

Self-adaptive PLL for general QAM constellations.

Brossier Jean-Marc *

Amblard Pierre-Olivier †

Geller Benoît ‡

ABSTRACT

This paper deals with the QAM phase estimation problem. After reviewing some classical solutions (Fourth power estimator and Costas loop), we propose several original improvements: self-adaptive first and second order loops. Furthermore, the performance are compared to the performance of the optimal loops.

1 Introduction

The output y_k of an ideal Additive White Gaussian Noise (AWGN) channel is given by $y_k = H a_k + n_k$ where the transmitted signal a_k is complex-valued, n_k is a zero mean, complex and circular Gaussian noise with variance $E |n_k|^2 = \sigma^2$ and H is a complex number, typically the value of the complex gain of a non selective channel. a_k is assumed to be independent of n_k . For the sake of clarity, we assume that $H = 1$ and $E |a_k|^2 = 1$. In order to ease the implementation, the real and imaginary component of a_k usually take discrete values (quantization) so that every transmitted symbol is represented by a point drawn from of a finite “constellation” of points. In the sequel, a_k are i.i.d. random variables drawn from a QAM constellation

* ST Microelectronics. 12, rue Jules Horowitz. BP 23. 38019 Grenoble cedex. France. jean-marc.brossier@st.com

† LIS/ENSIEG. BP 46. 38402 Saint-Martin d’Hères. France. bidou.amblard@lis.inpg.fr

‡ ESRR-LETIRC, Université Paris 12, 110-122, rue Paul Armandot, 94 400 Vitry sur Seine. France. geller@lesir.ens-cachan.fr

This ideal AWGN model does not take into account demodulation errors due to a phase mismatch between the transmitter and the receiver. In this case, the receiver has to proceed a signal such as $y_k = a_k e^{i\xi_k} + n_k$, and the aim of this paper is to provide an efficient adaptive algorithm to estimate the phase ξ_k .

2 Fourth power estimator and Costas loop.

2.1 Fourth power estimator.

Let us assume that the phase error is a constant $\xi_k = \xi$ over the observation duration. Among various available phase estimators, the so-called fourth power estimator [8] appears to offer a good trade-off between performance and complexity. It is based on the equality $\xi = \frac{1}{4} \arg [E (a_k^{*4}) E (y_k^4)]$. For QAM constellations, $E (a_k^{*4})$ is a negative number and the previous equality reduces to $\xi = \frac{1}{4} \arg (E (y_k^4)) + \frac{\pi}{4}$. Noting φ_n^{FP} the fourth power estimation of ξ based on n observed samples $\{y_0, \dots, y_{n-1}\}$, the sampled version of the fourth power estimator is given by:

$$\varphi_n^{\text{FP}} = \frac{1}{4} \arg \left[E (a_k^{*4}) \frac{1}{n} \sum_{k=0}^{n-1} y_k^4 \right] \xrightarrow{n \rightarrow \infty} \xi \bmod \frac{\pi}{2}$$

This estimator is well suited to the estimation of a constant phase error on a finite length observation window and its performance is close to the Cramér-Rao lower bound [9].

2.2 Costas loop.

The expression $\sum_{k=0}^{n-1} |y_k^4 e^{-i4\varphi_n} - 1|^2$ has global minima corresponding to $\sum_{k=0}^{n-1} \Im(y_k^4 e^{-i4\varphi_n}) = 0$, i.e.:

$$\tan(4\varphi_n) = \frac{\sum_{k=0}^{n-1} \Im(y_k^4)}{\sum_{k=0}^{n-1} \Re(y_k^4)} = \tan \left[4 \left(\varphi_n^{\text{FP}} - \frac{\pi}{4} \right) \right]$$

where $\Im(z)$ and $\Re(z)$ stand respectively for the real and imaginary parts of a complex number z .

In other words, φ_n is the fourth power estimator up to a $\pi/4$ phase shift: $\varphi_n^{\text{FP}} - \frac{\pi}{4} = \varphi_n \bmod \frac{\pi}{2}$. If the phase becomes time varying, this estimator can be implemented adaptively using a stochastic gradient scheme to minimize the objective function $J(\varphi) = E|y_k^4 e^{-i4\varphi} - 1|^2$. Noting $\varkappa_k = y_k^4 e^{-i4\varphi_{k-1}}$ and $\varkappa_k^{\Im} = \Im(\varkappa_k)$ its imaginary part, the resulting algorithm is:

$$\varphi_k = \varphi_{k-1} + \gamma_k \varkappa_k^{\Im} \quad (1)$$

This is the so-called Costas loop, a synchronization scheme which was originally devised for 4-PSK synchronization. Actually, it is also able to synchronize general QAM signals. Note that the stepsize choice $\gamma_k = 1/k$ insures that the Costas loop converges to the fourth power solution (with a $\pi/4$ shift ambiguity) when estimating a constant phase.

For a time varying phase, one must choose γ to realize a trade-off between precision and speed. The optimum value depends on the evolution of the true parameter ξ_k . To avoid the need of a strong *a priori* knowledge on phase variations and the associated unknown parameter estimation problem, we now present an algorithm that jointly estimates the phase and the optimum stepsize.

3 Self-adaptive loop for QAM constellations.

3.1 First order loop.

The steady-state Mean Square Error (MSE) depends on γ in the following way:

$$J_{ss}(\gamma) = \lim_{k \rightarrow \infty} E |y_k^4 e^{-i4\varphi_{k-1}} - 1|^2 \quad (2)$$

where $J_{ss}(\gamma)$ is evaluated on the algorithm trajectory $\{\varphi_k\}$. The aim is to minimize (2) under the constraint (1). The stochastic scheme for the minimization of γ is:

$$\gamma_k = \gamma_{k-1} + \alpha G_{k-1} \varkappa_k^{\Im}$$

where α is a small parameter to be chosen and $G_k = \partial\varphi_k/\partial\gamma$ can be updated using:

$$G_k = (1 - 4\gamma_k \varkappa_k^{\Re}) G_{k-1} + \varkappa_k^{\Im}$$

This equation is obtained from (1) by derivation with respect to γ . The self-optimized algorithm for QAM synchronization thus becomes:

$$\begin{aligned} \gamma_k &= [\gamma_{k-1} + \alpha G_{k-1} \varkappa_k^{\Im}]_{\gamma_{\min}}^{\gamma_{\max}} \\ G_k &= (1 - 4\gamma_k \varkappa_k^{\Re}) G_{k-1} + \varkappa_k^{\Im} \\ \varphi_k &= \varphi_{k-1} + \gamma_k \varkappa_k^{\Im} \end{aligned} \quad (3)$$

where

$$[x]_{\gamma_{\min}}^{\gamma_{\max}} = \begin{cases} \gamma_{\max} & \text{if } x > \gamma_{\max} \\ x & \text{if } x \in [\gamma_{\min}, \gamma_{\max}] \\ \gamma_{\min} & \text{if } x < \gamma_{\min} \end{cases}$$

and constraining γ to the interval $[\gamma_{\min}, \gamma_{\max}]$ prevents the estimation procedure to diverge [6]. For the evolution model:

$$\xi_k = \xi_{k-1} + w_k$$

where w_k is a zero-mean Gaussian, i.i.d. noise with variance σ_w^2 , the theoretical optimum value of γ for algorithm (3) can be calculated using the tools of [3], and writes:

$$\gamma_* \approx \frac{\sigma_w \sqrt{2}}{\sqrt{E|a_k|^8 - E(a_k)^8}}$$

in the limit of small noises.

In the sequel, our simulations will be performed using normalized 16-QAM. In this case, the optimum stepsize is given by:

$$\gamma_* \approx 1.47\sigma_w$$

Figure (1) shows a simulation of the behavior of algorithm (3) and a comparison with the theoretical optimum stepsize. The top panel shows

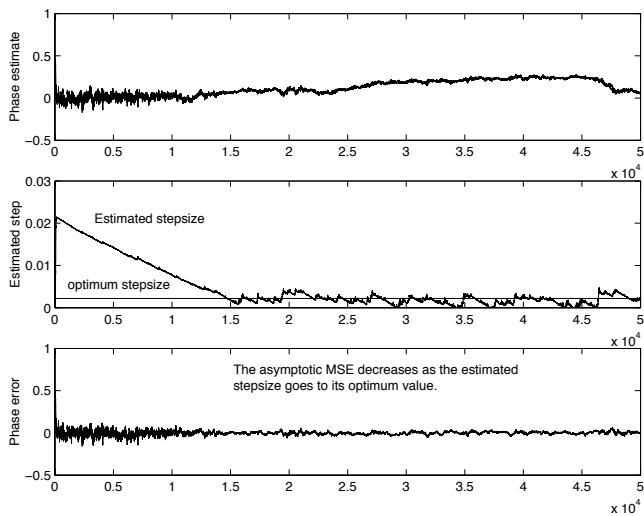


Figure 1: First order loop (3), 16-QAM, $\sigma = 0.01$, $\sigma_w = 0.001$, $\alpha = 10^{-5}$

the phase and its estimate, whereas the phase error appears in the bottom panel. The second panel depicts the evolution of the estimated stepsize and the optimal value γ_* . After convergence, the estimated stepsize fluctuates around the optimal value.

The main advantage of this kind of algorithm lies in its robustness versus α : a wide range of α yields to almost the same asymptotic MSE [2]. This robustness is illustrated in figure (2) where the estimated γ stepsize is plotted for three values of α . The effect of α on the estimated γ is the classical speed/precision trade-off of any adaptive algorithm. The self-adaptive algorithm is able to reach the performance of the best first order Costas loop. However, if some additional information about the phase variation is available, this should be introduced in the algorithm in order to improve the performance. As an example, we now consider the second order loop.

3.2 Second order loop.

Let us consider the realistic example of a phase evolution given by a random walk with a non zero drift ε : $\xi_k = \xi_{k-1} + \varepsilon + w_k$. This models a small frequency offset between the transmitter and the receiver or a small Doppler shift. This *a priori* information can be taken into account

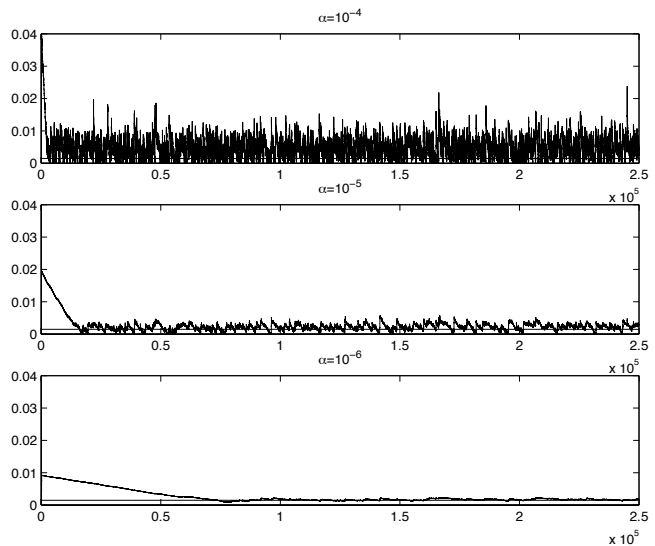


Figure 2: First order loop (3), 16-QAM, $\sigma = 0.01$, $\sigma_w = 0.001$, for $\alpha = 10^{-4}, 10^{-5}, 10^{-6}$.

using a second order loop [5]:

$$\varphi_k = \varphi_{k-1} + \left(\gamma_k^{(1)} + \frac{\gamma_k^{(2)}}{1 - z^{-1}} \right) \mathcal{X}_k^{\mathfrak{S}}$$

This is a symbolic notation corresponding to:

$$\begin{aligned} \varphi_k &= \varphi_{k-1} + s_{k-1} + \gamma_k^{(1)} \mathcal{X}_k^{\mathfrak{S}} \\ s_k &= s_{k-1} + \gamma_k^{(2)} \mathcal{X}_k^{\mathfrak{S}} \end{aligned}$$

where s_k is an estimation of the drift ε . In this algorithm, the filter $\left[\gamma_k^{(1)} + \frac{\gamma_k^{(2)}}{1 - z^{-1}} \right]$ is a direct representation of the state model of the unknown phase. A generalization of the automatic stepsize tuning algorithm (3) to this second order loop was presented in [5] for BPSK signals; the application to QAM synchronization is straightforward.

However, in some cases, it may be awkward to optimize both stepsizes. As an example, for a Brownian phase evolution with constant drift, it can be shown that the asymptotic optimum value $\gamma_{\infty}^{(2)}$ is zero [1]: the smaller $\gamma^{(2)}$, the better the asymptotic MSE. Thus, a quasi-optimum use of this algorithm can be achieved setting $\gamma^{(2)}$ to a small constant value¹ and using the

¹If the phase slope were exactly a constant, it would be possible to use a decreasing sequence for $\gamma_k^{(2)}$. A constant $\gamma^{(2)}$ allows small slope variations.

previous procedure for an on-line optimization of $\gamma_k^{(1)}$. Noting $D_k = \partial s_k / \partial \gamma$, the self-adaptive algorithm writes:

$$\begin{aligned} \gamma_k^{(1)} &= \left[\gamma_{k-1}^{(1)} + \alpha G_{k-1} \mathcal{X}_k^{\mathfrak{S}} \right]_{\gamma_{\min}}^{\gamma_{\max}} \\ G_k &= (1 - 4\gamma_k \mathcal{X}_k^{\mathfrak{R}}) G_{k-1} + D_{k-1} + \mathcal{X}_k^{\mathfrak{S}} \\ D_k &= D_{k-1} - \gamma^{(2)} G_{k-1} \mathcal{X}_k^{\mathfrak{R}} \\ \varphi_k &= \varphi_{k-1} + \left(\gamma_k^{(1)} + \frac{\gamma^{(2)}}{1-z^{-1}} \right) \mathcal{X}_k^{\mathfrak{S}} \end{aligned} \quad (4)$$

This algorithm is illustrated in figure (3) where we see the phase error (top), the estimated drift of the Brownian motion (middle) and the estimated stepsize γ^1 . This figure shows the good behavior of the self-adaptive second order loop. However, the theoretical performance remains to be studied.

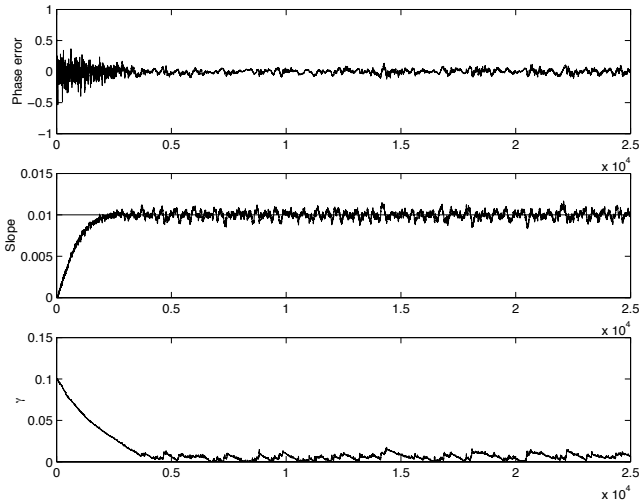


Figure 3: Second order loop (4), 16-QAM, $\sigma = 0.01$, $\sigma_w = 0.001$, $\varepsilon = 0.01$ for $\alpha = 10^{-4}$, $\gamma^{(2)} = 10^{-4}$.

4 Conclusion

The Costas loop, originally designed for PSK modulated signals, can also be used to synchronize general QAM constellations. But, in that case, the fluctuation of the algorithm are essentially due to a self-noise arising from an imperfect suppression of the modulation. However, we have shown that a self-adaptation of the loop allows to achieve good performance.

References

- [1] P.O. Amblard, J.M. Brossier, E. Moisan. *Phase tracking: what do we gain from optimality? Particle filtering versus phase-locked loops*. **Submitted to Signal Processing, 2001**.
- [2] J.M. Brossier, C. Adnet, G. Jourdain. *Self-Optimized Digital Phase-Locked Loops*. International Symposium on Signals, Systems and Electronics. **ISSSE-URSI, 1992**, Paris
- [3] A. Benveniste, M. Métivier, P. Priouret. *Adaptive algorithms and stochastic approximations*. Springer Verlag, 1990.
- [4] B. Geller, V. Capellano, J.M. Brossier, A. Essebbar, G. Jourdain. *Equalizer for Video Rate Transmission in Multipath Underwater Communications*. **IEEE, Journal of Oceanic Engineering**, vol. 21, N.2, pp. 150-155, April 1996
- [5] J.M. Brossier. *Signal et communication numérique. Égalisation et synchronisation*. 256 pages. **Hermès, 1997**.
- [6] H.J. Kushner, J. Yang. *Analysis of Adaptive Step-Size SA Algorithms for Parameter Tracking*. *IEEE transaction on Automatic Control*, vol. 40, N.8, pp. 1403-1410, August 1995.
- [7] J.L. Lacoume, P.O. Amblard, P. Comon. *Statistiques d'ordre supérieur pour le traitement du signal*. Masson, 1997.
- [8] M. Moeneclaey, G. de Jonghe. *ML oriented NDA carrier synchronization for general rotationally symmetric signal constellations*. **IEEE trans. Communication**, vol. 42, pp. 2531-2533, Aug. 1994.
- [9] E. Serpedin, P. Ciblat, G. Giannakis, P. Loubaton. *Performance analysis of blind carrier estimators for general QAM constellations*. **IEEE trans. Signal Processing**, vol. 49, N.8, pp. 1818-1824, August 2001.