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ANALYTICAL BLIND IDENTIFICATION OF A SISO COMMUNICATION CHANNEL

Olivier GRELLIER and Pierre COMON

Lab. I3S, Algorithmes-Euclide-B, 2000 route des Lucioles
BP 121, F-06903 Sophia-Antipolis cedex, France
`grellier@i3s.unice.fr` `comon@unice.fr`

ABSTRACT

In this paper, a novel analytical blind identification algorithm is presented, based on the non-circular second-order statistics of the output. It is shown that the channel taps need to satisfy a polynomial system of degree 2, and that identification amounts to solving the system. We describe the algorithm able to solve this particular system entirely analytically. Computer results demonstrate its efficiency.

1. INTRODUCTION

Blind identification methods depend on the characteristics of the input sources. For example, it is known that a system can only be identified up to an all-pass filter when its input is Gaussian circular. Consequently, a particular attention has been paid to the non-Gaussian input cases. In those situations the phase information can be accessed using high-order statistics of the observations, and in the SISO case the system is identified up to a scalar factor only. This has been studied in numerous papers among which one can cite the works of Shalvi-Weinstein [5], Tugnait [7].

An interesting class of non-Gaussian signals is the discrete one, which appears in wireless communications. The discrete character has been used by few authors such as Li [3] or Yellin and Porat [8], who were the first interested in an algebraic solution. The studied signals have also non zero cyclostationary statistics, which allows identification using second-order statistics only [4] [6].

The novelty of our contribution is two-fold. First, non-circular second-order moments are used. Second, an algebraic solution to a class of polynomial systems, constructed from a block of data, is introduced. Our approach is described in the case of MSK modulations, approximating well the digital modulation utilized in the GSM standard. In addition, block methods are well matched to burst-mode communication systems.

2. MODEL, NOTATION, AND ASSSUMPTIONS

Assume a finite sequence of input samples $x(m)$ is fed into a Finite Impulse Response (FIR) linear system of length M. Denote $y(n)$ the corresponding output sequence of length N, satisfying:

$$y(n) = \sum_{m=0}^{M-1} h(m)\, x(n-m) + w(n) \stackrel{\text{def}}{=} \mathbf{x}(n;M)^{\mathsf{T}}\mathbf{h} + w(n)$$

Multidimensional variables are stored in column vectors and denoted by boldface letters; for instance, $\mathbf{x}(n;M) = [x(n), \ldots x(n-M+1)]^{\mathsf{T}}$, by construction.

The input sequence is assumed to follow a discrete distribution, stemming from BPSK, MSK, or QPSK digital modulations, and the channel $\mathbf{h}$ is supposed time-invariant during the observation.

The key statistical property used in this paper is that discrete signals are non-circular at given orders. More precisely, for BPSK modulated signals :

$$\begin{aligned} \mathrm{E}\{x(n)x(n-\ell)|x(0)\} &= x(0)^2\delta(\ell) \\ \mathrm{E}\{x(n)x(n-\ell)^*\} &= \delta(\ell) \end{aligned}$$

for MSK signals :

$$\begin{aligned} \mathrm{E}\{x(n)x(n-\ell)|x(0)\} &= (-1)^n x(0)^2\delta(\ell) \\ \mathrm{E}\{x(n)x(n-\ell)^*|x(0)\} &= \delta(\ell) \end{aligned}$$

and for QPSK modulated signals:

$$\begin{aligned} \mathrm{E}\{Re\,[x(n)]\,Re\,[x(n-\ell)]\,|x(0)\} &= Re\,[x(0)]^2\,\delta(\ell) \\ \mathrm{E}\{Im\,[x(n)]\,Im\,[x(n-\ell)]\,|x(0)\} &= Im\,[x(0)]^2\,\delta(\ell) \\ \mathrm{E}\{x(n)x(n-k)x(n-\ell)x(n-m)|x(0)\} &= x(0)^4\delta(k+\ell+m) \\ \mathrm{E}\{x(n)x(n-\ell)^*\} &= \delta(\ell), \end{aligned}$$

and where $\delta(\ell) \stackrel{\text{def}}{=} 1$ if $\ell = 0$ and $\delta(\ell) = 0$ elsewhere. Note the conditional expectation, exhibiting cyclostationarity in the non-circular moment of MSK inputs.

Based on these properties, it is possible to derive a set of polynomial equations that the channel must satisfy. In the MSK case, we obtain :

$$E\left[y(n)y(n-\ell)|x(0)\right] = x(0)^2 \sum_{m=0}^{M-1} (-1)^m\, h(m)h(m+\ell)$$

In the BPSK case, we have :

$$E\left[y(n)y(n-\ell)|x(0)\right] = x(0)^2 \sum_{m=0}^{M-1} h(m)h(m+\ell)$$

lastly in the QPSK case :

$$\begin{aligned} &E\left[y(n)y(n-\ell_1)y(n-\ell_2)y(n-\ell_3)|x(0)\right] = \\ &\quad x(0)^4 \textstyle\sum_{m=0}^{M-1} h(m)h(m+\ell_1)h(m+\ell_2)h(m+\ell_3) \end{aligned}$$

3. SOLVING THE POLYNOMIAL SYSTEM

3.1. Example

In order to introduce in easy words our contribution, let's give a simple example. Let the input signal be MSK and the channel be real of length $M=3$. Then non circular statistics yield:

$$\left\{ \begin{array}{rcl} h(0)^2 - h(1)^2 + h(2)^2 & = & f_1 \\ h(0)h(1) - h(1)h(2) & = & f_2 \\ h(0)h(2) & = & f_3 \end{array} \right. \quad (1)$$

whereas circular ones yield:

$$\left\{ \begin{array}{rcl} h(0)^2 + h(1)^2 + h(2)^2 & = & g_1 \\ h(0)h(1) + h(1)h(2) & = & g_2 \\ h(0)h(2) & = & f_3 \end{array} \right.$$

where f_i and g_i are given (they depend on statistics of observations y). The grouping of those equations allows to obtain:

$$\left\{ \begin{array}{rcl} h(0)^2 + h(2)^2 & = & (f_1+g_1)/2 \\ h(0)h(1) & = & (f_2+g_2)/2 \\ h(0)h(2) & = & f_3 \end{array} \right.$$

Using the first and third equations, one gets:

$$\begin{aligned} (h(0)-ih(2))^2 &= h(0)^2 + h(2)^2 - 2ih(0)h(2) \\ &= (f_1+g_1)/2 - 2if_3 \end{aligned}$$

This equation eventually allows to calculate $h(0)$ and $h(2)$ up to a sign, and then $h(1)$.

Thus we have been able to identify a real channel by using the **non-circular second order** statistics together with **circular second order** ones. The general algorithm that is described in this section computes the finite set of solutions of the polynomial system built on the non-circular second-order statistics only. In the next section, the choice of the channel estimation is discussed.

3.2. Preliminaries

Consider the ring $\mathcal{R} = \mathbb{C}[\boldsymbol{\xi}]$ of polynomials in variables $\boldsymbol{\xi} \stackrel{\text{def}}{=} [h(0), h(1), \ldots h(M-1)]$ with coefficients in the complex field $\mathbb{C}$; the dual space of $\mathcal{R}$ is the set of linear forms from $\mathcal{R}$ to $\mathbb{C}$, denoted $\widehat{\mathcal{R}}$. The evaluation of a polynomial p at a point $\zeta \in \mathbb{C}^M$, denoted by $\mathbf{1}_\zeta : p \mapsto p(\zeta)$, is the linear form which we are most interested in.

Given a polynomial $a \in \mathcal{A}$, define the multiplication operator by a as the mapping $\mathcal{M}_a$ that associates q with aq :

$$\begin{array}{rcl} \mathcal{M}_a : \mathcal{A} & \rightarrow & \mathcal{A} \\ q & \mapsto & q\,a \end{array} \quad (2)$$

The transposed operator, $\mathcal{M}_a^{\mathsf{T}}$, is by definition the mapping from $\mathcal{A}$ onto itself so that $\langle q, M_a^{\mathsf{T}}\Lambda\rangle = \langle M_a q, \Lambda\rangle = \langle aq, \Lambda\rangle$, $\forall \Lambda \in \widehat{\mathcal{A}}$, $\forall q \in \mathcal{R}$ so that $\mathcal{M}_a^{\mathsf{T}}(\Lambda)(q) = \Lambda(q\,a)$.

3.3. Lemmas

Let $\mathcal{P}$ be the subset $\mathcal{R}$ of polynomials $\{f_1, \ldots, f_M\}$ of degree D and belonging to $\mathcal{R}$. Bézout's theorem [2, p.227] states that such a system

$$\mathcal{P} \ : \ \{f_m(\boldsymbol{\xi}) = 0,\ 1 \le m \le M\}. \quad (3)$$

where $\boldsymbol{\xi} \stackrel{\text{def}}{=} [\xi(0), \xi(2), \ldots \xi(M-1)]$, has an infinity of solutions, or a number of solutions smaller or equal to D^M.

When the system has a finite number of solutions, one conventional way to compute them is to reduce the problem to an eigenvector computation, as shown by the following lemma.

Lemma 3.1 *Linear forms $\mathbf{1}_{\boldsymbol{\xi}} : p \mapsto p(\boldsymbol{\xi})$, where $\boldsymbol{\xi}$ is any solution of $\mathcal{P}$, are the eigenvectors of all matrices $(M_a^{\mathsf{T}})_{a\in\mathcal{A}}$. The corresponding eigenvalues are $a(\boldsymbol{\xi})$.*

For a proof see [1].

Therefore, the computation of the multiplication matrix M_a appears as a key step in the proposed algorithm, since its eigen vectors allows to find the solutions of $\mathcal{P}$. Indeed, if we take for a basis of $\mathcal{A}$, $\mathcal{B} = \{1, h(0), h(1), \cdots, h(0)h(1), \cdots\}$, the entries of the eigenvectors are equal to $\{1, \xi(0), \xi(1), \cdots, \xi(0)\xi(1), \cdots\}$, where $\boldsymbol{\xi}$ stands for any possible solutions of $\mathcal{P}$.

3.4. Computation of matrix M_a

Matrix M_a can be directly computed from the Macaulay matrix associated with polynomials

$\{f_1, \ldots, f_M\}$ [1]. These matrices are the extension of the so-called Sylvester matrices to multivariate polynomials.

However, if we take into account the relationships between the monomials introduced in the polynomial system $\mathcal{P}$, there exists a much simpler procedure to compute M_a.

In order to simplify the discussion, we will use the following example. Suppose that a channel of length $M = 3$ is excited by a MSK input. System $\mathcal{P}$ is then equal to that in (1). In this case, a generic basis that can solve this kind of system is given by :

$$\begin{aligned}\mathcal{B}_3 \;=\; \{&1, h(0), h(1), h(2), h(0)h(1),\\ &h(0)h(2), h(1)h(2), h(0)h(1)h(2)\}\end{aligned}$$

However, this basis cannot be used in our problem unless we first apply a change in the variables. Thus, the computation of the multiplication matrix can be split into 4 steps.

First step : Change in variables

Suppose we use the following change in variables : $\boldsymbol{\xi} = T\mathbf{h}$ then the system in $\boldsymbol{\xi}$ becomes :

$$A\mathcal{H} = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix} \tag{4}$$

where the entries of A are functions of the entries of T, and :

$$\begin{aligned}\mathcal{H} \;=\; [&1, h(0), h(1), h(2), h(0)h(1),\\ &h(0)h(2), h(1)h(2), h(0)^2, h(1)^2, h(2)]\end{aligned}$$

. The matrix T must be chosen so that monomials $h(0)^2$, $h(1)^2$ and $h(2)^2$ can be directly expressed as functions of the basis (see the next step).

Second step : Expression of the second degree monomials

Suppose we want to find the matrix associated with the multiplication by $h(0)$. The monomials that we have to express are :

Monomials of the basis		Monomials to be expressed
1		$h(0)$
$h(0)$		$h(0)^2$
$h(1)$		$h(0)h(1)$
$h(2)$		$h(0)h(2)$
$h(0)h(1)$	$\xmapsto{\times h(0)}$	$h(0)^2h(1)$
$h(0)h(2)$		$h(0)^2h(2)$
$h(1)h(2)$		$h(0)h(1)h(2)$
$h(0)h(1)h(2)$		$h(0)^2h(1)h(2)$

Among these monomials, some are already in the basis. In our example, the monomials $h(0)$, $h(0)h(1)$, $h(0)h(2)$ et $h(0)h(1)h(2)$ are in the basis. The others monomials, $h(0)^2$, $h(0)^2h(1)$, $h(0)^2h(2)$ and $h(0)^2h(1)h(2)$, have to be expressed using the polynomial system.

According to equation (4), monomials $h(0)^2$, $h(1)^2$ and $h(2)^2$ can be expressed directly as a function of 1, $h(0)$, $h(1)$, $h(2)$, $h(0)h(1)$, $h(0)h(2)$ and $h(1)h(2)$, provided that T is chosen correctly. In other words, monomials $h(0)^2$, $h(1)^2$ and $h(2)^2$ can be expressed directly as a function of the basis using equation (4).

$$\begin{bmatrix} h(0)^2 \\ h(1)^2 \\ h(2)^2 \end{bmatrix} = B \begin{bmatrix} 1 \\ h(0) \\ h(1) \\ h(2) \\ h(0)h(1) \\ h(0)h(2) \\ h(1)h(2) \end{bmatrix} \tag{5}$$

Therefore, the monomial $h(0)^2$ is now expressed.

Third step : expression of the third degree monomials

We now care about monomials $h(0)^2h(1)$ and $h(0)^2h(2)$. These monomials can be expressed using the expression of the monomial $h(0)^2$ in equation (5). In this equation, if we multiply $h(0)^2$ by $h(1)$, monomials $h(0)^2h(1)$ appears in the left hand side and monomials $h(1)$, $h(0)h(1)$, $h(1)^2$, $h(2)h(1)$, $h(0)h(1)^2$, $h(0)h(1)h(2)$ and $h(1)^2h(2)$ appear in the right hand side. Among these monomials, one can distinguish those that are in the basis like $h(1)$, $h(0)h(1)$, $h(2)h(1)$ and $h(0)h(1)h(2)$, those that are already expressed in the basis like $h(1)^2$, and those that are unknown like $h(1)^2h(2)$ and $h(0)h(1)^2$. However, these unknown monomials are of the same kind as monomials $h(0)^2h(1)$ and $h(0)^2h(2)$, and one can show that the expression of monomials $h(0)^2h(1)$, $h(0)^2h(2)$, $h(1)^2h(0)$, $h(1)^2h(2)$, $h(2)^2h(0)$ and $h(2)^2h(1)$ using (5) leads to :

$$\begin{bmatrix} h(0)^2h(1) \\ h(0)^2h(2) \\ h(1)^2h(0) \\ h(1)^2h(2) \\ h(2)^2h(0) \\ h(2)^2h(1) \end{bmatrix} = C \begin{bmatrix} 1 \\ h(0) \\ h(1) \\ h(2) \\ h(0)h(1) \\ h(0)h(2) \\ h(1)h(2) \\ h(0)h(1)h(2) \end{bmatrix} \tag{6}$$

Fourth step : expression of the fourth degree monomials

Using the same method as before, one can express the monomials like $h(0)^2h(1)h(2)$ using equation (6).

We now have expressed all the monomials, and the multiplication matrix can then be built.

3.5. Choosing the channel estimate

Once the multiplication matrix is computed, the possible solutions are given by its eigenvectors. Then, the last step consists of choosing the solution that best matches the true channel.

The method used in this paper consists of comparing the circular-statistics of the observation with that given by each estimate. The solution that best matches is selected.

3.6. Identifiability

Lemma 3.2 *Suppose we look for a FIR channel H of length M from given second-order circular statistics, then the number of solutions is infinite because of a scalar phase indeterminacy. If the phase indeterminacy is fixed, the number of solutions is finite and equal to 2^{M-1} if H is causal and equal to 2^{2M-2} if H is not necessarily causal.*

Theorem 3.3 *Suppose we look for a FIR channel H of length M from given second-order circular and non-circular statistics, then the number of solutions is finite and equal to :*

- *2 if H has no real root,*
- *2^{Q+1} if H has Q real roots and is causal,*
- *2^{2Q+1} if H has Q real roots and is not necessarily causal.*

When the source is MSK, the channel can be identified up to a sign.

Proof. The z-transform of the circular covariance $c(n)$ of the output $y(n)$ is equal to $C(z) = H(z)H^*(1/z^*)$. This shows that if $H(z)$ is causal, it can be determined up to 2 indeterminacies. First, $H(z)$ can only be determined up to a multiplicative constant phase factor. Second, if $H(z)$ is transformed into $H(z)\Phi(z)$ where $\Phi(z)$ verifies $\Phi(z)\Phi^*(1/z^*) = 1$, $C(z)$ remains the same. It is well known that $\Phi(z)$ is an all pass filter, *i.e.* $\Phi(z)$ is of the form :

$$\Phi(z) = \prod_{i=1}^{Q} \frac{1 - a_i^* z^{-1}}{a_i - z^{-1}}$$

Since $H(z)$ must be FIR and $\Phi(z)$ is not FIR, $H(z)\Phi(z)$ is FIR only if each pole of $\Phi(z)$ is associated with a root of $H(z)$. As a consequence, there is a finite number of all-pass filters such that $H(z)\Phi(z)$ is FIR. Therefore, if the phase indeterminacy is fixed, there are 2^{M-1} possible FIR filters that correspond to $C(z)$. This gives the first part of lemma 3.2.

If $H(z)$ is non-causal, a third indeterminacy exists. $C(z)$ has $M-1$ pairs of roots $(b_i, 1/b_i^*)$ (if b_i is a root then $1/b_i^*$ is also a root). Thus any $H(z)$ built with $M-1$ roots, where each root h_j is equal to b_j or $1/b_j^*$, gives th same $C(z)$. The number of these channels is equal to 2^{M-1}. Thus, when $H(z)$ is non causal, the number of solutions is equal to 2^{2M-2}. This gives the second part of lemma 3.2.

Suppose now that we also use the non-circular covariance $\overline{c}(n)$ of $y(n)$. Its z-transform is equal to $\overline{C}(z) = H(z)H(1/z)$, if the input is white. This new constraint shows that the phase indeterminacy is reduced to a sign indeterminacy, and that the all-pass filter $\Phi(z)$ must have real poles. As a consequence, if $H(z)$ has no real roots, the all-pass filter $\Phi(z)$ must be equal to ± 1. In this case, there are only 2 solutions. If $H(z)$ has Q real roots, one can use the results of lemma 3.2. The number of solutions is then equal to 2^{Q+1} if H is causal, because the solutions are given up to a sign; if H is not necessarily causal the number of solutions is equal to 2^{2Q+1}.

When the input source is MSK, $\overline{C}(z) = H(z)H(-1/z)$, which no all-pass filter but $\Phi(z) = \pm 1$ satisfies. Therefore, the channel can be identified up to a sign if the input source is MSK. □

Corollary 3.4 *Suppose the circular and non-circular moments of the output $y(n)$ are known and that the channel $H(z)$ has no real roots. Then, there exists $2^{M-1} C_{2M-2}^{M-1}$ possible solutions matching circular moments, each of them known up to a constant phase indeterminacy. They can be computed directly from the covariance $C(z)$. For each solution, the phase indeterminacy can be fixed with the non-circular moments, and the channel estimate is the channel that best matches the non-circular moments.* This Corollary gives a new identification method.

4. COMPUTER RESULTS

The first tests have been run on a random FIR channel ($M = 5$). At each run the channel is a realization of a Clarke filter in the typical urban mode and is excited by a MSK input. The performances are presented as a function of the SNR and of the length N of the observation block, and averaged over 500 runs.

Figure 1 shows the average Bit Error Rate obtained at the output of a Viterbi algorithm that uses our channel estimate. The solid, dashed and dashdotted lines correspond to block lengths $N = 200$, $N = 500$ and

$N = 1000$ respectively. These performances are compared to the average BER obtained with the true channel (dotted line).

For high SNRs and $N = 200$ or $N = 500$ the results show the effects of statistic estimation errors. These effects disappear for $N = 1000$, where the performances exhibit a loss of $2dB$ compared to the true channel results.

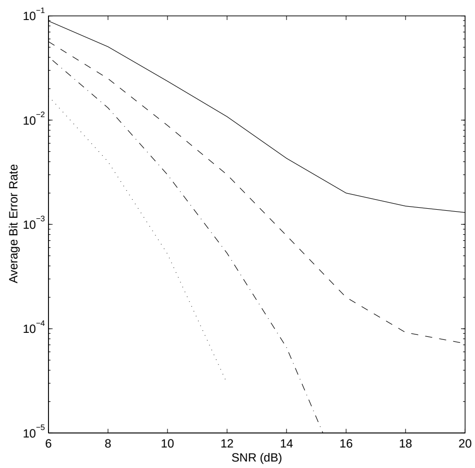

Figure 1: Average Bit Error Rate at the output of a Viterbi algorithm using our channel estimate.

A second test has been run on a random FIR channel of length $M = 3$. Figure 2 shows the average Bit Error Rate obtained at the output of a Viterbi algorithm that uses our channel estimate when $\hat{M} = 3$ (solid line) and $\hat{M} = 5$ (dashdotted line). These two performances are compared to the one obtained when the true channel is used (dotted line). Hence, this test illustrates the loss of performance encountered in presence of over-determination.

5. CONCLUDING REMARKS

In this paper, we presented a new blind identification method based on the non-circular moments of the observation. These moments yield a polynomial system that can be solved by computing the eigenvectors of a multiplication matrix. Identifiabilty results are presented when a FIR channel is searched for, from both circular and non-circular moments. Finally, computer results show that the behaviour of our algorithm depends on the estimation of the moments.

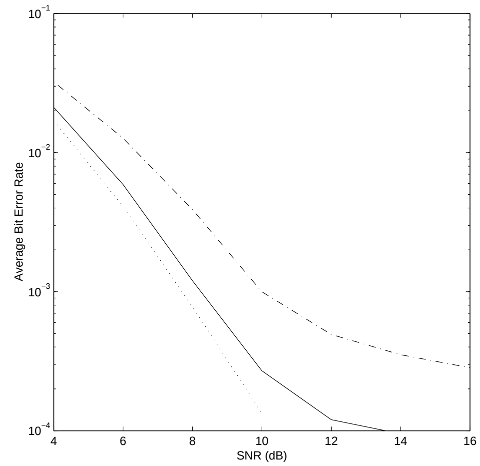

Figure 2: verage Bit Error Rate at the output of a Viterbi algorithm using our channel estimate in presence of over-determination

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