Opinion dynamics in social networks

Modelling, analysis, and control

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A survey of models and results

1. Opinion dynamics: to agree or not to agree
   - Deterministic or randomized interactions in a social network
   - Why to agree
     - Opinion diffusion & averaging
   - Why not to agree
     - Antagonistic interactions
     - Bounded confidence
     - Obstinacy and prejudices
2. Opinion control
   - System-theoretic approaches
   - Optimal stubborn placement

Models of opinion dynamics

A population of individuals, or **agents**, A is given

Agents have **opinions** $x_a(t)$

Opinions evolve through **interactions** between agents

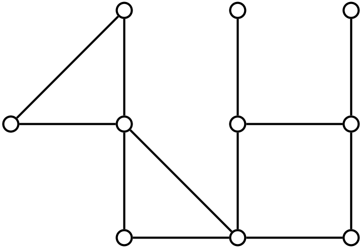
then, we have to model

- the set of allowed interactions: the **social network**
- the **interaction process**: discrete-time, deterministic/randomized
- the **effects of interactions**: positive/negative/no influence

Social network example

A social network is represented by a graph:

- **nodes** are individuals $a \in A$
- **edges** are potential interactions, *i.e.*, pairs $(a, b) \in A \times A$



Diffusive coupling: Deterministic updates

Assumption: interactions bring opinions closer to each other

$\Longrightarrow$ (discrete-time) dynamics: **local averaging**

$$x_a(t+1) = \sum_{b \in A} C_{ab}\, x_b(t)$$

positive couplings $C_{ab} \geq 0$, $\sum_b C_{ab} = 1$, $C_{ab} = 0$ if (a, b) is not an edge

Result:

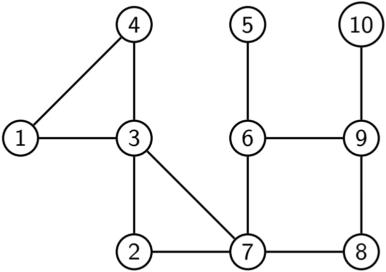
- $x(t)$ converges to a **consensus** on one opinion

J. R. P. French. A formal theory of social power. *Psychological Review*, 63:181–94, 1956

Diffusive coupling: SRW matrix (example)

If we choose equal coupling weights, then the matrix C corresponds to the *simple random walk*

$$C = \begin{bmatrix} 0 & 0 & .5 & .5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & .5 & 0 & 0 & 0 & .5 & 0 & 0 & 0 \\ .25 & .25 & 0 & .25 & 0 & 0 & .25 & 0 & 0 & 0 \\ .5 & 0 & .5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & .33 & 0 & .33 & 0 & .33 & 0 \\ 0 & .25 & .25 & 0 & 0 & .25 & 0 & .25 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & .5 & 0 & .5 & 0 \\ 0 & 0 & 0 & 0 & 0 & .33 & 0 & .33 & 0 & .33 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$



Diffusive coupling: Gossip updates

Synchronous rounds of updates are a poor description of real interaction processes: we can instead use sparse randomized interactions

Gossip approach: at each time t, choose a random edge (a, b) for interaction and update

$$
\begin{aligned}
x_a(t+1) =& \frac{1}{2}x_a(t) + \frac{1}{2}x_b(t) \\
x_b(t+1) =& \frac{1}{2}x_a(t) + \frac{1}{2}x_b(t) \\
x_c(t+1) =& x_c(t) \quad \text{if } c \notin \{a, b\}
\end{aligned}
$$

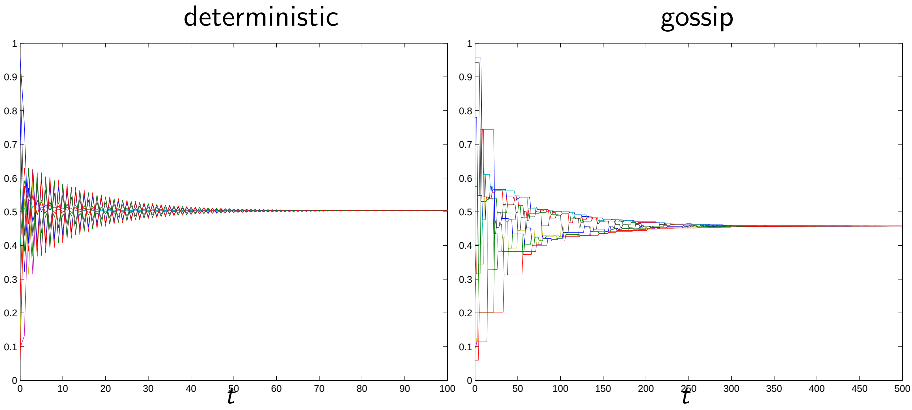
Result:

- $x(t)$ almost surely converges to a **consensus** on one opinion

The convergence analysis is based on the average dynamics

S. Boyd, A. Ghosh, B. Prabhakar, and D. Shah. Randomized gossip algorithms. *IEEE Transactions on Information Theory*, 52(6):2508–2530, 2006

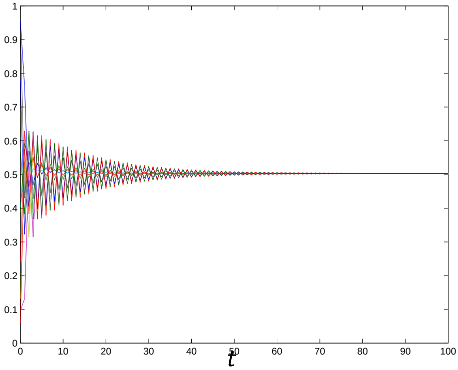
Diffusive coupling: Examples and discussion



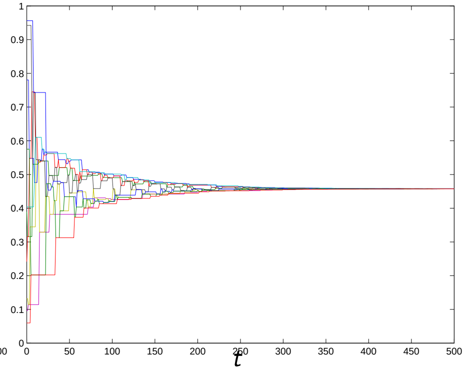
+ easy, well understood
− societies do not exhibit consensus

Diffusive coupling: Examples and discussion

deterministic



gossip



\+ easy, well understood

– societies do not exhibit consensus

We need to model the **reasons for persistent disagreement in societies**

Antagonistic interactions

Assumption: interactions bring opinions either closer to each other, or more apart from each other – depending on friendship or enmity

$$\Longrightarrow \qquad x_a(t+1) = \sum_{b \in A} C_{ab}\, x_b(t)$$

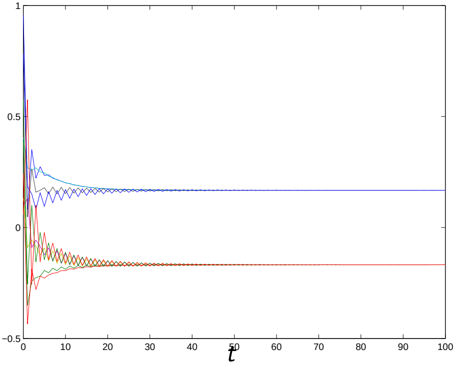
where now C_{ab} may also be negative!

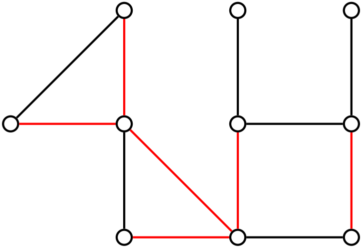
Result:

- $x(t)$ converges to a *polarization* with two opinion parties, if and only if the network is *structurally balanced*

C. Altafini. Consensus problems on networks with antagonistic interactions. *IEEE Transactions on Automatic Control*, 58(4):935–946, 2013

Antagonistic interactions: Examples and discussion

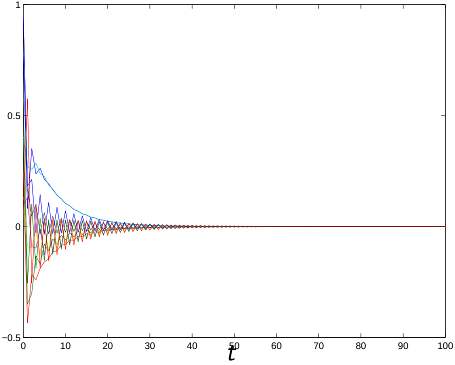


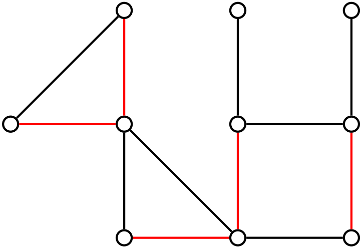


red edges connect enemies

- \+ opinion parties are formed
- − two opinion parties are too few
- − structural balance is a fragile property

Antagonistic interactions: Examples and discussion





red edges connect enemies

+ opinion parties are formed
− two opinion parties are too few
− structural balance is a fragile property

Bounded confidence

Assumption: interactions bring opinions closer to each other, if they are already close enough

Interaction graph depends on confidence threshold R:

$$x_a(t+1) = \frac{1}{|\{b : |x_a(t) - x_b(t)| \leq R\}|} \sum_{b:|x_a(t)-x_b(t)|\leq R} x_b(t)$$

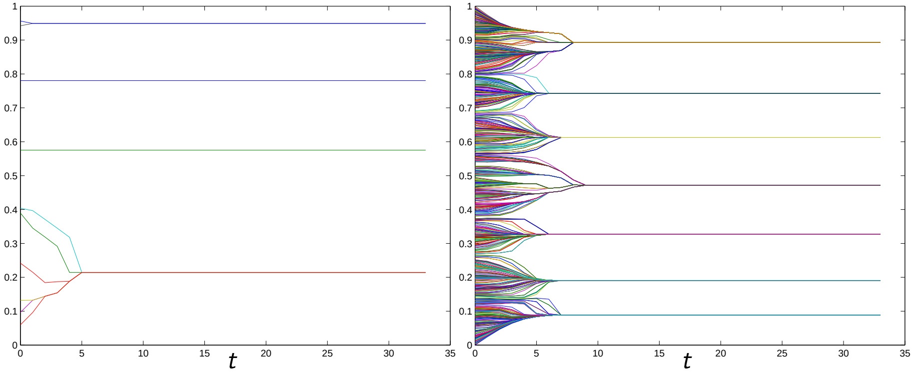
Result:

- $x(t)$ converges to a *clusterization* with **several** opinion parties;
- the number of parties is (roughly) $\propto \frac{1}{2R}$

V. D. Blondel, J. M. Hendrickx, and J. N. Tsitsiklis. On Krause's multi-agent consensus model with state-dependent connectivity. *IEEE Transactions on Automatic Control*, 54(11):2586–2597, 2009

F. Ceragioli and P. Frasca. Continuous and discontinuous opinion dynamics with bounded confidence. *Nonlinear Analysis: Real World Applications*, 13(3):1239–1251, 2012

Bounded confidence: Examples and discussion



- \+ many opinion parties
- − non-linear dynamics → difficult to study
- − opinion parties are disconnected from each other ($|x_1 - x_2| > R$)

Prejudices and stubborn agents

Assumption: interactions bring opinions closer to each other, but the initial opinions are never forgotten

$p \in \mathbb{R}^A$ is a vector of **prejudices**
$w \in [0,1]^A$ is a vector of **obstinacies**

$$x_a(0) = p_a$$
$$x_a(t+1) = (1 - w_a) \sum_{b \in A} C_{ab}\, x_b(t) + w_a p_a$$

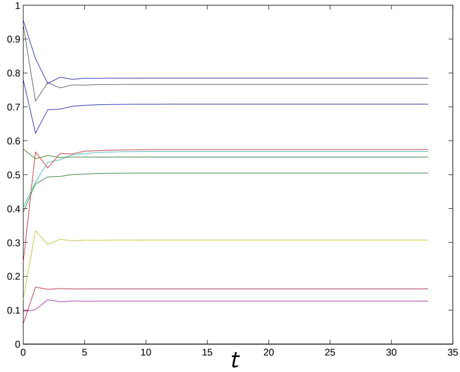
Result:

- $x(t)$ converges to a non-trivial opinion profile

$$x(\infty) = \big(I - (I - \mathrm{diag}(w))C\big)^{-1} \mathrm{diag}(w) p$$

N. E. Friedkin and E. C. Johnsen. Social influence networks and opinion change. In E. J. Lawler and M. W. Macy, editors, *Advances in Group Processes*, volume 16, pages 1–29. JAI Press, 1999

Prejudices: Example and discussion



+ linear dynamics → easy to study
+ complex limit opinion profiles (no consensus)

Steady-state analysis & electrical networks

Special case: $w \in \{0,1\}^A$: agents are either stubborn or open-minded

Result: the final opinions $x(\infty)$ can be described by an **electrical analogy**:

- consider the edges of the graph as **resistors** (with suitable resistance)
- define a potential $W : A \to \mathbb{R}$
  such that $W_s = p_s$ if $w_s = 1$ (s is stubborn)

Then, the opinions equal the induced potential: $x_a(\infty) = W_a \quad \forall a \in A$

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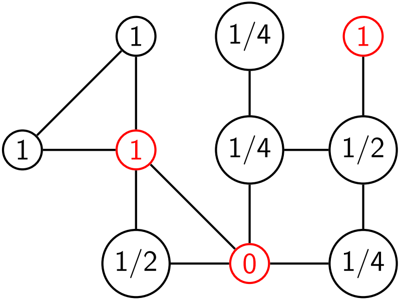


Gossips and prejudices

We can also define sparse random interactions:

for a randomly chosen edge (a, b)

$$
\begin{aligned}
x_a(t+1) =& (1-w_a)\Big(\frac{1}{2}x_a(t) + \frac{1}{2}x_b(t)\Big) + w_a p_a \\
x_b(t+1) =& (1-w_b)\Big(\frac{1}{2}x_b(t) + \frac{1}{2}x_a(t)\Big) + w_b p_b \\
x_c(t+1) =& x_c(t) \quad \text{if } c \notin \{a, b\}
\end{aligned}
$$

Result:

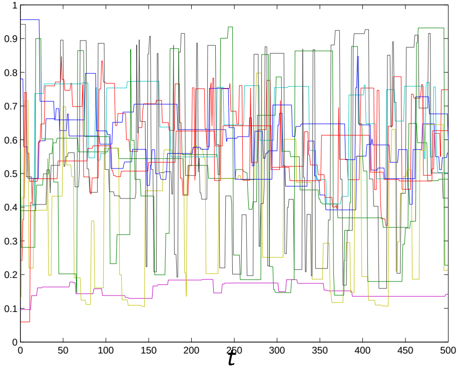
- $x(t)$ persistently oscillates
- oscillations are *ergodic* (around the average dynamics)
- oscillations can be smoothed away by *time-averaging*

D. Acemoglu, G. Como, F. Fagnani, and A. Ozdaglar. Opinion fluctuations and disagreement in social networks. *Mathematics of Operations Research*, 38(1):1–27, 2013

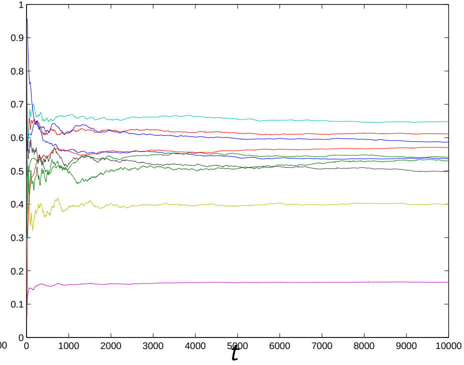
P. Frasca, C. Ravazzi, R. Tempo, and H. Ishii. Gossips and prejudices: Ergodic randomized dynamics in social networks. In *IFAC Workshop on Estimation and Control of Networked Systems*, pages 212–219, Koblenz, Germany, September 2013

Gossips and prejudices: Example

opinions $x(t)$



time-averages



Opinion control (?)

Open problems: dynamics

More complex models of opinion dynamics, including:

- concurrent obstinacy and bounded confidence
- asymmetric asynchronous interactions
- heterogeneous agents
- multidimensional opinions
- discrete or binary opinions

C. Castellano, S. Fortunato, and V. Loreto. Statistical physics of social dynamics. *Reviews of Modern Physics*, 81(2):591–646, 2009

A. Mirtabatabaei and F. Bullo. Opinion dynamics in heterogeneous networks: Convergence conjectures and theorems. *SIAM Journal on Control and Optimization*, 50(5):2763–2785, 2012

Open problems: control

Which control actions are allowable?
Only sparse controls (acting on few nodes/edges)

- inputs in selected nodes
- removal/addition of edges
- removal/addition of nodes

Which are the control goals?

- "classical" control of states to a prescribed vector
- qualitative changes to the limit profile (*e.g.*, merge clusters together)
- quantitative changes to some *observable* (*e.g.*, average opinion, target nodes)

R.D. Braatz. The management of social networks [from the editor]. *IEEE Control Systems Magazine*, 33(2):6–7, 2013

Controlling opinions: System-theoretic approaches

Which nodes can control the network?

General approaches based on system-theoretic notions of *controllability*:

- "driver nodes" are (often) those with low degree

  Y.Y. Liu, J.J.E. Slotine, and A.L. Barabasi. Controllability of complex networks. *Nature*, 473(7346), 2011
- controllability depends on graph topology (via "equitable partitions")

  M. Egerstedt, S. Martini, M. Cao, K. Camlibel, and A. Bicchi. Interacting with networks: How does structure relate to controllability in single-leader, consensus networks? *IEEE Control Systems Magazine*, 32(4):66–73, 2012
- more intuitive results on special graph topologies

  G. Parlangeli and G. Notarstefano. On the reachability and observability of path and cycle graphs. *IEEE Transactions on Automatic Control*, 57(3):743 –748, 2012
- finding the sparsest controller is hard

  A. Olshevsky. Minimal controllability problems. Available at `http://arxiv.org/abs/1304.3071`, 2014
- quantifying controllability

  F. Pasqualetti, S. Zampieri, and F. Bullo. Controllability metrics, limitations and algorithms for complex networks. *IEEE Transactions on Control of Network Systems*, 1(1):40–52, 2014

Optimization approach: stubborn placement

What is the most influential node?

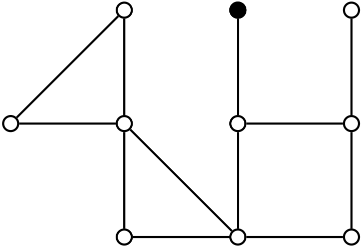
Optimization problem:

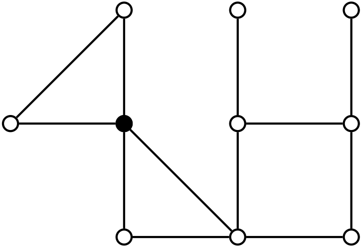
- Given a graph and a set of nodes which are **stubborn with state 0**
- we can choose **one** node to be stubborn **with state 1**
- find for this "controlled stubborn" the location on the graph which **maximizes the average opinion** $\frac{1}{|A|}\sum_a x_a(\infty)$

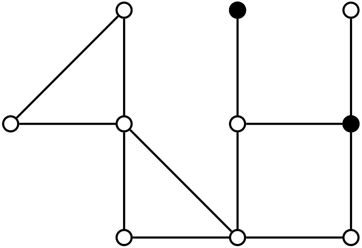
E. Yildiz, A. Ozdaglar, D. Acemoglu, A. Saberi, and A. Scaglione. Binary opinion dynamics with stubborn agents. *ACM Transactions on Economy and Computation*, 1(4):1–30, 2013

Optimal stubborn placement: Examples

Stubborn with state 0 are filled in black. Where would you put the agent with state 1?







Use the electrical analogy!

Optimal stubborn placement: Examples

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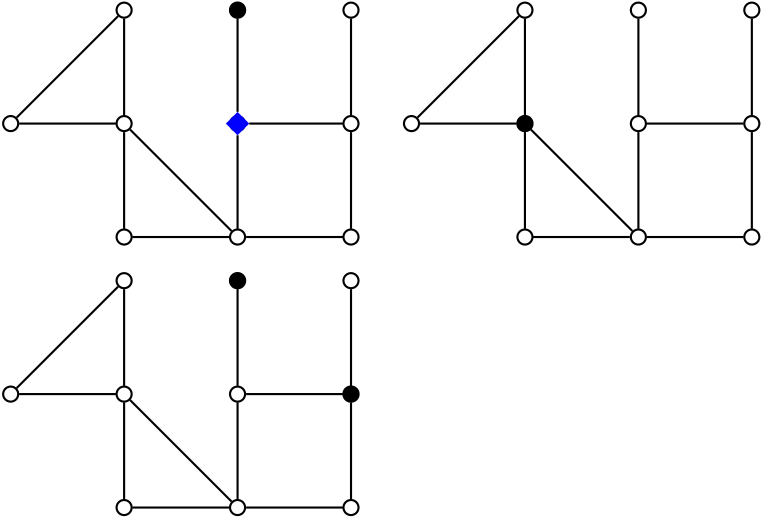

Optimal stubborn placement: Examples

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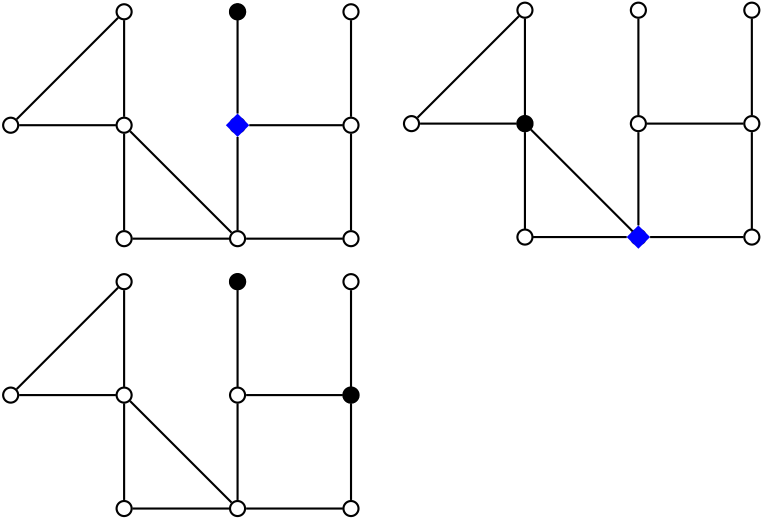

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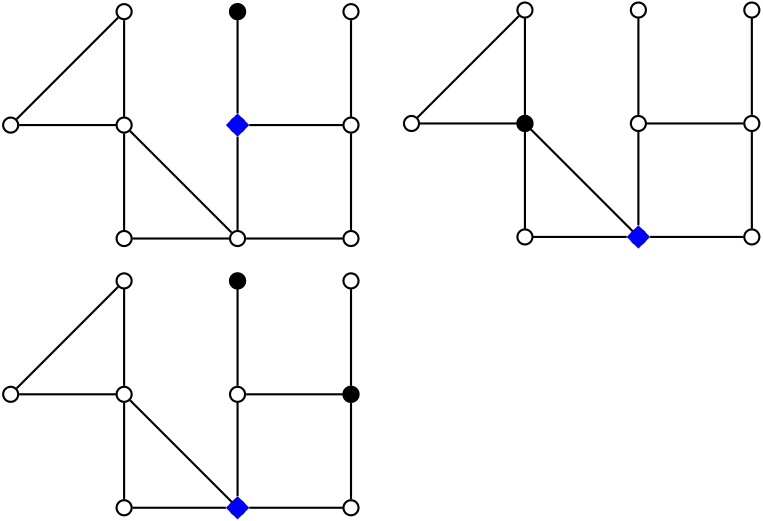

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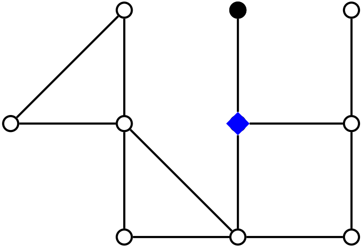

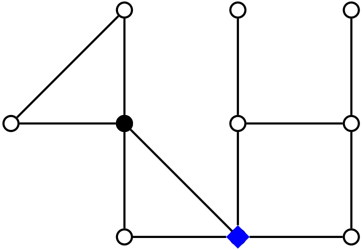

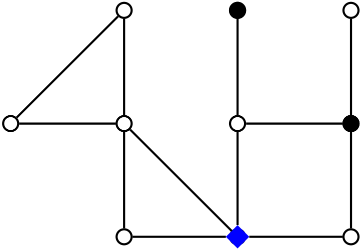

Can we use this intuition?

Algorithmic solution

Yes!
the electrical analogy leads to design an algorithm to solve the stubborn placement problem, which is

- **distributed**: agents can run it online, only communicating with neighbors
- **fast**: runs in $O(\text{diameter})$

To be presented at
- CWTS & UT workshop (next week)
- European Control Conference (in two weeks)
- Symposium on Mathematical Theory of Networks and Systems (next month)

L. Vassio, F. Fagnani, P. Frasca, and A. Ozdaglar. Message passing optimization of harmonic influence centrality. *IEEE Transactions on Control of Network Systems*, 1(1):109–120, 2014