.
- Optimality is defined by a cost or performance function.

Applications in Engineering and Biology



Animal territories

Warehouse automation



Environmental monitoring

Coordination via task and territory partitioning

Model: customers appear randomly in space/time
robotic network knows locations and provides service

Goal: minimize customer delay

Approach: assign customers to robots by partioning the space

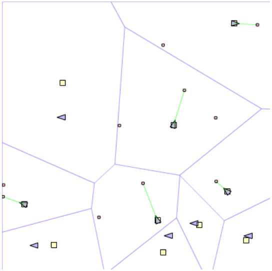
Expected wait time

$$H(P,x) = \int_{P_1} \|y - x_1\| dy \; + \; \cdots \; + \; \int_{P_n} \|y - x_n\| dy$$

- n robots at $x = \{x_1, \ldots, x_n\}$
- environment is partitioned into Voronoi regions $P = \{P_1, \ldots, P_n\}$

Breakthroughs & Limitations

Available algorithms can provide effective task servicing



F. Bullo, E. Frazzoli, M. Pavone, K. Savla, and S. L. Smith. Dynamic vehicle routing for robotic systems. *IEEE Proceedings*, May 2011. To appear

We want to remove restrictive assumptions:

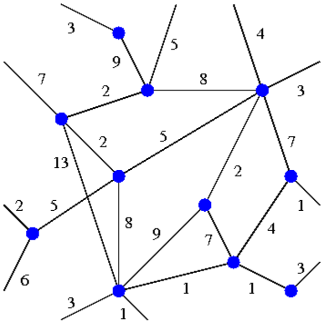
- convex environment
- synchronous reliable communication between neighbor robots

Outline

Graph coverage

In this work:

- the environment is a weighted graph.
- the robots sit on nodes and hop across edges:
  the weight is the time it takes for the move.
- optimality is minimizing a coverage cost:
  the expected response time to events happening at the nodes.



The environment (the graph) is divided into regions (node subsets):
each region is assigned to one robot, which is responsible for the covering.

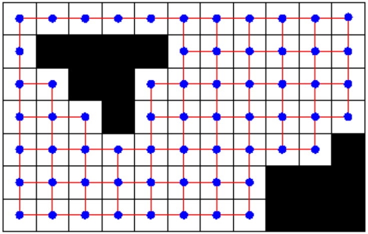
Motivations & models

Why a graph?

- Network environments (e.g. road/water/electricity networks)
- Discretization of continuous environments (e.g. a room)
  - by finding points of interest; or
  - by an occupancy grid → occupancy graph

Occupancy graph

- Free cells (where no obstacle) are vertices
- Adjacent free cells are neighbors
- All edge weights are grid resolution



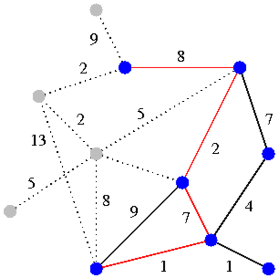
Formal problem statement

- a weighted graph $G = (X, E, A)$
- a set of robots moving on the graph, indexed in $I = \{1, \dots, n\}$

Goal: minimize the multi-center cost

$$H_{\mathrm{mc}}(P, \mathbf{x}) = \sum_{i \in I} \sum_{y \in P_i} d_{P_i}(x_i, y)\phi(y)$$

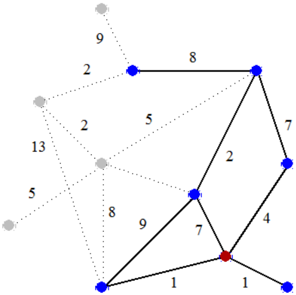
- $P = (P_i)_{i \in I}$ is any partition of X in regions $P_i \subset X$
- $\mathbf{x} \in X^I$ is the vector of robot locations
- $d_{P_i}(\cdot, \cdot)$ is a region-restricted graph distance
- $\phi : X \to \mathbb{R}_{>0}$ is the density of events



Preliminary remarks

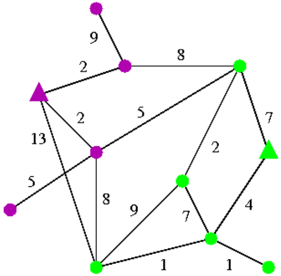
Optimization remarks:

1. for fixed regions, each robot can find one best location in its region, which we call the centroid



2. for fixed robot locations, it is optimal that each robot region contains the nodes which are closer to the robot location.

Voronoi-like split ⟶



The gossip algorithm

Aiming at the least communication needs: robots interact only in pairs

The algorithm has to

1. Ensure that neighbors meet frequently enough:
   $\Rightarrow$ Random Destination & Wait Motion Protocol
2. Update partition when two robots meet:
   $\Rightarrow$ Pairwise Partitioning Rule

J. W. Durham, R. Carli, P. Frasca, and F. Bullo. Discrete partitioning and coverage control for gossiping robots. *IEEE Trans Robotics*, November 2010. Submitted

When two robots interact,

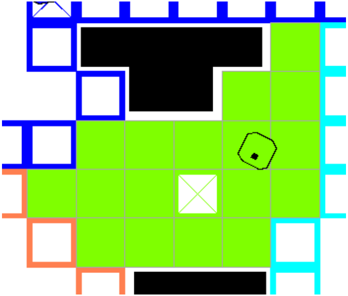
1. they communicate their regions to each other
2. they merge their regions and optimally split the union
3. they move to their new region centroids.

Discrete gossip algorithm I

Random Destination & Wait Motion Protocol

Each robot continuously executes:

1: select sample destination $z_i \in P_i$
2: move to z_i
3: wait at z_i for time $\tau > 0$



(Picture of an occupancy graph)

When robots meet ($=$ z_i and z_j get close enough),
$\Longrightarrow$ robots i and j can communicate

Discrete gossip algorithm II

Pairwise Partitioning Rule

Whenever robots i and j communicate:

$Q \leftarrow P_i \cup P_j$
while (computation time is available) **do**
    $(y_i, y_j) \leftarrow$ sample vertices in Q
    $(Q_i, Q_j) \leftarrow$ Voronoi of Q by (y_i, y_j)
    if $(H_1(Q_i, y_i) + H_1(Q_j, y_j)$ improves) **then**
        $(x_i, x_j) \leftarrow (y_i, y_j)$
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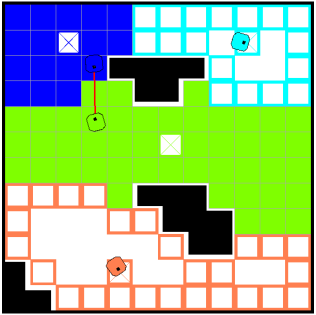
    end if
end while



$$H_1(P_i, x_i) = \sum_{y \in P_i} d_{P_i}(x_i, y)\phi(y)$$

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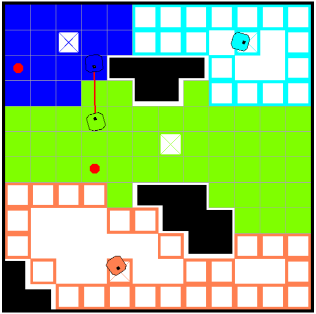

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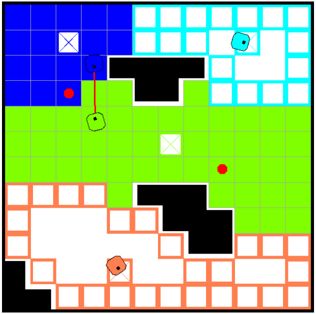

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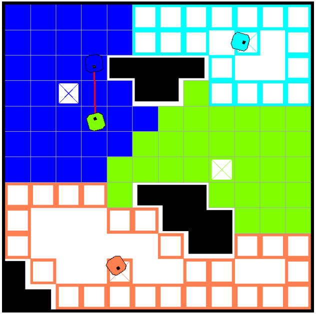

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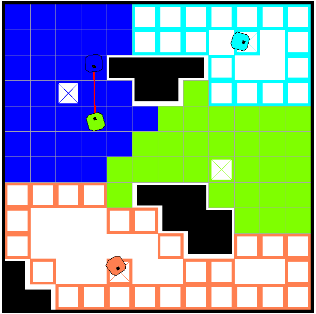


(combinatorial optimization) – interruptible anytime algorithm

Convergence result

Theorem

On any connected graph the discrete gossip coverage algorithm

1. monotonically optimizes the coverage cost;
2. keeps regions connected;
3. a.s. converges in a finite number of steps;
4. the limit configuration (P^*, x^*) is a "locally optimal" configuration: no further improvement of the cost is possible by moving centroids or adjusting boundaries with neighbors

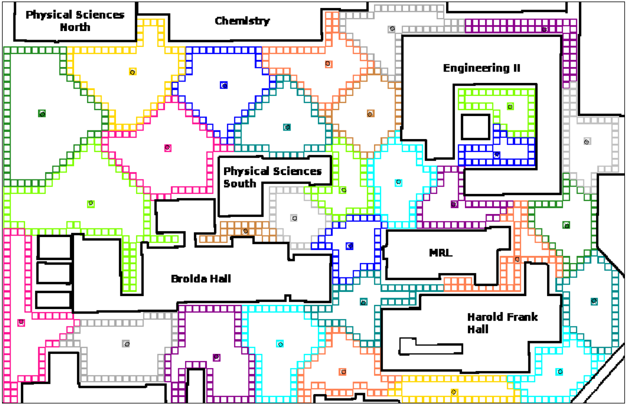
Proof

The coverage cost is Lyapunov function, and we apply a LaSalle invariance principle

Harware-in-the-loop experiment

Player/Stage robot simulation & control system: realistic robot models with integrated wireless network model & obstacle-avoidance planner.
Hardware-in-the-loop experiment: 3 physical and 6 simulated robots

Larger-scale simulation experiment



Player/Stage robot simulation & control system:
Simulation experiment: 30 robots; UCSB campus.

More movies at `http://www.youtube.com/user/control4robot`

Summary of contributions

A network of randomly roaming robots with limited communication capacities can autonomously deploy on a graph

The proposed algorithm has good features:

1. "Anytime" optimization of response time (monotonic decrease)
2. Immediate implementation (inherently discrete)
3. Only pairwise sporadic/unreliable communication required
4. No need for long-range expensive communication
5. Graph environments may represent either networks or complex non-convex regions
    - by a discretization/sampling,
    - by approximating the geodesic distance with a graph distance
6. The limit configuration is close to the optimum

Further and related research

1. Euclidean gossip coverage (convex environment, pairwise communication)

   F. Bullo, R. Carli, and P. Frasca. Gossip coverage control for robotic networks: Dynamical systems on the the space of partitions. *SIAM JCO*, August 2010. Submitted

2. Coverings of graphs (instead of partitions) allow for handling
   - appearing/disappearing robots
   - non reciprocal communication

   J. W. Durham, R. Carli, P. Frasca, and F. Bullo. Dynamic coverage control with asynchronous one-to-base-station communication. In *Proc CDC-ECC*, Orlando, FL, USA, December 2011. To be presented

3. Different cost functions (e.g., for effective patrolling)