OFDM synchronisation based on the phase rotation of sub-carriers

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Abstract - This paper proposes a method to simultaneously track symbol timing, carrier phase and frequency offset errors for an OFDM (Orthogonal Frequency Division Multiplex) communications system. Each of these three offsets rotates the OFDM symbol constellations in a conspicuous way. Their characters are illustrated and it is explained how can the receiver DFT (Discrete Fourier Transform) outputs be used for estimation. The method presented is based on the rotational impact of offsets on the individual sub-carriers. The technique employs the joint-ESPRIT (Estimation of Signal Parameters via Rotational Invariance Techniques) algorithm and can be used for data-aided and blind estimation, as well as for the combination of these two. Assuming a prior initial acquisition, it uses the information contained in the *phase* rotation between the correct and the received sub-symbols *after* the DFT.

I Introduction

The OFDM modulation scheme has matured into a well-established technology for wireless and wired broadband data delivery. It is used in the digital video broadcasting (DVB), digital audio broadcasting (DAB), IEEE 802.11a, HyperLAN II and xDSL standards. The main advantage of OFDM is allowing the transmission over highly frequency-selective channels at a low receiver implementation cost.

Complex and expensive equalizers needed in a single-carrier systems are dramatically simplified or even obsolete when differential modulation schemes are used. Nevertheless, OFDM is highly vulnerable to synchronisation errors. Differential detection is a widely used remedy in today radio communications. The drawbacks of the differential detection are the irreducible error probability obtained due to the fading, and the degraded performance compared to the ideal coherent detection. Coherent detection for OFDM, on which this paper concentrates, requires an accurate carrier phase reference. This can be done with or without the help of pilot symbols inserted periodically in the data stream. These symbols increase both the bandwidth and the required signal to noise ratio (SNR) for a given bit error probability, so their number should be kept to a minimum.

Recently, there were some works on tracking (see [1],[2]). In this paper, we investigate data-aided and blind tracking of symbol timing, carrier phase and frequency, by applying the joint-ESPRIT signal processing technique on the angle differences of individual sub-symbols from their correct position.

Let the complex envelope of OFDM signal after IDFT and up-conversion be given by

$$\begin{aligned} s(t) &= \sum_{k=-\infty}^{+\infty} s_k(t) \quad (1) \\ &= \sum_{k=-\infty}^{+\infty} \left[\sum_{i=-N/2}^{N/2-1} x_{i,k} e^{j\left[2\pi(t-kT)\frac{i}{T_{FFT}}+2\pi f_c t\right]} \right] \end{aligned}$$

where t is the absolute time, T duration of one OFDM symbol, N the number of carriers, $x_{i,k}$ the sub-symbol that modulates the i-th carrier of the k-th symbol, T_{FFT} the duration of the useful part of the OFDM symbol and f_c the carrier frequency at the transmitter.

After passing the signal through a multipath channel (assumed to be quasi-constant), down-conversion and sampling, the outputs of the DFT are expressed as (c.f. references [2],[3])

$$\widehat{x}_{i,k} = H_i \frac{\sin\left(\pi\delta f\, T_{FFT}\right)}{N \sin\left(\frac{\pi\delta f\, T_{FFT}}{N}\right)} x_{i,k} e^{j\Psi_{i,k}} + I_{i,k} + n_{i,k} \quad (2)$$

where H_i is the channel response, $\delta f = f_c - f_c^{receiver}$ is the frequency offset, $I_{i,k}$ describes the intercarrier interference as a function of δf, $n_{i,k}$ is noise, $\Psi_{i,k}$ characterises the rotation of $x_{i,k}$ and is given by

$$\Psi_{i,k} = 2\pi\left[i\frac{\tau}{N} + k\delta f\; T\right] + \left[2\pi\left(\tau - \frac{1}{2}\right)\frac{\delta f\; T_{FFT}}{N} + \phi_o\right] \quad (3)$$

in which τ is the symbol-timing offset normalised to $\frac{T_{FFT}}{N}$ and ϕ_o is the carrier phase offset.

From (2) and (3) it can be seen that offsets reduce the subcarrier amplitude, introduce inter-carrier interference (ICI) and rotate the sub-symbols. For instance,

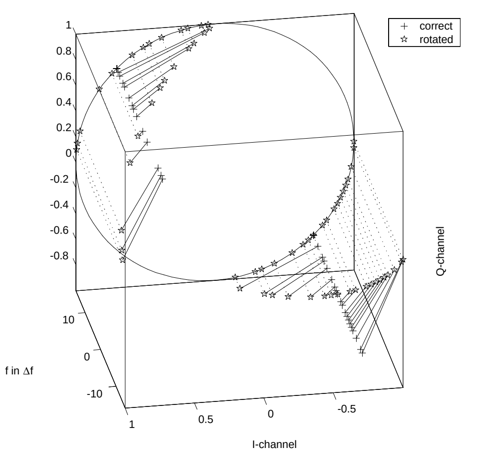

Figure 1: Rotation caused by symbol timing offset only

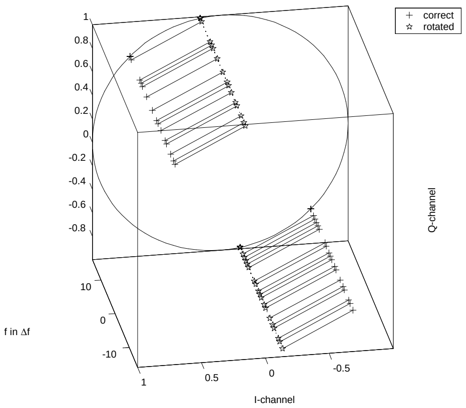

Figure 2: Rotation caused by carrier phase offset only

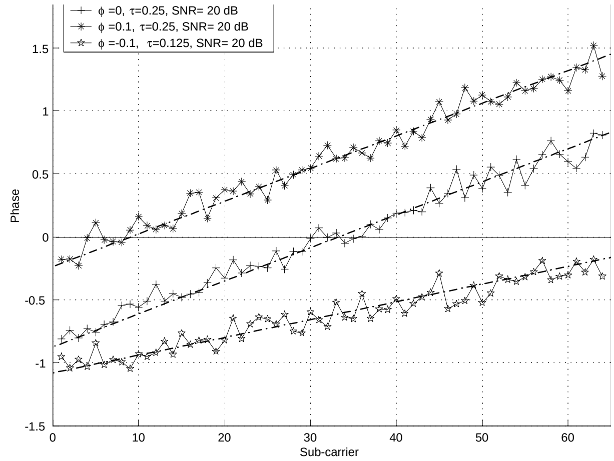

Figure 3: Angle differences for Data-aided estimation

δf has to be less than 2% of intercarrier distance, otherwise the effective SNR drops off by 30 dB [1].

Assuming perfect initial synchronisation, the initial offsets and channel response are known and compensated for, which enables the approximation $|I_{i,k}| \ll |x_{i,k}|$ (valid for very small offsets). This assumption is crucial for the method to work, because it is designed to keep the system in synchronisation by tracking the offsets under the bounds $\tau < 1$ and $\delta f < 0.02$. Then, offsets induce only rotations of the OFDM sub-symbols, displacing them from their correct constellation positions.

For the sake of simplicity of analysis, it will be assumed that the sampling frequency is correct and the only channel impairment is AWGN (making $H_i = 1$ and $\phi_o = 0$). In the multipath environment, some carriers can fade to a level where they become useless. These sub-channels are detected in the acquisition phase and should be discarded from calculations.

The technique proposed can be used both for systems with pilots or for blind estimation. It tracks the symbol timing, phase and frequency offsets by observing the angle differences $\widehat{\Psi}_{i,k}$ given by (4) between the received ($\widehat{x}_{i,k}$) and the correct ($x_{i,k}$) phases of sub-carriers (which in case of data-aided estimation are known; in case of blind estimation, correct means "estimate of correct") and applying the joint-ESPRIT [4] algorithm on them.

$$\widehat{\Psi}_{i,k} = \measuredangle(\widehat{x}_{i,k}/x_{i,k}) \tag{4}$$

Figures 1 and 2 show why is it important to know offsets for the coherent detection. The rotation is shown on an OFDM symbol having 32 sub-carriers modulated with BPSK. The frequency spacing between adjacent sub-carriers is $\Delta f = 1/T_{FFT}$. Figure 1 illustrates the rotation of the OFDM sub-carriers caused by symbol timing offset only ($\tau = 0.25$, $\phi_o = 0$, $\delta f = 0$), with a progressive rotation of sub-carriers proportional to the distance from the centre frequency. Figure 2 shows the effect of the carrier phase offset only ($\tau = 0$, $\phi_o = 0.1$, $\delta f = 0$), which rotates all sub-carriers with the same angle.

Figures 3 and 4 illustrate how differences in rotation between received and (estimated) correct sub-carrier phase, $\widehat{\Psi}_{i,k}$, can be used for estimation. It can be seen that there is an opportunity to fit a line, which can be used for tracking purposes. Its angle is proportional to the symbol timing offset and its elevation to the carrier phase offset.

Figure 3 shows examples of data-aided estimation. Figure 4 is a result of blind estimation, where only knowledge of of the modulation type (QPSK) was used, and therefore more prone to errors in estimation. It is very important to observe the slip of $\pi/2$. It should be also noted that if offsets are large, the trendline becomes saw-like, due to the rotations of sub-symbols larger than 2π. A practicable strategy to counteract this will be presented in the following.

The change of the angle difference $\widehat{\Psi}_{i,k}$ caused by the frequency offset is the same for all sub-carriers within one OFDM symbol, making it impossible to estimate δf with one trendline, but it introduces changes of phase between consecutive OFDM symbols, which can be detected if information over more symbols is used. Therefore, the idea shown in Figures 3 and 4 can be extended to include tracking of frequency offset as

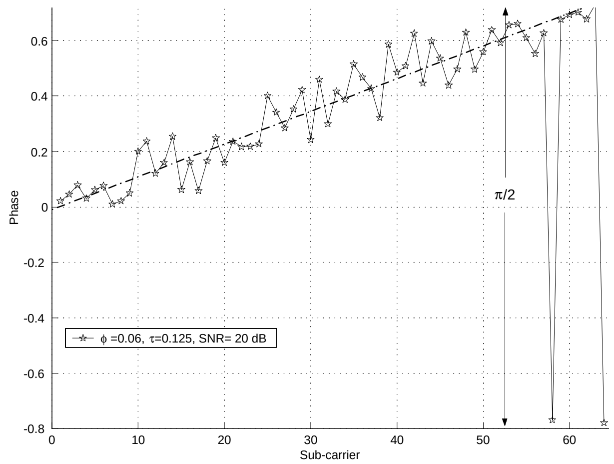

Figure 4: Angle differences for Blind estimation

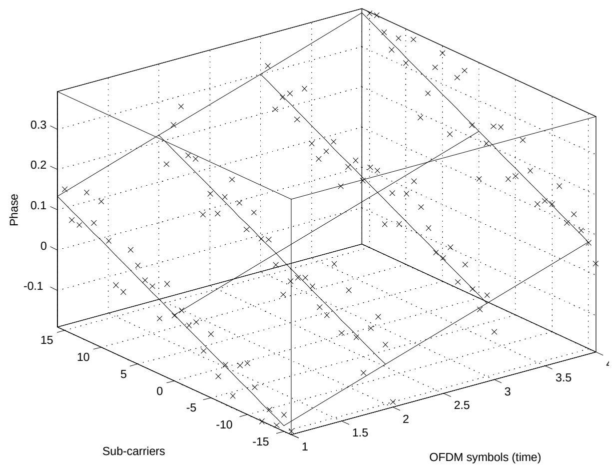

Figure 5: Carrier frequency offset causes evolution of the angle differences over 4 OFDM symbols (all three offsets and noise are present)

well. Figure 5 illustrates the evolution of the trendline over 4 OFDM symbols, with all three offsets and noise present.

II The proposed method

$I_{i,k}$ in Eq. (2) can be considered as noise, and when δf is less than 2% of Δf, independent from $x_{i,k}$, making it possible to recover $\{x_{i,k}, \tau, \delta f, \phi_o\}$. A simple rotated projection scheme on the nearest QPSK gives us the estimated $\widehat{x}_{i,k}$, which allows to estimate the offset effect $\widehat{o}_{i,k}$ defined by $\widehat{x}_{i,k} = \widehat{o}_{i,k} \cdot x_{i,k} + (noise)$.

If a single OFDM symbol is observed (one vector of information), the frequency offset is constant over the whole symbol duration, and Eq. (2) and (3) lead to $\widehat{o}_i = \alpha \exp\left(2\pi i \frac{\tau}{N} + \Psi_o\right) + n(i)$, which simplifies the estimation problem to the retrieval of a single harmonic (complex exponential) in the presence of noise. In this case, the maximum likelihood estimation gives very good results, whose the general formula in the presence of Gaussian noise is:

$$\arg \min_{\alpha,\tau,\Psi_o \in R} \|\mathbf{o}_k - \alpha \exp(j\Psi_o) \cdot \mathbf{a}_\tau\|^2 \tag{5}$$

where $\mathbf{o}_k = [o_{-N/2,k}, \ldots, o_{N/2-1,k}]^\intercal$, and $\mathbf{a}_\tau = \left[\exp\left(2\pi j \frac{\tau}{N} \cdot \frac{N}{2}\right), \ldots, \exp\left(-2\pi j \frac{\tau}{N} \cdot \left(\frac{N}{2} - 1\right)\right)\right]^\intercal$. Many algorithms exist in the literature to solve this problem, one of the most efficient being ESPRIT, which uses the rotational invariance between the consecutive samples of several series. Initially, it has been created to cope with several harmonics. A simplified version of it[1] is used here, for the simple case of one harmonic.

The simplification proposed here consists of estimating the rotational shift of the sub-carrier by looking at consecutive sub-carriers:

$$\hat{\tau} = \frac{N}{2\pi} \cdot (\mathbf{o}_k)^{\intercal *}_{1:N-1} \cdot (\mathbf{o}_k)_{2:N} \tag{6}$$

with $\mathbf{o}^{\intercal *}$ meaning the conjugate transpose of $\mathbf{o}$, and the subscript $\mathbf{o}_{2:N}$ denotes the vector $\mathbf{o}$ limited to the elements 2 to N. In fact, this inner vector product is a sum of all the phase differences between two sub-carriers, which is referred later to as a phase-estimator I. Unfortunately, if a symbol is wrongly detected, it adds an extra angular value before the addition. To compensate this effect, we propose an update: before summation, a test is performed to check the consistency of the differential phase, the inconsistent (too large) differences are discarded, and the average phase is calculated over the remaining ones. This is referred to as phase-estimator II. In order to have an alternative estimator to ESPRIT, $\widehat{\Psi}_{i,k}$ was fitted with a linear function; the slope of the fitting approximation is proportional to the time offset, which allows us to recover it. This will be referred to as a phase-estimator III.

In fact, these three methods are made to estimate the phase shift of a complex exponential, and therefore can be applied to any similar problem.

To get a frequency offset estimate, several OFDM symbols are needed. Eq. (5) transforms to (bold letters denote matrices or vectors):

$$\arg \min_{\alpha,\tau,\delta f,\Psi_o \in R} \|\mathbf{O} - \alpha \exp(j\Psi_o) \cdot \mathbf{a}_\tau \cdot \mathbf{b}_{\delta f}\|^2_{FRO}$$

where $\mathbf{b}_{\delta f} = [1, \exp(\delta f T), \ldots, \exp((k-1)\delta f T)]$ and $\mathbf{O} = [\mathbf{o}_1, \ldots, \mathbf{o}_k]$. This is clearly visible in Figure 5, where the slope of the plane in the directions of both axes. This kind of problem can be solved with joint diagonalisation (or joint-ESPRIT) as in [4]: from the Singular Value Decomposition (SVD) of the matrix $\mathbf{O}$, the largest singular values are kept together with their associated sub-spaces (row and column). These two sub-spaces have the rotational property used in ESPRIT, and are related to each other. However, assuming a single user OFDM signal, things are far simpler, since there is only one harmonic of interest and we deal with the strongest component only. The SVD will give us a row that will be proportional to $\mathbf{b}_{\delta f}$, and a column that is proportional to $\mathbf{a}_\tau$. In our scheme, the

[1] To get an in-depth view on ESPRIT, refer to [5].

SVD serves only to average the noise and to regularize the estimate of $\mathbf{a}_\tau$ and $\mathbf{b}_{\delta f}$: $\hat{\mathbf{a}}_\tau$ and $\hat{\mathbf{b}}_{\delta f}$.

We applied the three processing methods described in the previous sections on both vectors, which resulted in three estimates of the timing and the frequency offsets.

During simulations, it appeared that for $\tau = p/4$, $p \in \{1, 2, 3\}$, the method breaks down. To bring the timing offset within $(-0.25 < \tau < 0.25)$, we propose to use the first OFDM symbol for the coarse timing estimation. Then, we compensate the timing effect in the OFDM symbols using this result (as in [6] for frequency estimation), and apply the SVD method to the resulting time-frequency "carpet" for the fine estimation. We will reference to it as the cascaded estimation method.

III Results

The dstimators for one OFDM symbol in the blind and in the data-aided case were studied; as for K consecutive symbols for the one-step, and later for the cascaded method. Due to lack of space, only the best dstimator in each case is presdnted. The performance of these estimators was studied for 128 carriers OFDM with QPSK modulation, AWGN channel, with 1000 independent realizations per induced frequency offset $\delta f = 0.005 \times k\,[\Delta f]$, $k \in [0, 4]$, and timing offset $\tau = k/16\,[samples]$, $k \in [0, 15]$.

A single OFDM symbol can only estimate the timing offset τ. While the three phase estimators were of equal quality for data-aided case, only phase estimator II could endure the blind case. Due to their nature, the phase estimator I and III could not cope with too large timing deviations and produced estimates which were the remainder after division by 0.25. Figure 6 presents the absolute bias and the standard deviation of the timing offset, the various lines correspond to SNR values between 0 dB and 30 dB, by increments of 5 dB (the low SNR being on the top of the figure), the crosses correspond to data-aided, while the circles correspond to blind estimation.

This experiment clearly shows that there is a threshold at 15 dB where the estimates become consistent. For this value, the standard deviation falls under 10% of the correct value, which will be acceptable for the second stage of algorithm (explained in the following). It is interesting to notice that the blind and the data-aided estimation present the same performance for high SNR and small timing offset, while for larger timing offset the blind method abruptly performs worse. The explanation comes from the way the offset vector is estimated - with low timing offset the data can not spread over a phase difference of $\pi/2$, thus the blind data estimates are consistent. With larger spread, the blind estimation is not consistent anymore, and the offset $\mathbf{o}_k$ gets a jump of $\pi/2$ in phase, what destroys the phase-estimators I and III, while the phase-estimator II overcomes this jump, due to its construction, and delivers the right value. Due to the discarded points, this estimator has then a higher standard deviation.

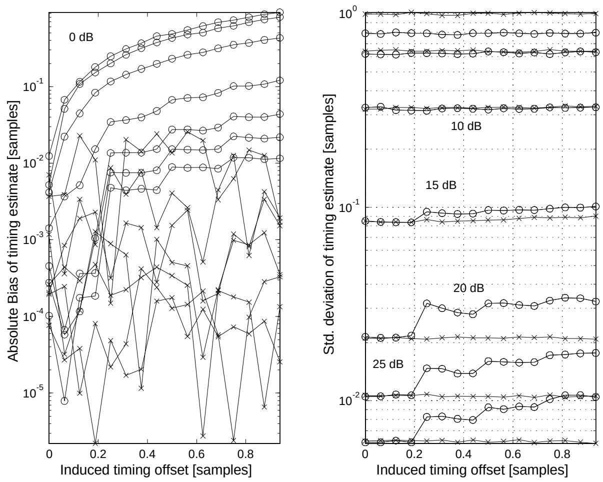

Figure 6: Data-aided (+) and Blind (o) timing estimation for a single OFDM symbol using phase-estimator II. $0 \leq SNR[dB] \leq 30$

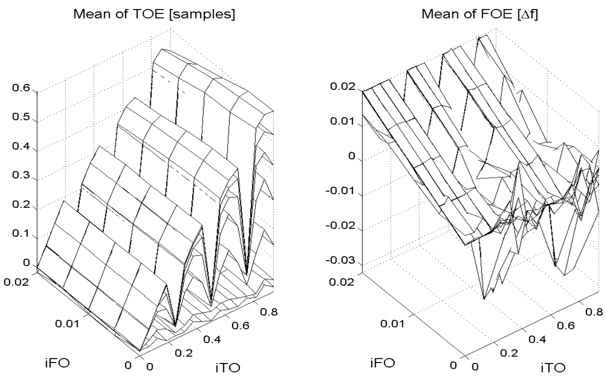

Figure 7: Timing offset estimate TOE (left) and the frequency offset estiamte FOE (right) in function of the induced timing (iTO) and the frequency offset (iFO) for various SNR. The SNR ranges form 0 dB (lowest surface) to 30 dB (upper surface), by step of 5 dB.

Several OFDM symbols allow us to observe the influence of the frequency offset (Figure 5). A batch of simulations were run on the same parameters as in the precedent paragraph, but with 2 to 8 symbols. Figure 7 presents a typical result with the best phase estimators for 6 symbols and shows that the method breaks down invariably for time offsets of 0.25, 0.5 and 0.75 sampling interval. These holes make the method usable only for small timing offsets and low SNR. These holes have the same origin as the sudden fall in timing performance for a single OFDM symbol.

Figure 7 shows that this simple scheme responded quite well for low timing offset and SNR $\geq$ 15 dB. We can get a good coarse timing estimate from the first OFDM symbol alone. A cascaded algorithm in which the timing offset is coarsely estimated and compensated for in the later OFDM symbols, allows the simple method to be used in the domain where its behavior is

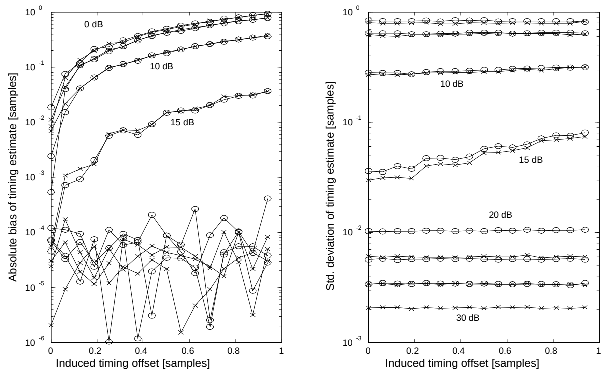

Figure 8: Absolute bias (left) and standard deviation (right) of the timing offset of cascaded estimation for 3 (o) and 8 (+) symbols.

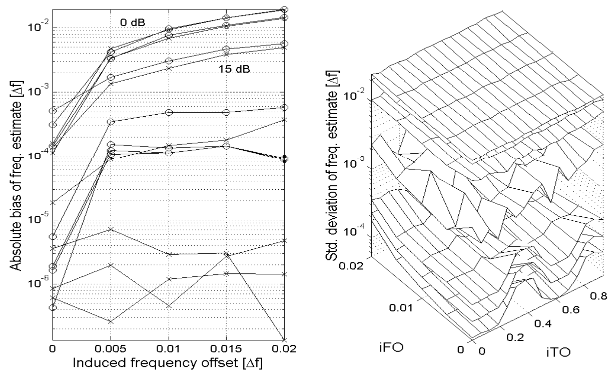

Figure 9: Left: Absolute bias for frequency offset of the cascaded estimation for 3 (o) and 8 (+) symbols. Right: the std deviation for the frequency offset with 8 symbols.

correct.

Figures 8 and 9 present the performance of the cascaded estimation for 3 and 8 symbols. It appears that for timing estimation the phase-estimation method I was the most efficient at low SNR, and the phase-estimation method III at high SNR. Due to its simplicity, we chose to present the method I for timing estimation (Figure 8). The phase-estimation method II appears to have the best performance for frequency estimation, and thus is the one presented (Figure 9).

Figure 8 demonstrates that the 15 dB barrier for good performance is also present here and can not be bypasssed. It shows as well that the number of symbols does not reduce the bias. This effect probably has its origin in the fact that the timing offset has been refined in the algorithm. Otherwise for high SNR ($SNR \geq 15\,dB$) the standard deviation with 8 OFDM symbols has the constant $\sqrt{8/3}$-ratio over the deviation for 3 symbols, since the noise was averaged over more symbols.

Figure 9 shows that more symbols do not improve the mean of the estimator for low SNR, but as soon as the SNR>15, the performance of 8 symbols is getting 20 dB improvement in quality. The standard deviation shows interesting dependency on timing and frequency.

IV Conclusions

A method for tracking small offsets in OFDM systems, based on phase differences and on the ESPRIT algorithm, was presented. The whole system enables precise correcting of fractional offset synchronisation errors for SNR values higher than 15 dB.

The analysis presented here did not include pulse shaping, but the method can also be used in the systems with pulse shaping. However, its precise effects need to be further analyzed The principle can be easily extended to the multipath environment, if the channel remains quasi-static between two training sequences. Also, it can be used for estimation in systems that incorporate pilot symbols, such as HyperLAN, DAB or DVB.

Acknowledgments

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